SOME EXPERIENCE IN R-CURVE TECHNIQUE

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ABSTRACT

Base of the R-curve method is the registration of stable crack growth of pre-cracked specimen with rising static load. This can be done by the compliance method or potential drop method. Both methods were used on CCT-specimen of different materials, thickness, width and crack length at beginning of the tests. Comparison of the R-curves determined with both methods shows the properties, advantages and disadvantages and also the difficulties in test equipment of both methods.

KEYWORDS

R-curve method; plane stress; elastic-plastic fracture mechanics; measurement of stable crack growth; compliance method; potential drop method; calibration curves; equations for data fitting.

INTRODUCTION

If the thickness $s$ of components is less than a certain thickness $s_{\text{min}}$ plane strain condition changes to a mixed stress condition or plane stress. Plane stress condition, however, is accompanied by large plastic zones in the ligament of precracked components and also stable crack growth occurring with rising static load.

This elastic-plastic behaviour cannot be described by a widely constant fracture toughness value like $K_c$ for linear-elastic fracture behaviour. Therefore $K_{IC}$ has to be replaced by a value $K_{IC}$ which depends on

- specimen thickness $s$
- specimen width
- crack length at beginning of static loading.

A reasonable method for determining $K_{IC}$ at plane stress is the resistance (R)-curve method which counts for larger plastic zones and stable crack growth. Krafft and coworkers (1961) postulated that for a given material and thickness there is a unique relationship between the amount a crack grows and the applied stress intensity factor - the R-curve.
The theoretical background of the R-curve method has been explained for many times e.g. by Sullivan (1974) and Schwalbe (1976). Main point of this paper, therefore, is to show the experimental work necessary for R-curve method and the experience of IABG in this field.

**TEST METHOD**

The R-curve method described in ASTM STP 527 (1973) and by ASTM E 561-76T (1976) is extending the K-concept to the range of stable crack growth using an effective crack length

\[ l_{\text{eff}} = l_0 + \Delta l + r_y \]

which is calculated from crack length \( l_0 \) at start of static loading, stable crack growth \( \Delta l \) and plastic zone \( r_y \).

Using \( l_{\text{eff}} \) analog to linear-elastic fracture mechanics an effective stress concentration factor

\[ K_{\text{eff}} = \sqrt{\pi \cdot l_{\text{eff}} \cdot \gamma} \]

is calculated.

Stable crack extension \( \Delta l \) is a function of crack growth resistance \( R \) respectively \( K_R \)

\[ R, K_R = f (\Delta l) \]

To determine \( \Delta l \) in the experiment two methods

- potential drop method
- compliance method

are recommended.

**TEST EQUIPMENT**

Both methods were used at IABG on CCT-specimen of different materials (Titanium and Aluminium alloys), different specimen thickness and width and with different ratios of \( l_c/W \). For comparison of results determined from the two methods most of the specimen were supplied with both test equipments, Fig. 1

![Fig. 1. Schematic test equipment for determining crack extension l](image-url)
Test preparation mainly was done due to the recommendations of ASTM E 561-76T and the German working group "Bruchverhalten dünnwandiger Strukturen".

To avoid buckling of the thin sheet anti-buckling guides were mounted to the specimen. The test for determining R-curve was done strain-controlled. The load ratio was limited to $G < 1 \, \text{N/mm}^2$ taking into account stiffness of the specimen and the expected maximum load. Using this load ratio time-dependent strain effects will be eliminated. The test equipment is shown in Fig. 2.

Fig. 2. Test equipment of 500 mm-wide CCT panel specimen

COMPLIANCE MEASUREMENTS

For load $F$ versus crack opening displacement an extensometer (traverse beam with strain gauges) is used. Load and displacement are plotted by a $X$-$Y$-plotter.

