EXPERIMENTAL INVESTIGATIONS OF THE PATH INDEPENDENCE OF THE J-INTEGRAL FOR LARGE PLASTIC ZONES

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ABSTRACT

The path independence of the J-Integral is investigated experimentally in regions of elastic and large plastic deformations.

The strains on the surface of the specimens are measured by photoelastic-coating technique. The stresses are computed by aid of the differential Prandtl-Reuss law. After estimating the furthermore required quantities, the J-Integral is computed on different paths.

The experiments show, that the J-Integral decreases with increasing path-length in the region of plastic material-behaviour. If the ligament is fully plastified, then the J-Integral tends again to a constant.
The deviations from a constant are small for all plastic zone sizes.
Moreover the approximative J-values evaluated with a load-displacement diagramm are compared with J-values of the line integral.

KEYWORDS

J-Integral, photoelastic-coating technique, large plastic deformations

INTRODUCTION

For plane crack-problems the J-Integral

$$J = \int_{\Gamma} (W dy - \vec{T} \frac{\partial \vec{u}}{\partial x} ds) \tag{1}$$

is often used as an elastic-plastic fracture criterion (Rice, 1968). The integration-path connects the lower with the upper crack-surface (see Fig. 1).

$\vec{T}$, $\vec{u}$ are the stress- and strain vector, W is the strain-energy density.
The J-Integral can only be valid as a fracture-criterion if it does not depend on the integration-path. As long as the condition

$$\sigma_{ij} = \partial W/\partial\varepsilon_{ij} \quad (\sigma_{ij}, \varepsilon_{ij} = \text{stress, strain components})$$

holds, i.e. for hyperelastic material-behaviour, the path-independence can be proved theoretically. This proof is impossible for plastic material behaviour. But it seems to be possible, that the path-independence is conserved, as long as there is no unloading. That means, that the stress-strain relation remains univocal.

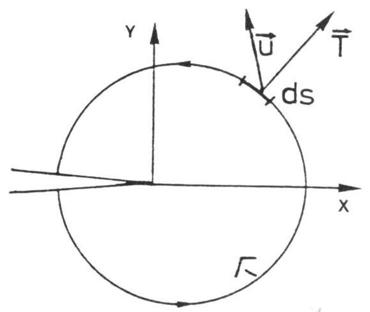

Fig. 1. Integration path surrounding the crack-tip.

The path-independence of the J-Integral can be examined either by numerical or by experimental methods. The experimental methods need less idealisations than the numerical methods. However there is a complete lack of experimental investigations of the path-dependence. The object of the present paper is the experimental determination of the J-Integral for different integration-paths starting from the elastic-material behaviour until large plastic deformations.

Experimental Method

Initially the strains on the surface of the loaded specimens are measured by the photoelastic coating technique (Wolf, 1961). For this purpose a photoelastic-coating is bonded on the surface of the specimen. After loading the specimen, the strains are transmitted to the coating.

The quantitative determination of the strain difference in the coating is done by aid of a Babinet-Soleil compensator. The direction of principal-axis can be estimated by using linear polarised light, which creates the isoclinic-lines. The seperation of the principal-strains occurs by measuring the strain-difference for two angles of light-incidence. With those three measurements, the components of the plane strain-tensor can be calculated.

SEN-specimens made of X 5 Cro Ni 18 9 were used. This steel has a low yielding stress and a large elongation, which makes it possible to produce large plastic zones. The uniaxial stress-strain relation is shown in Fig. 2.

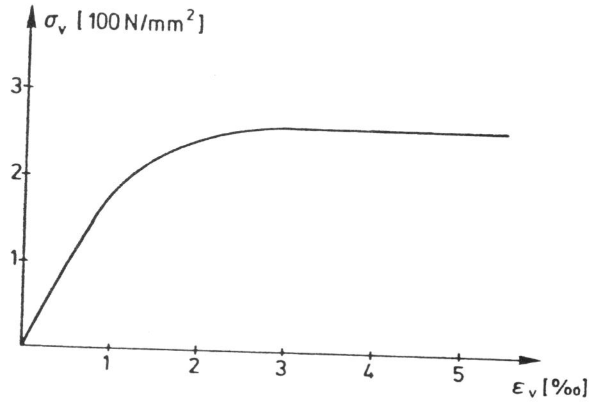

Fig. 2. Uniaxial stress-strain relation for the steel X 5 Cro Ni 18 9

The dimensions of the specimens are: width = 100 mm, thickness = 5 mm, length = 320 mm, crack-length = 20 - 36 mm. Instead of sharp cracks there are used saw-notches, because they can be created easily. In the regions where measurements are taken the strain field of the saw-notch and the field of the sharp crack are only slightly different.

