The Problem Of Crack Growth Simulation Under Random Loading

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Abstract. The paper shows the possibility of using Paris-Erdogan equation for simulation fatigue crack growth under random loading. In this equation was introduced the effective stress intensity factor range. The discussed methodology of crack growth simulation is based on the concept of a “basic” random loading and based on the experimental researches of fatigue crack growth under random loading that have been realized during specimens fatigue tests of two Al-based alloys (D16chAT - the same as 2024-T3, and B95ATB – 7075-T6). Overloads influence on the fatigue crack growth is considered by the discussed methodology. In all cases of specimen tests with center cracked panel the random loading has been considered as Gaussian processes of cyclic loading with introduced and discussed parameters of investigated processes. Good correlation between experimental data for crack growth period and simulated by the introduced methodology was shown in the different cases random cyclic loads.

Introduction.

Theoretical-experimental researches of crack grows duration usually include two stages. First stage is changing of real loads specter on the schematized one by the different models. The aim of such models is to decrease testing time for structures. Specters of random loads which represent several models, which used for test of aircraft structures, are shown in Fig. 1.

Harmonic loading (or constant amplitude of loads) and typical block loads (Fig.1a) used for comparing test results of aircraft structures of different design. The “typical flight”, shown in Fig.1,b, mainly used for lower wing sheet tests and calculations and, also, can be considered block of loads “TWIST”-type (Fig.1,f). Typical blocks of flight-type cyclic loads for wing lower sheets in wing-root-location used in tests for two different aircrafts area shown in Fig. 2.

Fig. 1,e shows wing lower panel tensometric stress record at “bumpy flight”. It is clear that specter of operated loads has principle difference with their modeled programs. That is why the first level of mistakes in crack grows duration estimation related to changing real operated loads specters of their modeling.

Second stage, mathematical models construction for crack grows duration estimation, which, took in account design philosophy of observed structures, loading conditions and in-flight operations. In constructed mathematical models used empirical parameters which should be experimentally estimated. That’s why the next possible mistakes in theoretical-experimental estimations of crack growth duration are inaccuracy in estimations of parameters of cyclic loading processes.

This article analyzed existent cracks growth estimation models under overloads introduced a new approach to crack grows modeling. The article shows good correlation of calculated results by the introduced model in comparison with experiment results performed under random loading. The principle of linear damages accumulation summering possibility to use for crack growth duration estimation is discussed for cases of “typical flight” program and stationary Gauss processes.

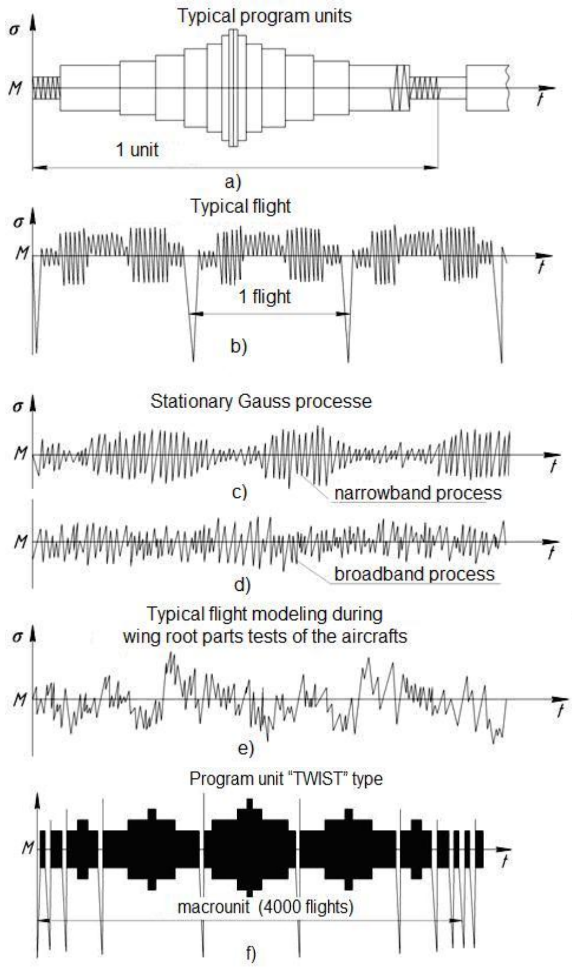


Fig. 1. Operational loads spectrum models.

