The effect on the mode I stress intensity factor of plastic dissipation in heat at the crack tip under cyclic loading

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Abstract. Plastic dissipation at the crack tip under cyclic loading is responsible for the creation of an heterogeneous temperature field around the crack tip. A thermomechanical model is proposed in this paper for two problems: (i) an infinite plate with a semi-infinite through crack and (ii) a centre cracked plate specimen, both under mode I cyclic loading. The heat source is assumed to be located in the reverse cyclic plastic zone (RCPZ). The solution of the thermomechanical problem shows that the crack tip is under compression due to thermal stresses coming from the heterogeneous temperature field around the crack tip. The effect of this stress field on the stress intensity factor (its maximum and its range) is calculated. The heat flux within the RCPZ is the key parameter to quantify the effect of dissipation at the crack tip on the stress intensity factor.

Introduction

During a cyclic loading of a crack, the cyclic plasticity is located in the reverse cyclic plastic zone (RCPZ) near the crack tip which was first explained by Paris in 1964 [1] and studied later by Rice in 1967 [2]. The effect of crack closure during cyclic loading [3] must also be considered. In metals, during plastic strain, a significant part of the plastic energy (around 90% [4, 5]) is converted into heat. The dissipated energy in the RCPZ also generates a heterogeneous temperature field which depends on the intensity of the heat source associated with the plasticity and the thermal boundary conditions of the cracked structure [6]. Due to the thermal expansion of the material, the temperature gradient near the crack tip creates thermal stresses which contribute to the stress field in this region. The objective of this work is to quantify the effect on the stress intensity factor of these heterogeneous temperature and stress induced fields for a crack under cyclic loading in mode I.

The proposed paper presents first the main results of Ranc et al. [6] about the theoretical problem of an infinite plate with a semi-infinite through crack loaded in fatigue in mode I where thermal losses due to convection and radiation are neglected. Then experimental data are presented and analysed with a FEA model which consider convective thermal losses. Finally, the consequences of the calculation on the range, the ratio and the maximum and the minimum values of the stress intensity factor are discussed. Some recommendations for further theoretical and experimental investigations are proposed.

General Considerations about Dissipation in Heat at the crack Tip under Cyclic Loading

During fatigue crack growth under cyclic loading, the cyclic plastic strains at each cycle are confined within the RCPZ. A proportion of the plastic strain energy is dissipated in heat and generates a temperature variation. Generally, the size of this RCPZ is very small compared to the crack length and the cracked structure dimensions. To determine the temperature field due to dissipation in heat at the crack tip, Ranc et al. [6] have shown that the hypothesis of a line heat source located within the RCPZ is correct. The temperature variation field obtained with the line heat source and the uniform heat source hypothesis within the RCPZ are very close together outside the RCPZ. Note that the computation of the temperature field within the RCPZ is not needed in this study.

The dissipated power per unit length of crack front is assumed to be proportional to the surface area of the RCPZ and the loading frequency *f* [7] $q = f\xi \; = f\eta r_R^2$, with ξ the dissipated energy per unit length of crack front during one cycle, r_R the radius of the RCPZ and η a material dependent proportionality factor. Since in plane stress or in plain strain the RCPZ radius is proportional to ΔK_I^2, where ΔK_I is the range of variation of the mode I stress intensity factor, it is clear that the dissipated power per unit length of crack front is therefore proportional to the variation of the stress intensity factor to the power four. These results have been already shown analytically [7] and numerically [8].

A constant heat source will be considered in this paper. Furthermore, note that in general the fatigue crack velocity is small, especially when the stress intensity range ΔK is close to the threshold value ΔK_{th}. Since q is proportional to ΔK^4, for a slow moving crack, ΔK and the heat source q can be assumed constant. Moreover, in such case the heat source associated with the fatigue crack propagation can also be considered to be motionless [6].