A first relationship for compliance $v/F$ versus crack length $l$ normalized to Young's modulus $E$ and specimen thickness $s$ was given by Irwin (1960). This relationship later on was modified to CCT-specimen by Liebowitz and Eftis (1972).

\[ 
\frac{E s}{2F} = \frac{R_{\text{eff}}}{W} \left( \frac{2s \text{ arcosh} \left( \frac{\sin \left( \frac{\pi y}{R_{\text{eff}}/W} \right)}{\cos \left( \frac{\pi y}{R_{\text{eff}}/W} \right)} \right)}{1 + \frac{1}{\sqrt{\frac{R_{\text{eff}}/W}{\sinh \left( \frac{R_{\text{eff}}/W}{s} \right)}}} \right)^2 \]

Valid for $0.2 \leq 2 \cdot l/W \leq 0.8$ and $y/W \leq 0.5$

Using this relation a load-crack growth $F$-$l$-curve can be calculated from the plotted load-displacement diagram. On the other hand the $F$-$v$-$l$ relationship can
also be determined experimentally from a calibration curve. The difference between calibration curve and calculated curve is less than 1%, Fig. 3. Therefore all tests had been analysed by the Liebowitz equation.

\[ K_{\text{eff}} = \frac{F}{s \cdot w} \cdot \sqrt{\frac{\pi \cdot l_{\text{eff}} \cdot \sec \frac{\pi \cdot l_{\text{eff}}}{w}}{w}} \]

The plot of \( K_{\text{eff}} \) vs \( \Delta l_{\text{eff}} \) is the R-curve. This R-curves should meet the \( \Delta l_{\text{eff}} \)-axis vertically at \( K_{\text{eff}} = 0 \). Deviation of this point of intersection from origin of coordinates are systematic differences. They are to eliminate. The R-curves determined by the compliance-method for different specimen width and ratios of \( l_{\text{eff}}/W \) show remarkable scatter for which no significant effect of the single parameters is recognized. Therefore this scatter seems to have its origin in systematic errors of the test method.

**POTENTIAL DROP METHOD**

Potential drop measurements are done with direct current using a power supply unit (10 A) of high stability and a low DC-compensation resistance. In this case constant DC is induced to the specimen. The potential \( U \) across the crack changes with crack extension proportionally to the variation of resistance. To make sure that the variation of the resistance in the elektric circuit is only a function of crack extension, the specimen has to be isolated to the test machine. Changes of the conductivity due to plasticity at the crack tip may be neglected.
The measured potential is amplified and filtered independent from absolute value. It is plotted against load \( F \) on a second X-Y-plotter parallel to the signals of the compliance measurement.

For the potential drop measurements a special contact device was developed which is pressed to the specimen by spring contacts. The contact points are located symmetrically to the crack plane, Fig. 1. Calibration tests have shown that the passive resistance is very constant.

DC should be fed in homogeneous over the whole width of the specimen. In the case of current induced via bond electrodes over the whole width of the specimen Johnson (1965) gave an analytical relationship \( I \) vs \( U \)

\[
1 = \frac{W}{\pi} \arccos \left( \frac{\cosh(\pi y/W)}{\cosh(y/\theta) \arccosh\left(\frac{\cosh(\pi y/W)}{\cos(\pi \theta/W)}\right)} \right)
\]

The use of this relationship postulates a good and homogeneous contact of the bond electrodes to the specimen over the whole width. If the conductivity of the specimen material and the copper wires is in the same order this requirement cannot be met sufficiently in the experiment e.g. for Aluminium alloys. In this case point electrodes far away from the crack plane are preferred, but no solution for punctual sources is available from theory of complex functions which satisfies the boundary conditions. Therefore calibration tests have to be done with this test equipment for different specimen widths.

![Figure 4](image.png)

Comparison of experimental and calculated calibration curves for potential drop method.
This calibration curves show very good agreement for a specimen width of $W = 160$, 300 and 500 mm. They can be fitted very accurate by polynomial regression using a square assumption, Fig. 4. The relationship between $l$ and $U$ for the test equipment used ($U = 0$ at $l = 0$) normalized to thickness $s$ is as follows

$$s \cdot U = 1385 \left( \frac{21}{W} \right) + 1377 \cdot \left( \frac{21}{W} \right)^2$$

and crack length

$$l = \frac{W}{2} \left( \sqrt{0.253 + \frac{s \cdot U}{1377} - 0.506} \right) \text{ mm}$$

A typical plot of a potential drop test is given in Fig. 5.