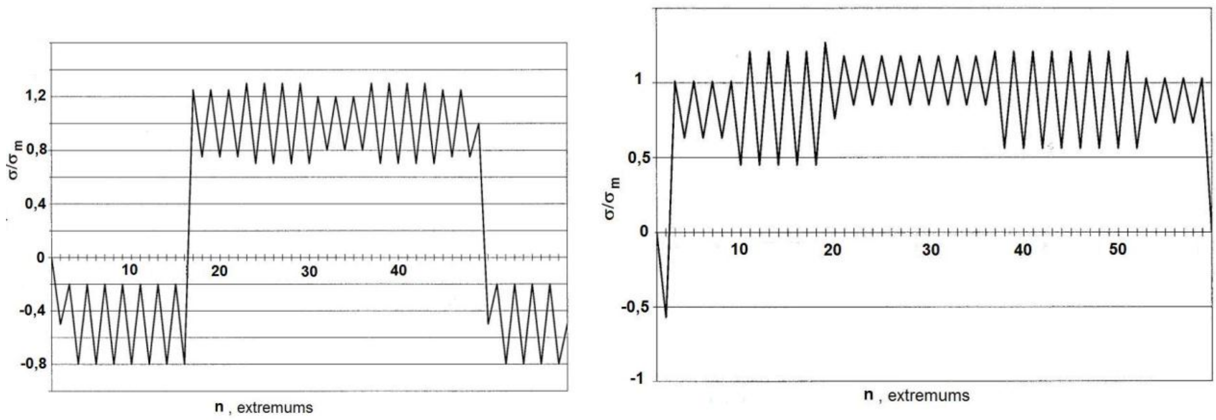


Fig. 2. Loads sequences used in tests for wing root zones lower panels' for (a) one and (b) another aircraft

Crack grows duration estimation at acting loads.

Fig. 3 shows total algorithm of crack grows duration [2]. First subject for the discussed methodology is choosing a model of acted in-service cyclic loads (for instance, comparing vitality estimation of "competitive" design philosophy, analysis of operational factors influence degree on crack grows duration, estimation of vitality results of crack growth modeling for structures to check intervals of their inspections and etc.). Models of cyclic loads sequences for two civil aircrafts wing root zone used for tests, for example, are shown in Fig. 2, [4, 5, 9].