The Case of an Infinite Plate with a Semi-Infinite Through Crack

The temperature variation field. For an infinite plate with a semi-infinite through crack under mode one cyclic loading with all the previous assumptions, the thermal problem is axisymetric and if the line heat source is along the z axis which is the normal direction to the surface of the plate, the associated heat transfer equation is (1) where ρ is the density of the material, C its heat capacity, λ its heat conductivity and $\delta(r)$ the Dirac function.

$$\rho C \frac{\partial T}{\partial t} = q\delta(0) + \lambda \frac{\partial^2 T}{\partial r^2} \tag{1}$$

Ranc et al. [6] have computed both analytically and numerically (with finite element analysis) the temperature variation field and the stress strain field due to the dissipation in heat in the RCPZ. Between time $t=0$ and time t, the temperature variation field $\vartheta(r,t) = T(r,t) - T_0$ is calculated assuming that before loading the specimen temperature is homogeneous and equal to the room temperature T_0. The key parameter is the heat flux q of the heat source; the temperature variation field and then the stresses are proportional to q.

Figure 1 illustrates the evolution of the temperature variation field for different times according to the radius r from the line heat source. For this calculation, standard thermal and physical properties for steel are used. The density, the heat capacity and the thermal conductivity are taken to be respectively ρ=7800 kg.m^{-3}, C=460 JK^{-1}kg^{-1} and λ=52 Wm^{-1}K^{-1}. The dissipated power per unit length of crack front is chosen equal to the unit (q=1 W.m^{-1}). The curve on Figure 1 shows that the temperature increases abruptly when the radius tends to zero.

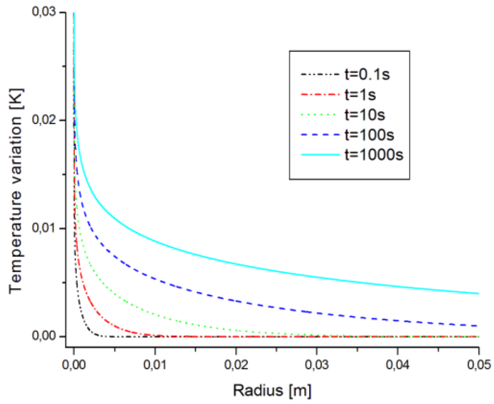

Fig. 1: The temperature variation field near the crack tip for q = 1 Wm^{-1}, for a semi-infinite plate with a semi-infinite through crack with its crack tip at x=0 (note that convection is neglected).

The stress field due to the dissipation in heat at the crack tip. The temperature variation field associated with the heat source in the RCPZ generates a temperature gradient varying with time outside this plastic zone and consequently thermal stresses due to the thermal expansion of the material. The thermo-mechanical problem is supposed to be bi-dimensional because the temperature field is axisymmetric. The material is assumed to be homogeneous and isotropic with an elastic plastic behaviour; plastic strain occurs only in the RCPZ. (cylinder domain with a radius r_R) consequently it is expected in first approximation, that the basic equations of thermo-elasticity will govern outside the RCPZ. Furthermore, the stress boundary conditions are the following. With alternating plasticity for r equal to the radius r_R of the RCPZ the mean stress will tend toward to zero (i.e. mean stress relaxation), so the normal radial stress for $r=r_R$ is null and the radial normal stress when r tends to infinity is equal to zero too.

With the previous assumptions, the authors have computed both analytically (2, 3) and numerically the problem of the semi-infinite plate with a through crack considering a heat source at the crack tip [6]. In these equations *Ei* is the exponential integral function, E'=E and α'=α for plane stress, whereas $E' = E/(1-\nu^2)$ and $\alpha' = \alpha(1+\nu)$ for plane strain.

$$\sigma_r(r,t) = \frac{\alpha' E' aq}{\pi \lambda r^2}\left[\frac{t}{2}\left(e^{\frac{-r^2}{4at}} - e^{\frac{-r_R^2}{4at}}\right) + \frac{1}{8a}\left(r^2 Ei\left(\frac{-r^2}{4at}\right) - r_R^2 Ei\left(\frac{-r_R^2}{4at}\right)\right)\right] \tag{2}$$

$$\sigma_\theta(r,t) = \frac{-\alpha' E' qa}{\pi \lambda r^2}\left[\frac{t}{2}\left(e^{\frac{-r^2}{4at}} - e^{\frac{-r_R^2}{4at}}\right) + \frac{1}{8a}\left[-r_R^2 Ei\left(\frac{-r_R^2}{4at}\right) - r^2 Ei\left(\frac{-r^2}{4at}\right)\right]\right] \tag{3}$$

