The Damage Mechanics Approach and of Bone Considering the Mechanical-Electric-magnetic Coupling Effects

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Abstract. *The damage behavior of bone tissues has attracted many interests of those scientists and engineers due to their load bearing function. Our daily life can bring a large number of cyclic mechanical loadings to the brittle bone and make it very vulnerable to the fatigue damage and fracture. Fortunately, bone, as a living tissue, it can repair itself. It means that our bones can operate under conditions of high loading, subjected to stresses and strains that cause damage, which, if not repaired, would lead to failure in a relatively short time. This renewing process is performed by bone remodeling. On the other hand, bone remodeling is far more than repairing the damaged bone tissues. It can adapt its internal microstructure and subsequently its associated mechanical properties to its specific mechanical and physiological environment. So the bone damage behavior can be easily influenced by many factors as long as these factors can affect bone remodeling or damage accumulation. Besides mechanical loadings, electromagnetic fields and hormones also play an important role in bone functional remodeling process. But what effect they have on the bone damage behaviours is still an opening question. In the present paper, we establish a mechanism-based damage model to rationalize the damage evolution law of bone under simultaneously applied mechanical and electromagnetic fields. Some crucial physical and biological mechanisms underlying the sophisticated damage, remodeling and healing process of bone tissues have been incorporated in the model. Especially, an evolutionary law of bone damage and the corresponding modeling methodology are presented to analyze the complicated damage/repair behavior of bone under multi-fields. Then the numerical examples are considered to verify the present theoretical model and the influence of those mechanical and non-mechanical factors on the bone damage and healing are simulated based on the new damage model. Those factors include mechanical overload, electromagnetic field, mechanical disuse, estrogen deficiency and bisphosphonate.*

Introduction

Bone is a rigid tissue which is subjected to complicated and varying mechanical loadings during its whole life. Repeated loads of a high magnitude may cause progressive damage or even failure of bone through a process known as fatigue, as in engineering materials. Fortunately, bone has an amazing self-repairing property to enable it operate under certain conditions of stresses and strains that are sufficiently high to cause damage. It has been widely accepted that damage repair is performed by a continual process called remodeling [1, 2,3]. Bone continuously checks the presence of cracks and other types of damage to its structure, and deliberately replaces damaged regions with new material by remodeling. In addition to removing fatigue damage, the remodeling process of bone can adjust its microstructure in response to imposed loads. Therefore, a certain amount of microdamage is not only a negative aspect for bone as a supporting material but, sometimes, a positive aspect to strengthen and renew the bone also. Based on these fundamental understandings, some models have been developed to predict the damage, fracture, and implant loosening of bone [6,7,8].Recently, much effort has been directed towards investigating the response of bone tissue to an extremely low frequency electromagnetic field. Electromagnetic stimulations of bone have been shown to promote osteogenesis both in experiments using cultured cells [9,10] and in vivo experiments using animals [11]. In addition, electromagnetic field-based techniques have been applied in the treatment of skeletal diseases such as osteoporosis, tendonitis, osteonecrosis, fracture and nonunion. Therefore, it is reasonable to presume that the damage and remodeling behavior of bone tissue could be obviously altered by the presence of an electromagnetic field. There is, however, a shortage of theoretical model that can simulate the complicated remodeling process of bone under simultaneous mechanical, electrical, and magnetic fields. In this paper, an attempt is made to establish such a model for the damage/repair process of bone based on the concepts of continuum damage mechanics [12] and the understanding of the biophysical mechanisms of bone remodeling and healing. The coupling and interaction effect of mechanical, electrical, and magnetic fields are integrated into the model. Several illustrative simulations are presented to validate the proposed theoretical model.

Bone damage model

In this section, the main concepts and the equations of the bone damage model will be described. The adaptation of the internal structure of bone is formulated in terms of bone porosity and damage level, and the bone tissue modeled as a structural material is characterized by several internal variables.

First, an internal scalar variable is defined to rationalize the macroscopic degradation of mechanical properties induced by the microscopical structural damage, as in continuum damage mechanics pioneered by Kachanov [12]. Considering the fact that the microstructural damage of bone always induces a loss of elastic modulus, the following damage factor is used to quantify the magnitude of bone damage [1,2]

$$D = 1 - \frac{E}{E_0} \tag{1}$$

where E and E_0 denote the elastic moduli of bone with and without damage, respectively. Following the suggestion of Martin [5], the fatigue damage accretion rate is

$$\frac{\mathrm{d}D}{\mathrm{d}t} = \frac{\mathrm{d}D_f}{\mathrm{d}t} - \frac{\mathrm{d}D_h}{\mathrm{d}t} \tag{2}$$