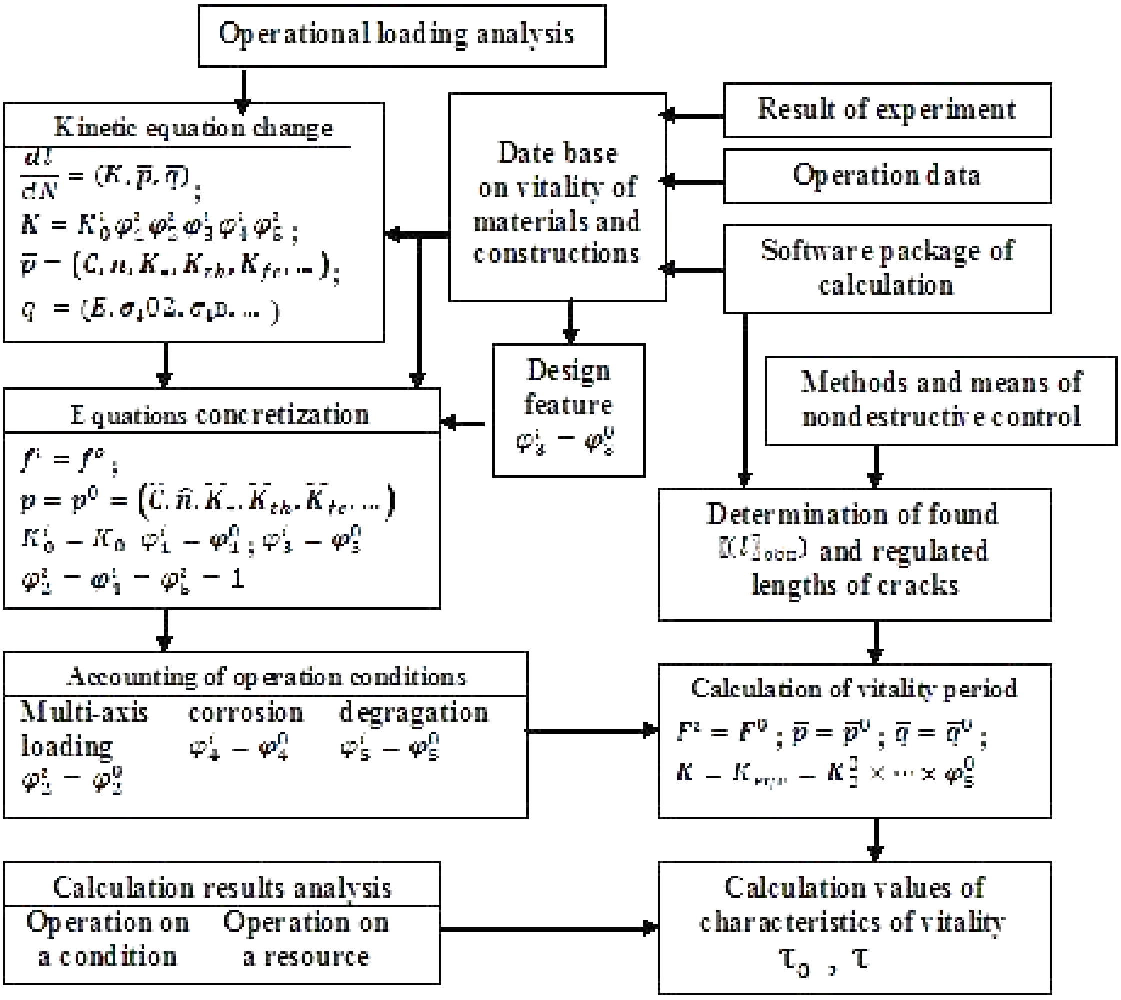


Fig. 3. Scheme of crack grows duration determination in elements subjected to in-service loads.

Model for crack growth simulation is chosen in dependence on in-service acting cyclic loads. In models can be used equations, which can considered simply case of cyclic loading without cycles loads interaction effects (equation types of Paris, Foreman et al), or models, which take into account these effects (models of Willer, Willenborg, Matsuoka and etc).
In generally case, crack grows equation can be written as

$$\frac{dl}{dN} = f(K, \bar{p}, \bar{q}). \quad (1)$$

In equation (1), l - crack length; N– loading cycles quantity; $\bar{p} = (c, n, K_*, K_{th}, K_{fc} \ldots)$ -vector of crack cycle closing ability parameters; c, n – experimentally determined parameters of kinetic equation; K_* - critical value of stress intensity factor (SIF); K_{th} – threshold of SIF; K_{fc} – maximum value of SIF for regular crack growth; $\bar{q} = (E, \sigma_{02}, \sigma_B \ldots)$ -vector, which define material mechanical properties; E– Modulus elasticity; σ_{02}– yield strength; σ_B – ultimate tensile stress. K - value has meaning of SIF range, or SIF maximal value and determines by relation

$$K = K_0 \varphi_1(N, l) \varphi_2(K_I K_{II}) \varphi_3(l, \bar{\Gamma}) \varphi_4 \varphi_5 \quad , \quad (2)$$

K_0 – SIF which is determining in the basically conditions (without interaction of cycles, geometric singularities and different operating factors, for example, during calculations of wing lower panel thin-sheets $K_0 = \Delta\sigma\sqrt{\pi\frac{l}{2}}$; φ_1 – functional correction, which determined cycles interaction effects; φ_2 – functional correction, which depend on biaxial loads ratio, $\varphi_3(l, \bar{\Gamma})$ - functional correction on geometric singularities of element, φ_4, φ_5 – functional corrections, which estimated environmental deterioration effects.

It is rationally to divide problem of crack growth simulation, firstly, estimating inaccuracy because of real spectrum changing by program unit, and, then, estimating inaccuracy, inserted by used model. But it is practically impossible to perform without experimental data of materials properties under cyclic loading and tests results for structure subjected random loading. Such estimation possible to perform for specimens test under loading spectrum being not far from the real of in-service loads sequence. In fact, inaccuracy of used method of crack growth simulation based on inserted program for acting loads modeling, mainly determines by amplitudes allocation and, in less, average value of considered process with its standard error. According this, it is possible to use tests results at forced loading.