These radial and circumferential normal stresses are plotted Fig. 2 for two times t=1 s and t=10 s and for r_R=4 μm with the same typical material parameters for steel as for Fig. 1 and with a thermal expansion coefficient of the material $\alpha=1.2\times10^{-5}$ K^{-1}, and q=1 $W.m^{-1}$. Note that the radial stress is always negative because the material is under compression due to the thermal expansion of the material near the crack tip. Near the RCPZ (r_R=4 μm) the circumferential normal stress is negative too because of the temperature and through the circumferential direction, the material is under compression due to the thermal expansion and the constraint effect. Further from this zone, the temperature is lower and the circumferential stress becomes positive (tension) due to the confinement of the material near the crack tip.

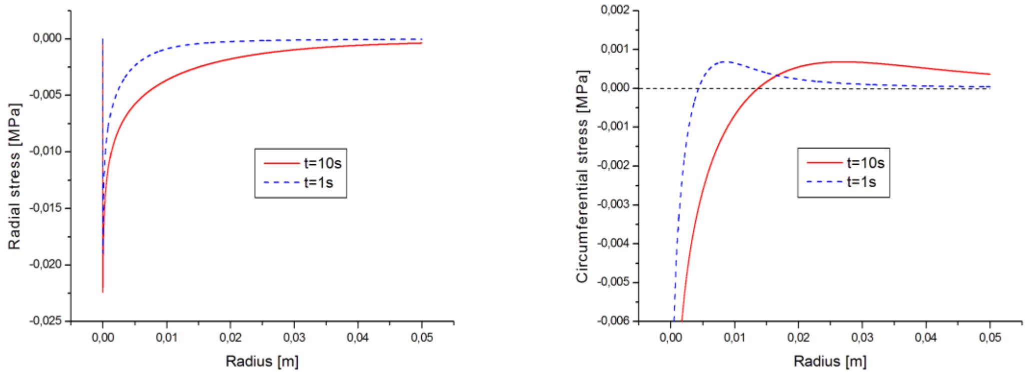

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Fig. 2: The distribution along the radius from the crack tip of a) the radial normal stress and b) the circumferential normal stress due to temperature for various times for a unit heat source q = 1 Wm^{-1} and r_R = 4 µm.

The effect of the thermal stresses on the stress intensity factor under cyclic loading. Within the heterogeneous stress field due to the thermal stresses, for the theoretical case of an infinite plate with a semi-infinite through crack along a radial line from r_R to $+\infty$, the associated stress intensity factor, $K_{I,temp}$, due to the temperature gradient (i.e. thermal correction on K_I) can be determined from the wedge force (Green's function) solution (see [10] page 87) as:

$$K_{I,temp}(t) = \sqrt{\frac{2}{\pi}} \int \frac{\sigma_\theta(r,t)}{\sqrt{r - r_R}} dr. \tag{4}$$

This integral was computed both analytically and numerically by Ranc et al. [6]. The analytical solution using hypergeometric functions shows that both in plane stress or in plane strain, $K_{I,temp}$ is proportional to the heat flux q. The analytical solution being too long for this conference text, the reader will go to ref. [6] for details. The evolution of $K_{I,temp}$ versus time is illustrated Fig. 3 for a unit line heat source and the same previous typical material parameters for steel. For instance, after times of 10s and 100 s the value of $K_{I,temp}$ is respectively -1.2×10^{-3} MPa√m and -2.1×10^{-3} MPa√m for a RCPZ radius of 4 µm and q=1 Wm^{-1}. These values are negative because the temperature field generates compressive circumferential normal stresses near the crack tip.