![Fig. 5. Typical plot for $F$ vs $2v$ and $F$ vs $\Delta U$](image)

This plot shows the significant plateau of the potential above which the electrical resistance of the specimens is only dependent from the ligament. Using this plateau-value the plot is analysed relatively ($\Delta U$). The main advantage of this method of measuring the potential is that one can amplify and measure the signal independent from absolute value of $U$. Taking in account crack length $l_0$ at start of the test the normalized voltage $s \cdot U_0$ which is calculated from relation 1 has to be added

$$l = \frac{W}{2} \left( \sqrt{0.253 + \frac{s \cdot U_0 + s \cdot \Delta U}{1377} - 0.506} \right)$$

Crack length $l$ calculated from this equation is only the physical length without plastic zone at the crack tip. To get $l_{\text{eff}}$, the radius of the plastic zone $r_\text{eff}$ has to be added. Using $l_{\text{eff}} = l + r$, the effective stress intensity factor $K_{\text{eff}}$ is calculated in the same manner as using compliance method and the $R$-curve $K_{\text{eff}}$ is plotted with $K_{\text{eff}}$ versus $\Delta l_{\text{eff}}$. 
Fig. 6. R-curves for different specimen width and thickness

EVALUATION OF CRITICAL FRACTURE TOUGHNESS VALUES FROM R-CURVE

Fracture of cracked components occurs at the "instability point" that means when the crack growth resistance energy $R$ is equal to energy release rate $G$.

$$G = R$$

The energy values $G$ and $R$, related to stress intensity $K$ in a simple manner, are normally replaced by the stress intensity $K_0$ and $K_R$. $R$ equal to $G$ therefore can be measured as the value $K_R$.

As shown before the $R$ ($K_R$)-curve is only effected by material thickness. Therefore for different crack lengths, and specimen widths but constant thickness the same R-curve may be used.

Broek (1974) made a proposal to describe the R-curve analytically by an exponential function. In the tested range the following relationship showed very good agreement to the experimental R-curve:

$$K_R = \exp\left( a_0 + a_1 \ln (1 - \frac{\Delta L}{L_0}) + a_2 \left(\ln (1-\frac{\Delta L}{L_0})\right)^2 \right)$$
The coefficients $a_0$, $a_1$, and $a_2$ are determined by calculating the regression line. Differentiation of this relation gives the slope of the $R$-curve:

$$\frac{\partial K_R}{\partial l} = K_R \left( a_1 + 2a_2 \cdot \ln \left( 1 - \frac{l_0}{l} \right) \right)$$

The linear-elastic stress intensity $K_G$ for the CCT-specimen is given by

$$K_G = G \sqrt{\frac{\pi \cdot l}{\cos \left( \frac{\pi \cdot l}{W} \right)}}$$

The slope of the $G$-curve therefore is

$$\frac{\partial K_G}{\partial l} = \frac{K_G}{2l} \left( 1 + \frac{\pi \cdot l}{W} \cdot \tan \frac{\pi \cdot l}{W} \right)$$

Using the requirement for instability,

$$K_R = K_G; \quad \frac{\partial K_R}{\partial l} = \frac{\partial K_G}{\partial l}$$

the point of tangency between both curves that means $K_R$ or critical crack length $l_c$ can be calculated for every $l$ or width $W$. Figure 7 e.g. shows this point of instability for an Aluminium alloy of 3 mm thickness, $2l_0/W = 0.33$ and different specimen widths.

All these tests had been conducted at ambient temperature, tests at liquid-nitrogen-temperature will follow this year.

Fig. 7. Prediction of fracture toughness $K_c$ and critical crack length $l_c$ by R-Curves.
RESULTS AND CONCLUSIONS

- R-curves calculated from potential drop measurements show significant smaller scatter than R-curves calculated from compliance measurements. Significant effects of specimen width and initial crack length l, are not noticeable. This confirms the assumption that the R-curve is only effected by specimen thickness, Fig.6.

- Tests with Aluminium material have shown that homogeneous feed in of the current over the whole width of the specimen is difficult for this material. In this case current induced by point electrodes is preferable.

- For determining critical fracture values from R-curve the instability point can be calculated analytically. This method is preferred to the graphic method, because at similar slope of both curves graphic determination of the intersection point is very unsafe.

REFERENCES