As material conditions seriously influenced methodical error, material design philosophy, it needs to choose rationally test-analogue with account this circumstance. Before calculations for main acting loads it can be recommended to perform test calculations for modeled loading and, if necessary, perform model correction various parameters, included in calculating proportions.

Cracks kinetic calculation at random stress.

Great influence on crack grows rate and duration provides peak loads in random loading spectrum [3, 8]. Input of clipper factor K_{Π} (Fig. 4b) in considered process provides conservative crack grows duration estimation [8]. It was shown that crack grows maximal speed achieves at = (2 - 2.5) [7]. Processes with such shearing ratio will name basic.

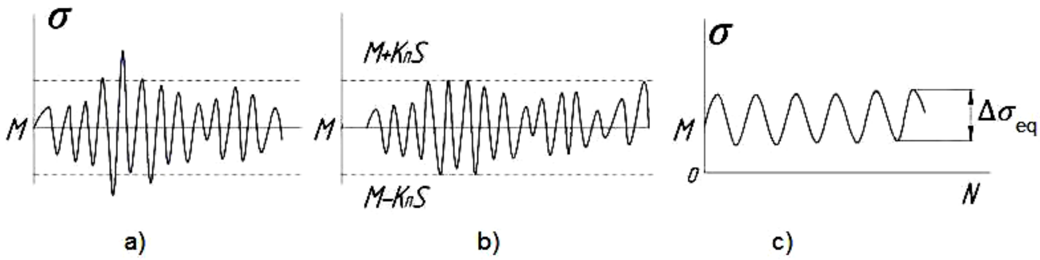


Fig. 4. Random working loading process a); shortened process b) (M – average of random process, S – standard error, K_{Π} – clipper factor); c) - equivalent harmonic loading.

Because of at basically loading modes interaction effects of cyclic loads are minimal, in Eqs. (1), (2) the functional correction φl(N,l,..)=1. In many cases SIF K_0 determines by the relation [5, 7, 15]

$$K_0 = \varphi_{01}(\sigma)\varphi_{02}(l) \quad . \tag{3}$$

Let be introduced symbol $f(l) = K_0\varphi_2(K_I K_{II})\varphi_3(l, \bar{\Gamma})\varphi_4\varphi_5$. Then let be used Paris-Erdohan equation

$$\frac{dl}{dN} = C\Delta K^n \quad . \tag{4}$$

At the introduced symbols Eq. (4) can be rewritten as

$$\frac{dl}{dN} = c\varphi_{01}(\sigma)^n f^n(l). \tag{5}$$

Let be introduced new variables

$$D = \left[\int_{l_0}^{l} f^{-n}(x)dx\right] \cdot \left[\int_{l_0}^{l_*} f^{-n}(x)dx\right]^{-1}. \tag{6}$$

In the Eq. (6) l_* is the critical length; l_0 – minimal value of crack length.
Then, Eq. (5) transforms to

$$\dot{D} = cB\varphi_{01}^n(\sigma), \tag{7}$$

where $B = \left[\int_{l_0}^{l_*} f^{-n}(x)dx\right]^{-1}$.

Transformations (6), (7) are allowable, because integrals in (6) exist, and critical (allowable) length of thin-wall elements regulates. Function *D* satisfy conditions *D(0)=0, D(t*)=1*, and agree with damage accumulation value defined according with rule of liner damage summation (N_* number of cycles for crack growth up to critical length l_*). D-parameter is analogous of introduced by V.V.Bolotin [1] parameter for damage accumulation estimation. As followed from Eq. (7) the hypotheses of linier damages accumulation is possible in estimation crack growth kinetics using well-known relations.

Let be $\varphi_{01} = \Delta\sigma$, then at constant loading amplitude the crack growth period can be estimated as

$$N_* \Delta\sigma^n = \gamma, \tag{8}$$

where $\gamma = C^{-1}B^{-1}$. It is not difficult to see, that Eq. (8) is the same as with S-N curve.

As soon as for clipper factor $K_п$ = 2…2.5, main statistics of random process practically don't change, so for stationary narrowband loading process average durability (at positive differential at zero) defines by equation

$$\bar{N}_{0+} = 2^{-\frac{sn}{2}}\Gamma(n/2+1)^{-1}\gamma \cdot S^{-n}. \tag{9}$$

For broadband random process meaning of cycle does not uniformly define, and durability calculations related to allowable schematization methods.