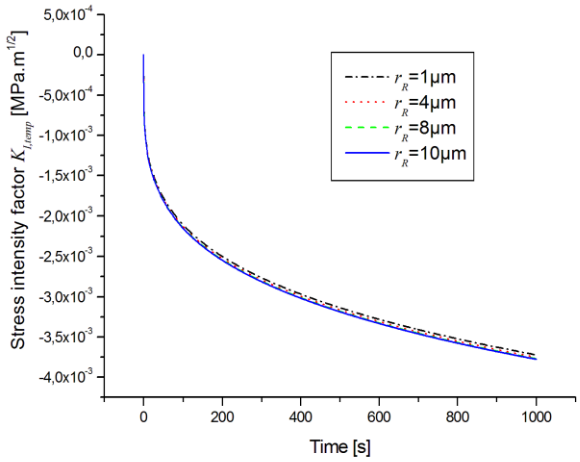

Fig. 3: The stress intensity factor $K_{I,temp}$ due to thermal stresses versus time for different radius of the RCPZ.

Figure 3 shows that $K_{I,temp}$ is not very sensitive to the size of the RCPZ. This is due to the very large dimensions of the plate (infinite plate in this case), compared to the size of the RCPZ. The thermal boundary conditions do not consider any heat exchange between the specimen and the environment that is the reason why no thermal equilibrium is reached even after a long time. Further theoretical work in the analytic solution has to be done to take this phenomenon into account for being representative of small specimens (finite dimensions) for which thermal equilibrium is reached when a fatigue crack growth test is running during several hours in laboratory. This is the case of slow fatigue crack growth typically when the range of the stress intensity is close to the threshold.

As written before, due to the compressive thermal stresses around the crack tip, it has been shown that the stress intensity factor during a cyclic loading has to be corrected by the factor $K_{I,temp}$. This correction factor would be a value superimposed on the usual stress intensity factor due to the fatigue cyclic loading noted $K_{I,cyc}$ in mode I: $K_I(t)=K_{I,temp}(t)+K_{I,cyc}(t)$. $K_{I,temp}$ varies with time but can be considered as constant for long time. In the very beginning of cyclic loading, the value of $K_{I,temp}$ is small compared with $K_{I,cyc}$ and $\Delta K_I(t) \approx \Delta K_{I,cyc}$. There is no significant effect of the temperature on the range of the stress intensity factor per load cycle. For long time (*t>>0),* $K_{I,temp}$ can be considered as constant during a loading period at usual testing frequency (10 Hz and more). Consequently the temperature has no effect on $\Delta K_I(t)$ but it has an effect on $K_{I,max} = K_{I,cyc,max} + K_{I,temp}$ and $K_{I,min} = K_{I,cyc,min} + K_{I,temp}$, where $K_{I,min}$ and $K_{I,max}$ are the minimum and the maximum value of $K_I(t)$ over a loading period. However, $K_{I,temp}$ can affect crack closure interpretation by changing the load ratio $R_K = K_{I,min} / K_{I,max}$. Indeed, the R_K ratio is affected by the temperature correction:

$$R_K = \frac{K_{I,min}}{K_{I,max}} = \frac{K_{I,cyc,min} + K_{I,temp}}{K_{I,cyc,max} + K_{I,temp}} \neq \frac{K_{I,cyc,min}}{K_{I,cyc,max}} \tag{5}$$

The evaluation of this correction needs a precise quantification of the heat source associated with the plastic dissipation and the thermal boundary conditions on the surfaces of the plate (heat losses if any). Experimental measurements of the temperature field, for instance by using pyrometry technique, need to be carried out in this way.

The Case of a Finite Cracked Plate with a Finite Through Crack

Experimental conditions. Fatigue crack growth tests were carried out on a centre cracked plate specimen (132 mm width, 400 mm long, 4 mm thick, with a central initial crack *2a*=26.6 mm) made of the mild steel C40 with the following mechanical characteristics (E= 210 GPa, $Rp_{0.2}$= 350 MPa, UTS = 600 MPa, ν=0.29). A resonant fatigue testing machine (vibrophore) was used at a loading frequency of ~100 Hz. The crack growth was measured with a high resolution optical digital camera and the temperature field at the specimen surface was recorded with an infrared camera (CEDIP Jade III, spectral range between 3.9 and 4.5 µm). The acquisition frequency and the aperture time of the IR camera were respectively 5 Hz and 1100 µs. In order to neglect the effect of the surface emissivity on the determination of the temperature, the specimen was covered with a fine coat of mat black paint. At the loading frequency of 100 Hz it was not possible to control the stress intensity range ΔK in real time, that is why the test was conducted step by step under force control. The amplitude and mean values of the tension sinusoidal force was calculated at the beginning of each loading block for obtaining the desired ΔK and R ratio, depending on the initial crack length. The measure of the crack length at the end of each loading block allowed us to compute the evolution of ΔK because of the crack propagation. If this increasing was smaller than 7% the block is considered as valid in agreement with ASTM standard. At the end of each loading block the specimen is unloaded and the specimen cools freely under the effect of natural convection in the laboratory.