The J-Integral is determined on rectangular paths for different increasing external loads. The percentage of the plastified ligament lies between zero and ninty percent, which shows, that the measurements were taken over a wide loading-range.

Determination of the J-Integral

To estimate the J-Integral, it is neccessary to compute the still unknown quantities in Eq. 1. It is presumed, that plane stress state exists, because the specimens are quite thin. For an elastic-plastic stress-strain relation one can use Prandtl-Reuss law (Szabó, 1958):

$$d\varepsilon_{ij} = \frac{1+\nu}{E} d\sigma_{ij} - \frac{\nu}{E} d\sigma_{KK} \delta_{ij} + \frac{3}{2} \frac{d\varepsilon_V^p}{\sigma_V} (\sigma_{ij} - \frac{1}{3} \sigma_{KK} \delta_{ij}) \quad .$$

Starting from a known initial state the stresses σ_{ij} and the strain ε_3 can be computed for each loading step by aid of the measured strain increments $d\varepsilon_{ij}$. σ_V and ε_V are the equivalent stress and strain. The Mises yield-criterion is used. ε_V^p is the plastic part of the equivalent strain.

The strain energy density is also calculated incrementally, using the computed stresses and measured strain increments:

$$dW = \sigma_1 d\varepsilon_1 + \sigma_2 d\varepsilon_2 .$$

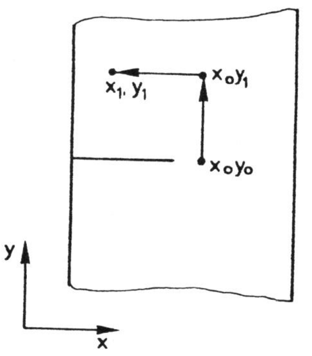

Fig. 3. Integration-path at the determination of $\partial v/\partial x$ (schematic)

In order to known the derivative $\partial v/\partial x$ (u, v = displacement in x, y-direction) first the displacement v is estimated, starting from the ligament, where v = 0 (see Fig. 3):

$$v\,(x_0, y_1) = \int_{y_0}^{y_1} \varepsilon_y \, dy$$

After the computation of v, the derivative $\partial v/\partial x$ can be calculated. For those points, which cannot be reached from the ligament by integration in y-direction, it is neccessary to compute first the derivative $\partial u/\partial y$. From the edge of the region, where $\partial v/\partial x$ is already determined, the integral

$$\left.\frac{\partial u}{\partial y}\right|_{x_1, y_1} = \int_{x_0}^{x_1} \frac{\partial \varepsilon_x}{\partial y} \, dx + \left.\frac{\partial u}{\partial y}\right|_{x_0, y_1}$$

is estimated. The constant of integration can be obtained from the equation:

$$\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = 2\,\varepsilon_{xy}$$

The derivative $\partial v/\partial y|_{x,y}$ can also be computed from the above equation.

Results

Path-Independence of the J-Integral. Figure 4a and Figure 5a show the J-values for two specimens as a function of the integration-path's length. The accompanying

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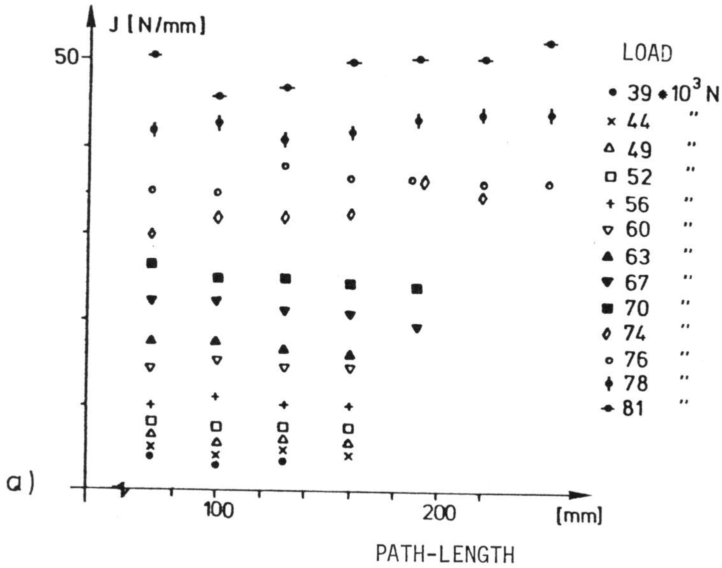

