The stress intensity factor Green's function for a crack interacting with a dislocation

S. Teyar[1] and M. Chabaat[2]

[1] Doctorate, Built Env. Res. Lab., Civil Engineering. Faculty, U.S.T.H.B., B.P. 32 El Alia, Bab Ezzouar, 16111 Algiers, Algeria. teyars@yahoo.fr

[2] Professor Built Env. Res. Lab., Civil Engineering Faculty, U.S.T.H.B., B.P. 32 El Alia, Bab Ezzouar, 16111 Algiers, Algeria. mchabaat2002@yahoo.com or mchabaat@usthb.dz

Keywords: Crack, microcrack, dislocation, Green's function, crack amplification, crack shielding, Stress intensity factor.

This paper presents the results of the analysis of Green's functions for the problem of interaction between a crack and a surrounding dislocation. It consists in the determination of Green's functions based on the formalism of the complex potentials of Muskhelishvili. On the basis of this latest, Green's functions are constructed for the Stress Intensity Factor (SIF) in Mode I and Mode II due to the unit dipole force applied to the main crack surfaces in the presence of a dislocation in front of the crack tip. Various stress fields have been obtained taking into consideration interactions between the crack and the microcrack for various configurations such as the orientation as well as the position of the microcrack with respect of the main crack. The effect of amplification and shielding on the resulting stress field is shown, through a study of mode I and mode II SIF. Obtained results are compared and agreed with those of other researchers.

Introduction

It is well known that in many brittle materials, crack propagation is accompanied by the formation of a damage surrounding the crack tip. This damage develops in a nearby tip-zone called also Fracture Process Zone (FPZ). This latest can reveal itself as the nucleation of many microcracks around the tip of a propagating crack [1, 2]. These microcracks can have a significant influence on the propagation of the main crack. They can either cause crack amplification or crack shielding. Crack shielding reduces stress intensity factors of the main crack while crack amplification increases those values. These effects have been investigated by several researchers using exact analytical methods for some particular cases [3], and with analytical approximations under certain assumptions [4, 5]. Because this damage can constitute an important toughening mechanism, problems dealing with crack microcracks interactions have received considerable research attention since they were introduced to fracture mechanics. As a result, a wide body of literature, on this topic, exists [6]. Solutions obtained are mostly based on the complex variable technique [7], or on numerical procedures [8], or asymptotic estimates for remotely located cracks [9]. Those techniques are usually different to a degree, but the basic principals remain the same.

In this paper, interaction between a macrocrack and a microcrack represented by a distribution of dislocations dipole is considered. A stress field distribution induced during these interactions is obtained using Muskhelshvili's complex variables formalism which relies on the Green's functions, solution to the multiple crack interaction problems. Contours of equal level of normalized stress intensity factor are determined for different orientations and positions of the dislocation-dipole with respect to the main crack. It is also shown that the existence of these microcracks affects the propagation of the crack appreciably and can possibly lead to the deterioration of the material. On the other hand, the effects of amplification and shielding of the stress field at the level of the vicinity of the main crack will be the subject of a meticulous study to elucidate the phenomena of the propagation of cracks.

Method of analysis

The analysis is based on the extension of the results obtained by Lo [4] and Denda [5] considering the interaction of the crack and a surrounding dislocation. Using Muskhelishvili complex variable formalism for plane isotropic elasticity where the two analytic functions or complex potential functions, $\emptyset(z)$ and $\psi(z)$, of a complex variable z = x + iy, are derived to express the stress and displacement fields according to

$$\frac{(\sigma_{xx}+\sigma_{yy})}{2} = 2\,Re[\emptyset(z)]$$

$$\frac{(\sigma_{yy}-\sigma_{xx})}{2} + i\,\sigma_{xy} = [\bar{z}\acute{\emptyset}(z)+\psi(z)] \qquad (1)$$

$$2\mu\,(u_x+u_y) = k\,\emptyset(z) - z\acute{\emptyset}(z) - \psi(z)$$

where μ is the shear modulus, κ is related to Poisson's ratio (κ = (3 – ν)/(1 + ν) for plane stress and κ = 3 - 4ν for plane strain). A prime indicates the differentiation with respect to z and a bar a complex conjugate.

It is known that for a point force applied at ξ with respect to the main crack-tip, the following potential functions are given by [6];

$$\emptyset^s(z,\xi) = -\gamma\,\{log(z-\xi)\}$$

$$\psi^s(z,\xi) = -k\bar{\gamma}\,\{log(z-\xi)\} + \gamma\left\{\frac{\bar{\xi}}{z-\xi}\right\} \qquad (2)$$

where γ stands for the dislocation data and is given by [7];

$$\gamma = \frac{i\,\mu\,b}{\pi\,(k+1))} \text{ and } \bar{\gamma} = \frac{i\,\mu\,\bar{b}}{\pi\,(k+1))} \qquad (3)$$

On the other hand, for a force dipole corresponding to a dislocation representing a displacement discontinuity over the infinitesimal line segment as shown in Fig.1, previous potential functions Eqs. (2) can be written as;

$$\emptyset^d(z,\xi) = -\gamma d\,\{log(z-\xi)\}$$

$$\psi^d(z,\xi) = -k\bar{\gamma}\,d\,\{log(z-\xi)\} + \gamma d\left\{\frac{\bar{\xi}}{z-\xi}\right\} \qquad (4)$$

and using the following differential operator;

$$d\,(\ldots) = \frac{\partial}{\partial \xi}(\ldots)\,d\xi + \frac{\partial}{\partial \bar{\xi}}(\ldots)\,d\bar{\xi} \tag{5}$$

Eqs. (4) become;

$$\emptyset^{d}(\xi, z) = \frac{\gamma}{(z-\xi)}$$

$$\psi^{d}(z, \xi) = \frac{k\bar{\gamma}}{z-\xi} + \frac{\gamma}{z-\xi} \tag{6}$$

Note that the potentials $\Phi\,(z)$ and $\psi(z)$ as defined by Eqs. (6) satisfy the traction-free boundary condition on the crack surface.

Crack dislocation interactions

The stress field generated in the vicinity of the main crack-tip interacting with a dislocation-dipole is obtained by the substitution of Eqs. (6) into Eqs. (1) under the following form;

$$\frac{\sigma_{xx} + \sigma_{yy}}{2} = -2\,Re\left\{\frac{i\,\mu\,b}{\pi\,(k+1))}\,\frac{d\xi}{(z-\xi)^2}\right\}$$

$$\frac{\sigma_{yy} - \sigma_{xx}}{2} + i\,\sigma_{xy} = \frac{i\,\mu}{\pi\,(k+1))}\left\{b\left[\frac{-d\bar{\xi}}{(z-\xi)^2} + \frac{2\,\overline{(z-\xi)}\,d\xi}{(z-\xi)^3}\right] + \bar{b}\frac{d\xi}{(z-\xi)^2}\right\} \tag{7}$$

Green's function for the SIF analysis

The SIF for both modes are expressed by the following relation;

$$K_I - iK_{II} = \lim_{\sqrt{z}\to 0} \sqrt{2\pi z}\,(\sigma_{yy} - i\sigma_{xy}) \tag{8}$$

which leads to;

$$K_I - iK_{II} = \xi^{\frac{-3}{2}}\,d\xi\sqrt{2\pi}\begin{bmatrix}\gamma\cos(\theta - \frac{3\beta}{2}) + \frac{3}{2}\bar{\gamma}sin\beta\,\sin\,(\frac{5\beta}{2} - \theta)\\ -\frac{3}{2}i\,\bar{\gamma}\sin\beta\,\cos\left(\theta - \frac{5\beta}{2}\right) + i\bar{\gamma}sin\theta e^{\frac{3i\beta}{2}}\end{bmatrix} \tag{9}$$

Each mode I and II can be written separately as;

$$\frac{K_I}{K_0} = b_x\left[3sin\beta\cos\left(\theta - \frac{5\beta}{2}\right) - 2\,sin\theta\,cos\frac{3\beta}{2}\right] + b_y\left[2cos\left(\theta - \frac{3\beta}{2}\right) - 2sin\theta cos\frac{3\beta}{2} + 3sin\beta\,\sin\left(\frac{5\beta}{2} - \theta\right)\right] \tag{10}$$

$$\frac{K_{II}}{K_0} = b_x \left[2\cos\left(\theta - \frac{3\beta}{2}\right) + 2sin\theta\, sin\frac{3\beta}{2} - 3sin\beta\, \sin(\frac{5\beta}{2} - \theta)\right]$$
$$+b_y \left[3sin\beta\, \cos(\theta - \frac{5\beta}{2}) - 2sin\theta cos\frac{3\beta}{2}\right] \qquad (11)$$

Where $\mathrm{K}_0 = \frac{\xi^{\frac{-3}{2}} d\xi \sqrt{2\pi}\mu}{2\pi(\kappa+1)}$

To avoid laborious computations, Eqs. (10) and (11) are normalized under the form;

For mode I $$K_I^* = \frac{K_I}{K_0 |b|} \qquad (12)$$

For mode II $$K_{II}^* = \frac{K_{II}}{K_0 |b|} \qquad (13)$$

where $|b| = \sqrt{b_x^2 + b_y^2}$ is the magnitude of the burgers vector.

In this work, Green's functions for the SIF due to a dislocation dipole at an arbitrary point z located at a distance ξ from the crack tip are illustrated for the followings cases:

Case of a unit discontinuity in vertical direction across a horizontal element

Plots of the contours of equal values of Green's functions near the crack tip are shown in Fig. 4a. The main feature and valuable information obtained from these results is the predominant shielding effect the microcrack produces on the main crack. Notice also that when the crack propagates from the left to the right, the microcrack will play a role, first, of shielding, and then, of enhancing the stress. Besides, the borderline between the two effects is found at an angle of about ± 68 degrees with respect to the crack. Plots very similar were obtained by Shiue and Lee [7] and later by Rose [8] and Rubinstein [6].

Case of a unit discontinuity in horizontal direction across a vertical element

Contours of equal values of Green's functions are shown in Fig. 4b. For this case, the amplification effect is more intense than the shielding effect and the borderline is located at about ± 35 degrees and ± 110 degrees with respect to the crack. As crack propagates from the left to the right, while the microcrack is stationary, Green's function will switch sign depending on the location of the discontinuity. These results agree with those obtained by Shiue and Lee [7]. It is shown in Fig. 5, the graph of the ratio (K_I^*) versus the arbitrarily orientation of the micro crack. As evident, a micro crack located closer to the main crack dominate the resulting interaction effect and reflect an anti-shielding of the damage while a reduction constitutes a material toughness.

All the results obtained are summarized in table 1. As one can notice that for both cases, amplification effect is predominant once a microcrack gets closer to the tip of the main crack meaning in the active part of the damage zone. On the other hand, a shielding effect occurs once the microcrack is takes position in the awake part of the damage zone.

Table 1. Delimitations between amplification and shielding zones for Mode I and II.

Case $b_x = 0$

Modes	Dislocation-position with respect to the main crack.	Delimitation between amplification and shielding zones (in degree).	
		Amplification	Shielding
Mode I	θ = 0°	β < 68	68 < β < 180
	θ = 90°	36 < β < 108	β < 36 108 < β < 180
Mode II	θ = 0°	β < 36 108 < β < 180	36 < β < 108
	θ = 90°	30 < β < 80 123 < β < 180	β < 30 80 < β < 123

Case $b_y = 0$

Modes	Dislocation-position with respect to the main crack.	Delimitation between amplification and shielding zones (in degree).	
		Amplification	Reduction
Mode I	θ = 0°	β < 36 108 < β < 180	36 < β < 108
	θ = 90°	30 < β < 80 123 < β < 180	β < 30 80 < β < 123
Mode II	θ = 0°	β < 30 80 < β < 124	30 < β < 80 124 < β < 180
	θ = 90°	β < 35 108 < β < 180	35 < β < 108

4- Conclusion

It is shown in this study by using a potential complex functions based on Muskhelishvili's formalism, one can determine the interaction between a macro crack and a surrounding dislocation. Green's functions for the Stress Intensity Factor are employed in this analysis to quantify the effects on a crack of a micro crack represented by a dislocation dipole. For a variety of configuration (position as well as orientation) of the dislocation, amplification and shielding effects have been described trough the study of normalized SIF K^*. It is proven, herein, that the intensity of a of microcrack increases significantly with increasing number of discontinuities. The overall effect of the damage is identified as being an amplifying effect. Since there is no toughening, the resulting local stress field would direct the propagation of the main crack.

References

[1] M. T. Takemori and R. P. Kambour, *Journal of Materials Science,* **16** (1981).
[2] J. Boots's, A. Chudnovsky and A. Moet, *Int. Journal of Fracture Mechanics,* ***33***, (1987).

[3] M. Chabaat and S. Djouder, *International Journal of Materials Science and Engineering A*, 387-389 (2004) 361-366.
[4] K. K. Lo, *Journal of Applied Mechanics*, 45 (1978) 797-802.
[5] M. Denda and P. Quick, *Electronic Journal of Boundary Element*, **2** (2004).
[6] A. A. Rubinstein, *International Journal of Fracture*, 27 (1985) 113-119.
[7] S. Shiue and A. Lee, *International Journal of Fracture*, **28**, (1985)
[8] L. R. F. Rose, *International Journal of Fracture*, **31**, (1986).
[9] N. Mushkelishvili: *Some basic problems of mathematical theory of elasticity*, Holland, (1953).

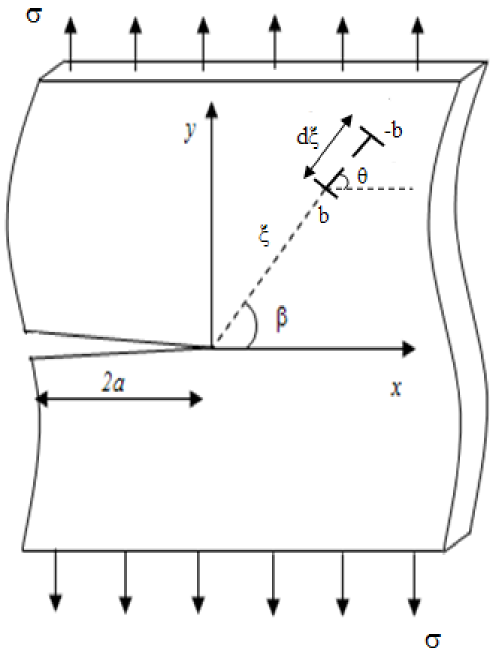


Fig 1. Dislocation dipole interacting with a crack

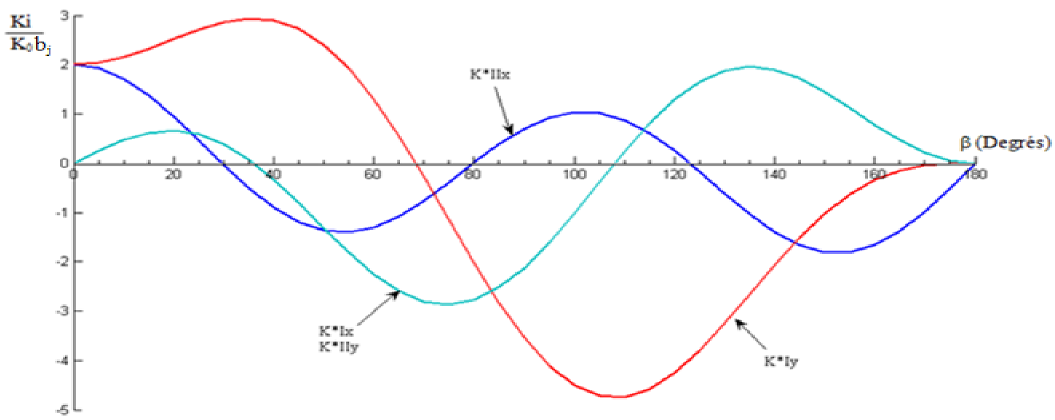


Fig. 2. Angular variation of stress intensity factor for $\theta = 0$.