Temperature variation field. The temperature evolution versus time at a point exhibits a superposition of a high frequency evolution of the temperature due to the thermo-elasticity and a low frequency signal evolution due to the energy dissipated in heat in the RCPZ. This last effect is only of interest in this paper since this is due to dissipation. The total increase of the temperature in the specimen between the beginning and the end of loading block corresponds to the heat source related to the plastic dissipation in the RCPZ. This heat diffuses in all the specimen and generates an increase of its temperature. Figure 4a shows the temperature variation field around the crack tip between the beginning and the end of block of 690 s under ΔK=20 MPa.√m with R_K=0.1. The total increase in the temperature is about 2.5 °C at 5 mm in front of the crack tip. It has to be noted that for this material and the investigated loaded conditions the self-heading on a smooth uncracked specimen was measured, it is neglected (under 0.2°C).

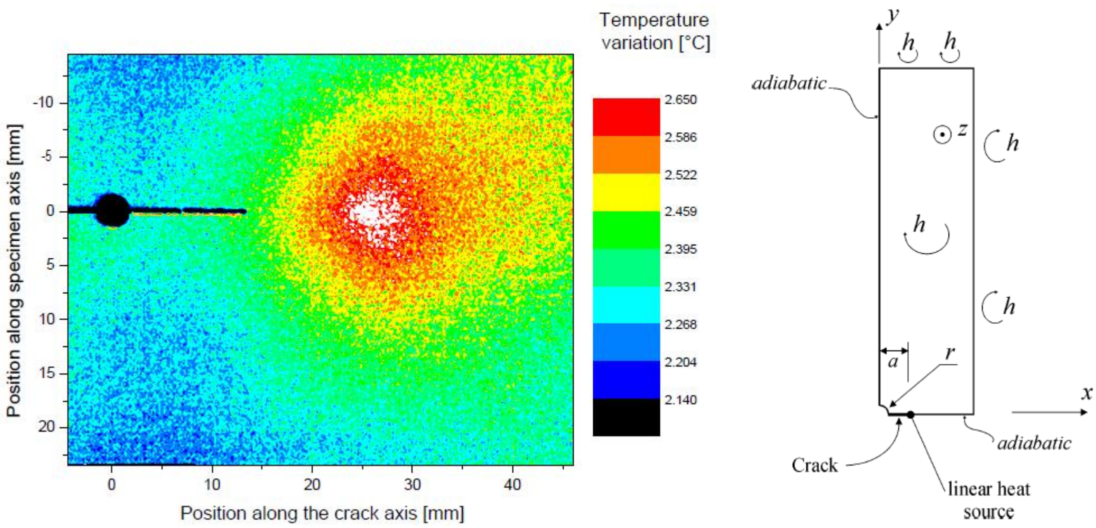

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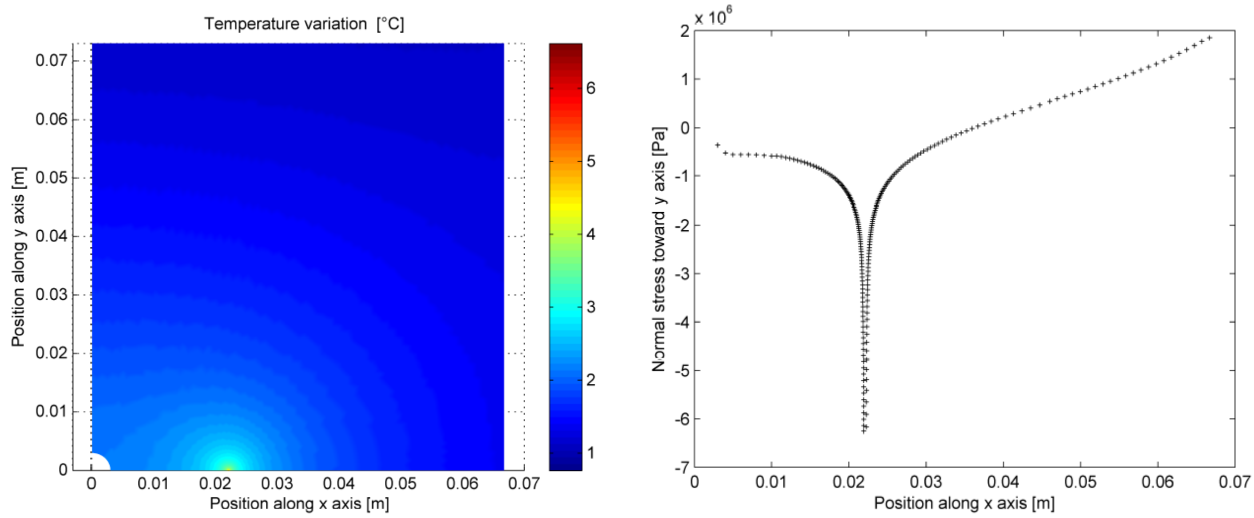
Fig. 4.: a) Temperature variation field measured near the crack tip after 690 s under ΔK=20MPa√m, R = 0.1), b) geometry and boundary conditions of the simulated thermal problem

Estimation of the power dissipated in heat in the RCPZ. In order to estimate the power dissipated in heat in the RCPZ, a numerical (FEA) thermo-mechanical model of the specimen was carried out. Due to the symmetries of the problem only one forth of the specimen was modelled (Fig. 4.b). The temperature was supposed to be homogeneous in the thickness of the specimen because the Biot number is small compared to one in this configuration [6]; consequently a two dimensional model was used. Heat losses by convection were taken into account with the appropriate boundary conditions all around the specimen (edge of the specimen