where D_f and D_h represent the formation and removal of fatigue damage, respectively. Martin [5] assumed that the changing rate of D_f is proportional to the product of the strain range (ε, microstrain) raised to a power and the loading rate (R_L, cycle number per unit time), i.e.,

$$\frac{\mathrm{d}D_f}{\mathrm{d}t} = k\varepsilon^{q_1} R_L \tag{3}$$

In general, the stress-induced damage of bone involves the nucleation, growth, and coalescence of microcracks. The microcracks can be repaired when they are sensed by osteocytes. The osteocytes can also distinguish cracks of different sizes [13,14]. Here it is hypothesized that only when the damage level is high enough to break the network of the osteocytes, it can be sensed and the repair mechanism can be triggered. Therefore, let D_s denote a damage level that can be detected by the bone tissue. One has

$$D_s = \begin{cases} 0 & \text{when } D \le D_0 \\ D - D_0 & \text{when } D > D_0 \end{cases} \tag{4}$$

where the threshold D_0 is the minimal damage that can be detected. Here the value of D_0 is related to the mechanical loadings via the following relation

$$D_0 = (1 - e^{k_1 \varepsilon})D \tag{5}$$

Once the damage of bone is detected, a signal will be released to trigger the damage removal and repair process. The remodeling task is achieved by the so-called basic multi-cellular units (BMUs) composed of tissue-resorbing osteoclasts and tissue-forming osteoblasts, which work in a coupled manner. The osteoclasts resorb the damaged tissue, and the osteoblasts refill the induced cavities. Here the number of the BMUs resorbing the damaged tissue is expressed by

$$N_R = N_{R(\max)} \left[1 - \exp\left(\frac{k_r D_s}{1 - D_s} \right) \right] \tag{6}$$

where $N_{R(\max)}$ is the maximum number of the BMUs that can be produced by the bone tissue. Then the number of the refilling BMUs is written as

$$N_F = k_f N_R \tag{7}$$

where the coefficient k_f correlates the refilling and the resorbing processes. In this study, k_f is taken in the form of a piecewise function:

$$k_f = \begin{cases} c_0 & \Phi_L \le \Phi \le \Phi_U \\ c_0 + (c_{\max} - c_0)\left[1 - e^{k_s(\Phi - \Phi_U)}\right] & \Phi > \Phi_U \end{cases} \tag{8}$$

where $k_s = -1.8$ defines the shape of the curve. The parameter Φ stands for the intensity of the environmental stimulus and is expressed as

$$\Phi = C_{ij}\varepsilon^{q_2} R_L + (C_i E_i + G_i H_i) f_e \tag{9}$$

where C_{ij}, C_i and G_i are the damage rate coefficients, ε, E_i and H_i are the intensities of the applied strain, electrical field and magnetic field, respectively. The exponent q_2 is set as a nominal value of $2/3$. The mechanical loading rate, R_L, is assumed to be 3000 cycles per day (cpd), and f_e is the frequency of the electromagnetic field. Considering that the quantity of the growth factors retained in osteocytes changes along with the environmental loads, as aforementioned, Φ_L and Φ_U can be considered as the thresholds of remodeling and modeling, respectively. In the range of $\Phi_L \le \Phi \le \Phi_U$, the growth factors keep constant ($k_f = c_0 = 1.0$) and the bone tissue is in the remodeling state. When $\Phi > \Phi_U$ (the upper limit of Φ), more growth factors will be generated, resulting in a process of bone modeling. In equation (9), the parameter $c_{\max}$ denotes the maximum number of the BMUs that can be produced by bone tissues. With the refilling of the cavities by osteoblasts, the damage of bone will be gradually healed. It is reasonable to assume that the damage healing rate is proportional to the number of BMUs that participate in the refill process, that is,

$$\frac{\mathrm{d}D_h}{\mathrm{d}t} = k_h N_F \tag{10}$$

where $k_h = 1.5 \times 10^{-3}$.In addition, it is well known that the mechanical property of bone sensitively depend on its porosity and the latter varies with the accumulation and repair of damage. Therefore, the porosity, p, is another internal variable to describe the relative change of mass density of bone. The elastic modulus is approximately related to the porosity by [12]

$$E_0 = (8.83\times10^5)p^6 - (2.99\times10^6)p^5 + (3.99\times10^6)p^4 - (2.64\times10^6)p^3 + (9.08\times10^5)p^2 - (1.68\times10^5)p + 2.37\times10^4 \quad (11)$$

As proposed by Hart, Davy and Martin [12], the changing rate of porosity is also a function of the mean bone resorbing (Q_R) and refilling (Q_f) rates of each BMU, as well as the density of resorbing (N_R) and refilling (N_F) BMUs/area. Then, one has

$$\frac{\mathrm{d}p}{\mathrm{d}t} = