The standardized Brazilian disc test as a contact problem: Quantifying the friction at the disc-jaw interface

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Introduction: Although the limitations of the applicability of the Brazilian disc test due to the friction developed at the disc-jaw interface have long been recognized, the main difficulty in confronting the problem is the inability to quantitatively relate the generated frictional forces with the imposed radial pressure in accordance with a physically acceptable law. A novel approach to this problem is discussed here based on the idea that friction is somehow related to the mismatch between the tangential components of displacement of the disc and jaw along their common contact length due to the different deformability of the two constituent materials.

The contact problem: A disc of radius R_1 is compressed between two curved jaws (Fig.1). For smooth contact the displacements of both disc and jaw at points τ along the contact length, 2ℓ, are obtained, from the Mixed Contact Fundamental Problem, as:

$$u_{1,2}^{-,+}(\tau) = -\frac{\kappa_{1,2}-1}{24R_1\mu_{1,2}K}\left(\tau\sqrt{\ell^2-\tau^2}+\ell^2 \operatorname{Arc}\tan\frac{\tau}{\sqrt{\ell^2-\tau^2}}\right) \qquad (1)$$

Indices (1,-) and (2,+) are used for the disc and the jaw, respectively. The real constant K is equal to $[(\kappa_1+1)/4\mu_1]+[(\kappa_2+1)/4\mu_2]$. κ_1, κ_2 are Muskhelishvili's constants and μ_1, μ_2 the shear moduli.

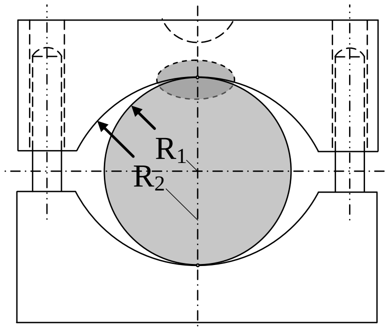

Fig.1 The ISRM device for the Brazilian disc test

The friction law: It is reasonable to assume now that friction must be proportional to both the normal radial stress $P(\tau)$ as well as to the displacement mismatch $U(\tau)=|u_1^-(\tau)|-|u_2^+(\tau)|$ as follows:

$$T(\tau) = f\ U(\tau)P(\tau) \qquad (2)$$

In Eq.(2), f is a constant somehow related to the coefficient of friction, and $P(\tau)=(\ell^2-\tau^2)^{1/2}/(3R_1K)$ is the distribution of radial pressure along the contact arc which is found to be of cyclic form [1].

Results and discussion: The distribution of frictional stresses along the contact semi-arc is plotted in Fig.2 for a disc (radius R_1=0.05 m) made from marble (E=75 GPa, ν=0.25) and jaws made from steel (E=210 GPa, ν=0.30). The variation is strongly skewed: it is zeroed at the centre and the end-points of the contact arc while its maximum value occurs at around two-thirds of the contact semi-arc length. These results are in good agreement with those obtained by Conway and Engel [2] and Hooper [3] using different methods.

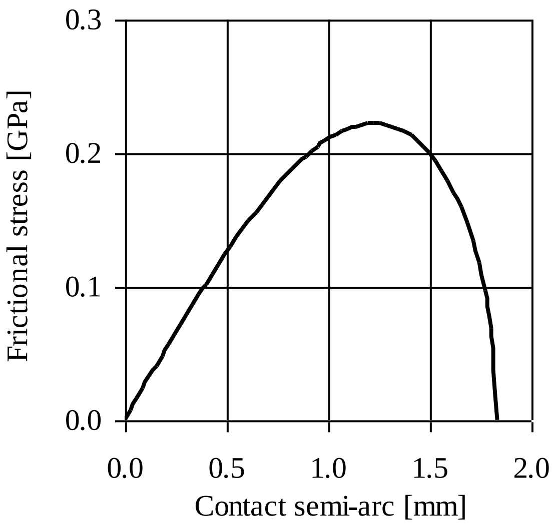

Fig.2 Variation of frictional stress along the contact arc

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