If random loading process schematizes by ranges methods, then schematized density of amplitude distribution specifies by formula

$$f_a(x) = \frac{x}{\varkappa^2 S^2}\exp\left(-\frac{x^2}{2S^2\varkappa^2}\right), \tag{10}$$

and average durability, in terms of numbers of positive extremums, is given by equation:

$$\bar{N}_{э+} = 2^{-\frac{sn}{2}} S^{-n}\varkappa^{-n} \cdot \Gamma^{-1}(n/2+1)\gamma, \tag{11}$$

Eq. (11) is transformed in (9) when æ = 1.

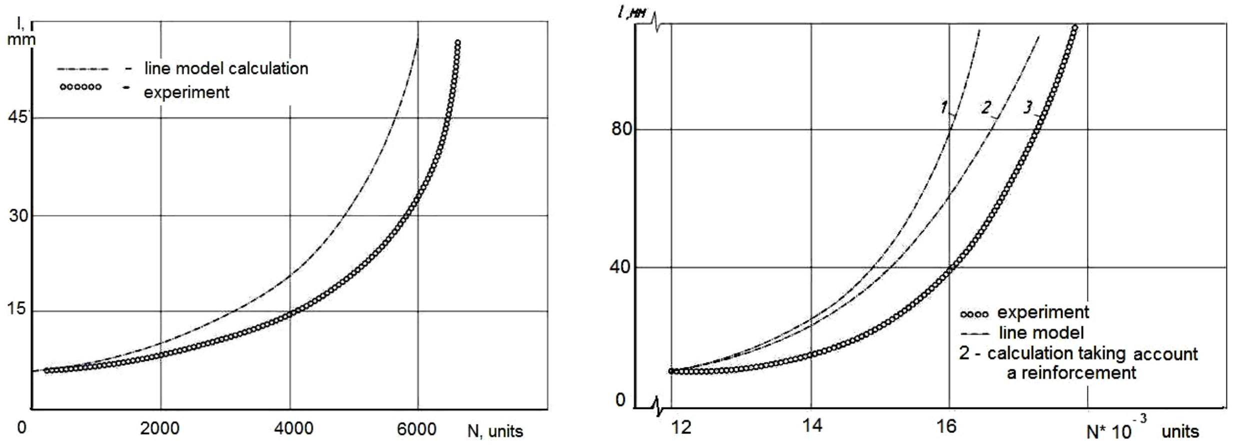
Using Eqs (7), (8) it can be introduced equation for estimations of crack growth period at block-program loading modes

$$N_{БЛ} = \left(\frac{1}{k_1}\sum_{i=1}^{k_1} k_{3i}\Delta\sigma_i^n\right)^{-1}\gamma/k_2. \tag{12}$$

In Eq. (12) k_1 – number of steps in program unit, k_2 – quantity of cycles in program unit, $\Delta\sigma_i$ – the range of stress in program unit i-step, k_{3i} – number of cycles in program unit i-step.

Note, if in the Eqs (9), (11), (12) l_* has variation then it is possible to have fatigue crack grows curves depended on operating time.

Fig. 5, 6 show possibility to use Eq. (12) applicably to different cases of block-programs loading. Fig. 5,a presents experimental and calculated curves of crack grows, based on program unit, which use for fatigue tests of aircraft root chord wing panel smooth sample manufactured from alloy D16chAT. Correlations between stresses maximums in program unit don't exceed 1.25, that's why cycles interactions effects visualize insignificantly, that confirmed by results shown in Fig. 5,a. Correlation between calculated and experimental crack growth period estimations NP/NЭ ≈ 0,88, that gives insignificant margin of vitality.



a) b)

Fig. 5. Modeled and experimental kinetic curves in (a), (b) two cases of programs during "typical flight" loading with considering life-time tests for aircraft wing root nervure area panel.

Fig. 5,b shows calculated results for aircraft crack dangerous zone located in 10-11 nervure area in comparison with an experimental data at block-programs).