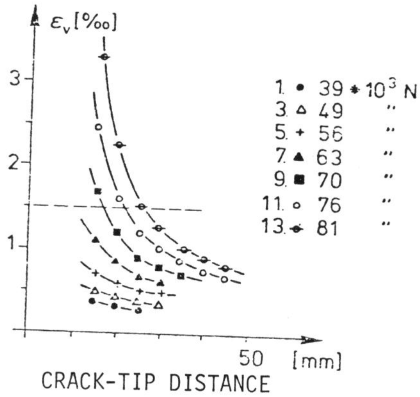

Fig. 4 a: J-Values as a function of the path-length
4 b: Equivalent strain (the dotted line shows the beginning of plastic deformation)

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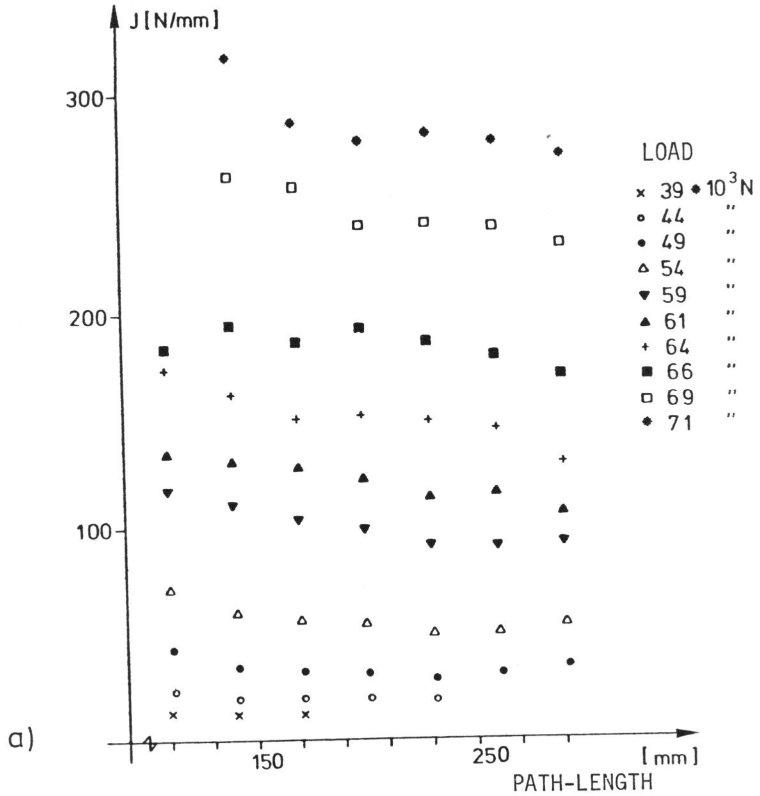

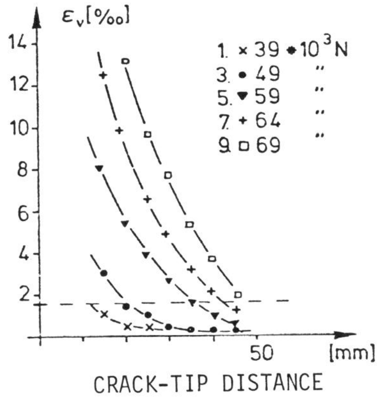

Fig. 5 a: J-Values as a function of the path-length
5 b: Equivalent strain (the dotted line shows the beginning of plastic deformation)

Fig. 4b and 5b show the equivalent strain on the ligament as a function of cracktip distance. Therewith the extent of plastification can be approximated. Specimen 11 is mainly stressed elastically. Beginning from the second load level specimen 16 is stressed plastically and reaches large plastic zone sizes. For specimen 11 the J-values are nearly constant for all loading steps. For specimen 16 the J-values are constant for the first and second loading step. With increasing load one can observe a small decrease of the J-values for increasing path-length. For shorter paths first the J-values decrease and finally approach a constant for increasing path-length.