and on the specimen faces); whereas radiative losses were neglected. The dimensions of the simulated specimen are similar to those of the tested specimen. The convective heat transfer coefficient, h, was taken equal to 10 $W.m^{-2}.K^{-1}$. For the C40 steel, the density, the heat capacity and the thermal conductivity are taken equal to respectively 7800 $kg.m^{-3}$, 460 $J.K^{-1}.kg^{-1}$ and 52$W.m^{-1}.K^{-1}$. The initial temperature of the specimen was assumed to be homogeneous and the ambient temperature is taken both equal to 20°C.

Since the aim of the numerical model is not to compute the temperature and the stress/strain inside the RCPZ but outside only, and because the radius of the RCPZ was small compared to the crack length and to the specimen dimensions in our experiments, the dissipated power in heat within the RCPZ was modelled by a linear heat source along the crack front [6]. Furthermore, the thermal problem is linear, so a line heat source of 1 $W.m^{-1}$ was applied. The thermal problem is consider as stationary and independent of time too. Note that this assumption is only correct for the tests with a stress intensity factor range low enough for which the crack growth (da/dt not da/dN) is small compared to the characteristic time of the heat transfer. In the future, a transient calculation could be carried out.

Figure 5a shows the temperature variation field calculated on the specimen surface for a crack length *a*= 21.9mm. This geometrical configuration corresponds to the beginning of the loading block with ΔK=20 MPa.√m and R=0.1. In this case, at a distance of 5mm from the crack tip, the calculated temperature increase is of 0.0163°C for a unit dissipated power. Since for this value of ΔK the temperature variation measured by the IR camera at the corresponding distance from the crack tip is about 2.5°C, the estimation of the dissipated power is of 153 $W.m^{-1}$ (per unit of specimen thickness, thus 0.612 W for 4 mm).