In capacity of calculating model to define corrective function φ_3, which accounts element design philosophy, stiffened plate was accepted with width having distance between spars axis (1420 mm). Sheet thickness δ was equal 3.5 mm, stringers step $\tau_{стр}$ = 125 mm, but fasteners step $\tau_{зак}$, their diameter d and another geometrical adjectives defined by proportions:

$$\frac{F_{стр}}{\delta_{стр}} = 1{,}25; \quad \frac{t}{t_{стр}} = 0{,}25; \quad \frac{d}{t} = 0{,}2. \tag{13}$$

Crack initial length was accepted equal 10 mm, and its critical length was 110 mm. Sheets and stringers were made from material D16ATV (σ_B =460 MPa, σ_{02} = 340MPa, E = 73000 MPa, μ = 0,3). To describe correlation between fatigue crack growth speed and SIF range was used equation of Paris-Herdogan (4). Parameters C and m were defined by testing results (at harmonic loading with different cycle asymmetry) of flat samples from the more resistant for cracking D16AT Al-alloy.

In SIF range calculations was considered stringers stress-state influence by inserting corrective function φ_3. Calculation was performed in two variants: plate with stringers stiffening and without stringers influence.

Modified results graphical interpretation shown possibility of fatigue crack grows speed calculations by linear model. At this, given estimations of life-time period have acceptable reserve. (1 case: $N_p/N_Э$ = 0,776, 2 case: $N_p/N_Э$ = 0,928). It should to note also that life-time period estimation accuracy materially increases (≈ 20% up) in case of influence on the fatigue crack kinetic of stiffener elements (stringers).

Equivalent stress range estimation

As soon as in stationary loading Gauss processes influence conditions during base modes cycle interaction minimizing, and curves, which show crack length dependence from cycles quantity or time, are smooth, it is possible to declare allowance of base process modeling by harmonic loading with tension span $\Delta\sigma_{eqv}$ (Fig. 4, c). At this, crack grows duration calculations carrying out by cracks kinetic linear equations Paris-Herdogan type.

Using hypothesis of damages linear summering, it is possible to write

$$\bar{n}_{\Delta} \int_0^{\infty} \frac{f(\Delta\sigma)d\Delta\sigma}{N_*(\Delta\sigma)} = \frac{\bar{n}_{\Pi}}{N_*(\Delta\sigma_{eqv})}, \tag{14}$$

Where $f(\Delta\sigma_{eqv})$ - density of stress range distribution at chosen method of a process schematization, $N_*(\Delta\sigma_{eqv})$ – number of cycles up to fracture at $\Delta\sigma_{eqv}$, n_{Δ} - frequency of cyclic loads (number of zero per unit time, number of extremums, full number of cycles etc), n_{Π} - harmonic tension equivalent frequency. Inserting in (14) proportion (8), receive

$$\bar{n}_{\Delta} \int_0^{\infty} \Delta\sigma^n f(\Delta\sigma) = \bar{n}_{\Pi}\Delta\sigma_{eqv} \tag{15}$$

or

$$\bar{n}_{\Pi}\Delta\sigma_{eqv}^n = \bar{n}_{\Delta}\langle\Delta\sigma^n\rangle. \tag{16}$$

In Eqs (15) and (16) three parameters n, n_{Δ} и n_{Π} used which choice mainly defines according degree of equivalent harmonic and operational loading.

Let be process schematized by spans method. In this case of cycles frequency n_{Δ} accords working loading maximums frequency. Let chose frequency n_n equal to n_{0+} crossings of average load level random process with positive derivative. In this case proportion (16) takes view

$$\Delta\sigma_{eqv} = \varkappa^{-\frac{1}{n}} \sqrt[n]{\langle\Delta\sigma^n\rangle}. \tag{17}$$

For stationary Gauss process at n = 2 independently from irregularity ratio $\Delta\sigma_{eqv} = 2\sqrt{2}S$.

Fig. 6 gives cracks closing ability experimental diagrams confrontation of samples from alloy D16chAT of 3 mm in thick at base stationary loading processes and harmonic loadings with different tension $\Delta\sigma_{eqv}$.

Specimens fracture surface analysis has shown that knee-point on the diagrams correlates with transition from flat-to-slant fatigue crack growth. Cycles of numbers N at diagrams drawing identified with number of average process level crossings (with positive derivative). Diagram (Fig. 6a) for harmonic loading constructed on results of samples tests with $\Delta\sigma_{eqv}$ = 80 MPa ≈ $2\sqrt{2}S$ and M = 70 MPa.