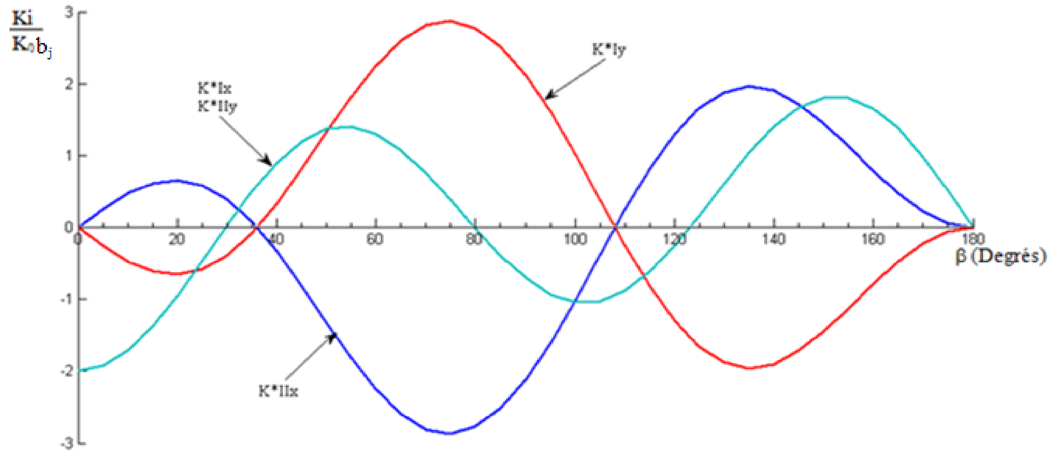
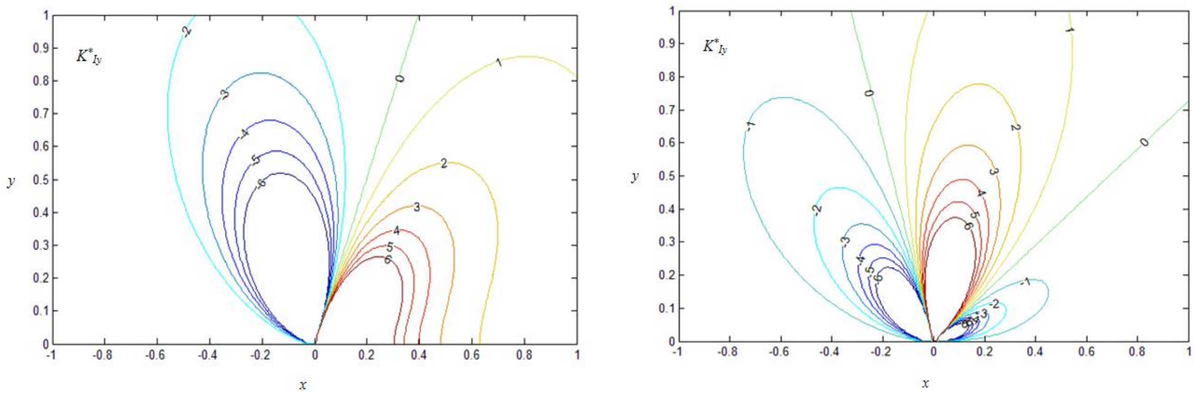


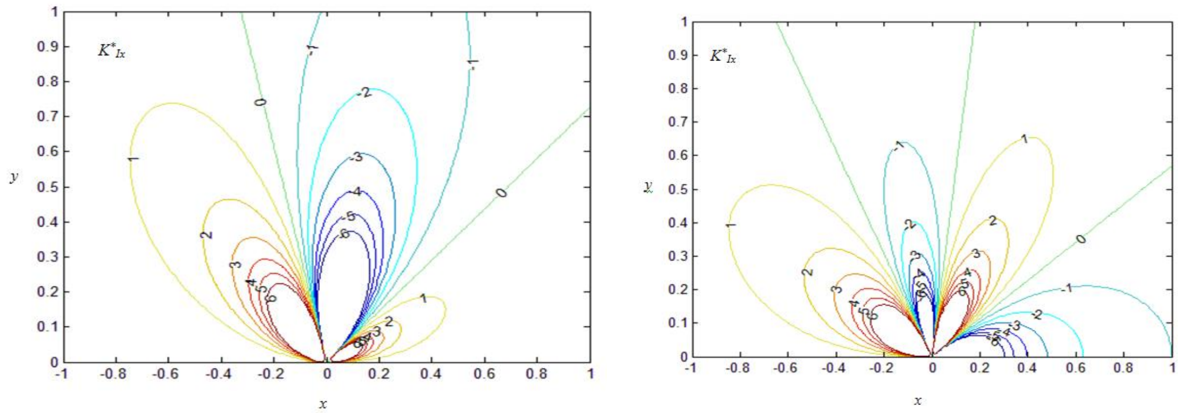
Fig. 3. Angular variation of stress intensity factor for $\theta = 90$.