-a- -b-

Fig. 5: For ΔK=20MPa√m and R = 0.1, simulated a) temperature field on the specimen surface for a unit heat source and b) σ_{yy} along the x axis from the crack tip

The stress field and the stress intensity factor due to the heterogeneous temperature variation field. Because of the specimen dimensions and loading, this problem is supposed to be in plane stress. Furthermore as assumed by Ranc et al. [6], with cyclic plasticity in the RCPZ, the mean stress will tend toward to zero. That is why the boundary condition in the RCPZ radius is that the radial normal stress is equal to zero [6]. Also only the stress field outside of the RCPZ is of interest to compute the stress intensity factor due to the thermal stresses. That is why, a thermo-elastic behavior law is used as first approximation outside the RCPZ. The density and the thermal expansion coefficient of the material are respectively equal to 7800 $kg.m^{-3}$, and 1.2×10^{-5} K^{-1}.

This thermo-mechanical problem can be decomposed into two problems: the first one (pure mechanical problem) is the classic cracked specimen under the mode I cyclic loading without heat source. The stress field associated with this classic problem leads to the usual mode I stress intensity factor $K_{I,cyc}(t)$. The second problem (pure thermal problem) is the cracked specimen subjected to the line heat source q. The thermal stresses associated with this thermal loading are responsible of the stress intensity factor, named $K_{I,temp}$. As demonstrated in [6] the thermal effect generates a compressive stress field (Fig. 5b) near the crack front and thus creates a negative contribution on the stress intensity factor ($K_{I,temp} < 0$). The calculation of the stress intensity factor $K_{I,temp}$ was done by using the Green's function [10]:

$$K_{I,temp} = \frac{2}{\sqrt{\pi}} \int \sigma_{yy}(u) \frac{\sqrt{a}}{\sqrt{a^2 - u^2}} du \,. \tag{6}$$

For the test shown Fig. 5a, the stress intensity factor due to the temperature field $K_{I,temp}$ is

$-0.32\,MPa\sqrt{m}$. Other values of $K_{I,temp}$ are given in Table 1. These values are small. Other tests have to be carried out especially with higher R ratio to avoid any crack closure (not considered here).

ΔK [MPa√m]	Temperature variation 5mm in front of the crack tip [°C]	Radius of the RCPZ [µm]	Dissipated power [W.m^{-1}]	$K_{I,temp}$ [MPa√m]
12	0.3	58	17.7	-0.04
15	1.0	90	62.2	-0.14
17	1.4	116	84.1	-0.18
20	2.5	160	153	-0.32

Table 1: Temperature variation 5 mm in front of the crack tip, radius of the RCPZ, dissipated power and stress intensity factor correction due to the temperature for 4 stress intensity factor ranges.

Conclusions and Prospects

It has been shown that under cyclic loading the effect, on the stress intensity factor in mode I, due to the heat source at the crack tip within the RCPZ is proportional to the heat flux. The quantification of this, is a key factor which is probably depending on the material behaviour (plasticity, visco-plasticity if any). Another consequence of the thermal stresses is due to the fact that q is proportional to ΔK^4 [7,8]. When ΔK is changing significantly, for instance from the threshold 5 MPa√m up to 50 MPa√m (10 times more), the effect on the heat source is 10^4 times! The effect on the correction due to thermal stresses is thus significant. Furthermore, since the value of q is also proportional to the loading frequency, a frequency effect on the crack growth may be also linked with the heat source. This opens interesting investigations for further studies.

However, it has to be kept in mind that for the two problems considered here the heat source was assumed to be motionless. This means that the proposed solution is physically correct for slow crack growth. This is the case when ΔK is close to the threshold value. For instance, with a crack growth rate ~10^{-9} m/cycle at a loading frequency between 1 Hz and 100 Hz the velocity of the crack tip (i.e. heat source velocity) is between 10^{-6} mm.s^{-1} and 10^{-4} mm.s^{-1}. This means that for a crack with a characteristic length between 1 mm and 10 mm that the Peclet number is small compared to the unity. In such a case, the motionless heat source hypothesis is correct and all the proposed results correct too. According to the authors, considering the effect of thermal stresses may be a very important point for studying the crack growth close to the threshold and for the physical phenomena including crack closure and frequency effect. Experimental investigations have to be carried out to quantify the heat source at the crack tip which should be a key factor in fracture mechanics. Further studies should also be carried out in thermo-mechanics to consider more precisely the temperature field effect on fracture mechanics considerations.

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