To establish the path-dependence of the J-Integral the following procedure was chosen, which includes the results of all specimens: The J-Integral as a function of the path-length was fitted by a linear approximation curve. The J-value for the longest path was estimated with the fitted curve. The deviation of this J-value from the mean value for the observed load step was calculated. There is a decrease of 3 percent for all specimens and load steps without any plastic deformation on the integration-paths. This decrease lies within the accuracy of measurement. For all load steps with elastic-plastic deformations on the integration-paths there is a decrease of 6.6 percent. This exceeds insignificantly the accuracy of measurement. It can be assumed, that the path-dependence is affected by the extent of plastification. Fig. 6 shows on one axis the deviation Y of the J-value for the longest path from the mean value of the load step. On the other axis there is shown the percentage X of the measuring points where plastification occurs. To use the results of all specimens, the average values of Y for all loading steps lying in a certain range of plastification were computed.

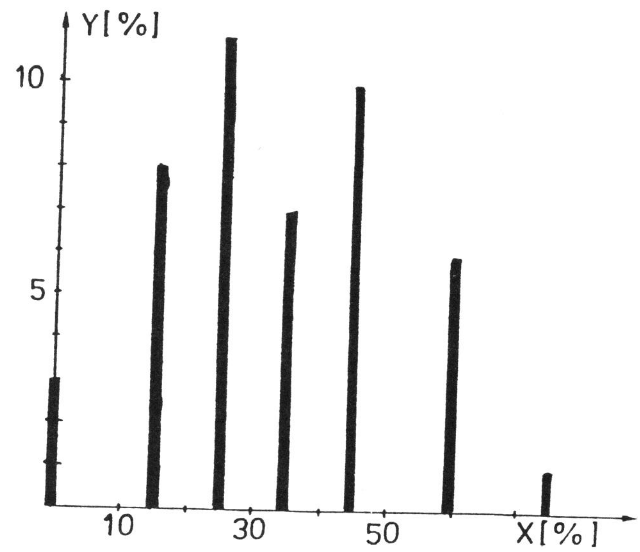

Fig. 6. Y: decrease of the J-value for the longest integration-path from the mean-value of the load step

X: percentage of the plastified measuring points

For elastic material behaviour the J-Intergral is path-independent within the accuracy of measurement. With increasing plastification the J-values decrease for increasing path-length. For large plastic zones the J-Integral is again constant.

J-values determined by load-displacement curves. It is interesting to know, whether the J-value determined by load-displacement diagramms approximately yield the same results in the range of plastic material behaviour as the J-value by evaluating the line integral Eq. (1). For specimens, which are mainly loaded by bending, Rice et al give the approximation

$$J^{*} = \frac{U}{B(w-a)};B = \text{thickness},\ W = \text{width},\ a = \text{crack-length},\ U = \text{work}.$$

Table 1 shows the J-values J by evaluating the line integral Eq. (1) and the values J^*determined by aid of a load-displacement curve. a^*corresponds to length, which includes the length of the plastified ligament. It can be seen, that there is a good agreement of J and J^*. That is a surprising fact, if one takes into account, that $J^*$is only an approximation.

TABLE 1

External load [N]	J [N/mm]	J^* [N/mm]	J/J^*	a^*/W
549oo	3o	27	1.11	.48
588oo	4o	36	1.11	.52
637oo	53	55	o.96	.59
656oo	61	62	o.98	.61

Conclusion

As expected the J-Integral is constant, if the integration-paths are passing through the elastic region. This is valid too, if there are plastic deformations outside the integration-paths.
If some of the paths pass through plastic region, one can notice a decrease of the J-values for increasing path-length. At further increase of the path-length and therewith decreasing of the plastification, the J-value approaches a constant value again. The observed trends are insignificantly larger than the accuracy of measurement.
Considering the accuracy of determining as well critical as actual values of the J-Integral in practical field of application, the noticed decrease of the J-Integral can be regarded as being small. For the conditions given in the experiments, the J-Integral is therefore nearly path-independent in the region of large plastic zones too.
The comparison of the J-Integral (Eq. 1) with the J-values determined by aid of load-displacement curves indicates the applicability of the load-displacement method for elastic-plastic crack problems too.

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