a- Case where $\theta = 0$

b- Case where $\theta = 90$

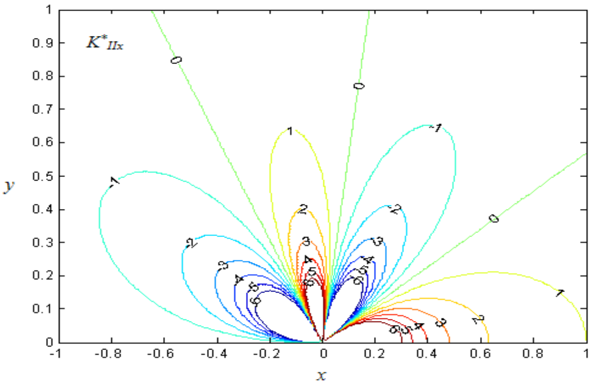
Fig. 4. Contours of equal levels of mode I stress intensity factor due to b_y.



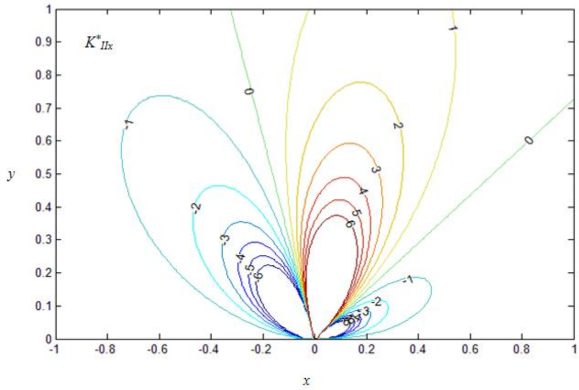
a- Case where $\theta = 0$

b- Case where $\theta = 90$

Fig. 5. Contours of equal levels of mode I stress intensity factor due to b_x.

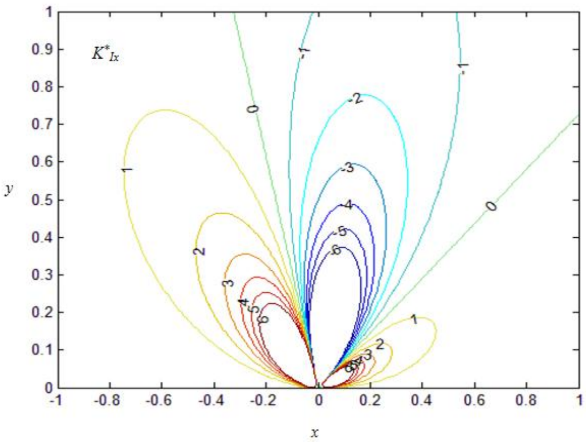


a- Case of $\theta = 0$

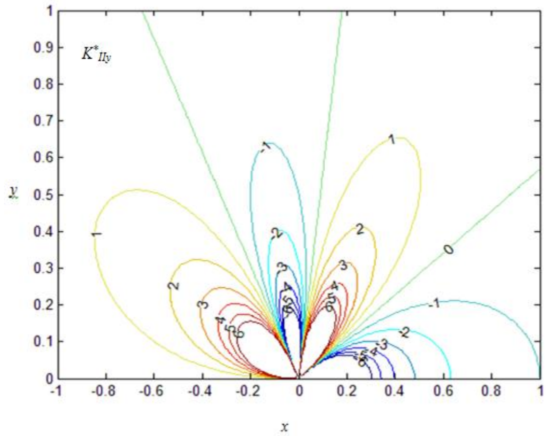


b- Case of $\theta = 90$

Fig. 6. Contours of equal levels of mode II stress intensity factor due to b_x



a- Case of $\theta = 0$



b- Case of $\theta = 90$

Fig. 7. Contours of equal levels of mode II stress intensity factor due to b_y