Figure 6b illustrates experimental checks possibilities of application calculating methods, based on calculation ΔK_{eqv} through two first span moments, received by "rain" method process schematization. Diagrams at harmonic loading constructed on tested samples results with amplitudes σ_a = 20 MPa and σ_a = 30 MPa as the nearest to according random processes statistics.

Modified results analysis certifies, that for crack grows duration calculation at working loading, approximated by basic narrowband process, it is possible to use equation type (4) with parameters of crack closing ability c and n, defined at harmonic loading; at this, K_{eqv} should be calculated through $\Delta\sigma_{eqv} = \langle\Delta\sigma\rangle$ or through $\Delta\sigma_{eqv} = 2\sqrt{2}S$. If loading is broadband, then satisfactory upper and lower grack grows speed estimation can be received at calculation ΔK_{eqv} through $\Delta\sigma_{eqv} = \sqrt{\langle\Delta\sigma^2\rangle}$ and $\Delta\sigma_{eqv} = \langle\Delta\sigma\rangle$ accordingly.

a) ● narrowband
○ broadband
□ constant amplitude
$\Delta\sigma_{eq} = 2\sqrt{2}S$

b) ●○ broadband proc
□ constant amplitude
● $\Delta\sigma_{eq} = \sqrt{\Delta\bar{\sigma}_c^2}$
○ $\Delta\sigma_{eq} = \Delta\bar{\sigma}_c$

c) ●○ narrowband proc
□ constant amplitude
● $\Delta\sigma_{eq} = \sqrt{\Delta\bar{\sigma}_c^2}$
○ $\Delta\sigma_{eq} = \Delta\bar{\sigma}_c$

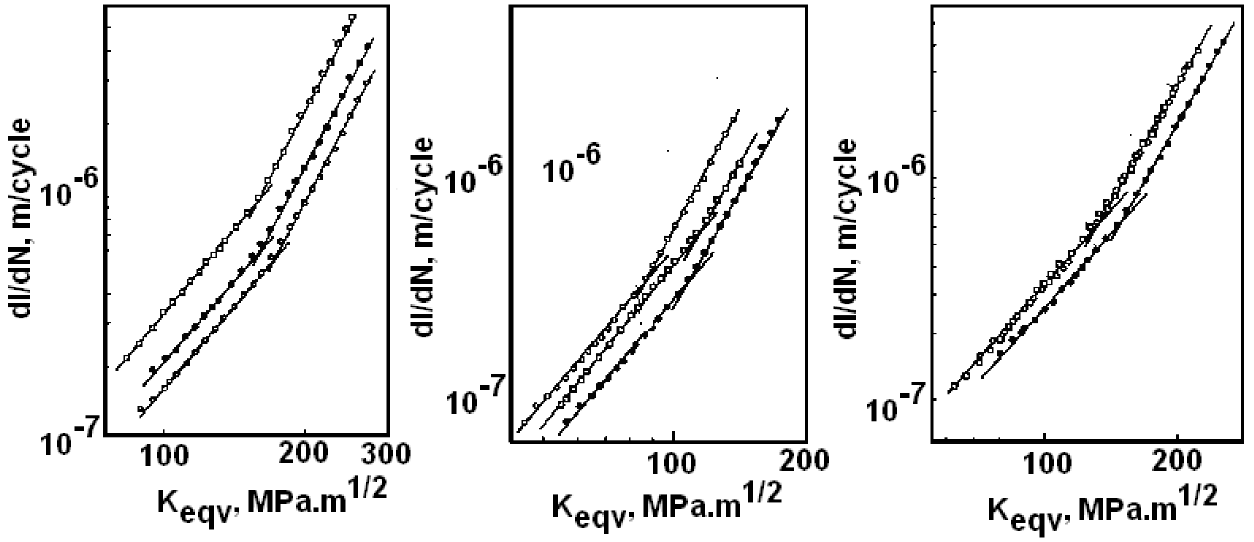


Fig. 6. Kinetic curves for different (a)-(c) cases studied in comparing with data for constant amplitude of cyclic loads

Basic loading processes with the same accuracy for practical usage are modeling by harmonic load, and experiments results at base loading presents as generally accepted diagrams of cycle crack stopping ability. If acting loadings have a gypseous value, then it is possible that calculations will give a conservative lifetime estimation. In the cases of random processes and extremal loads influence on material cracking specified lifetime period estimations based on nonlinear models [3].

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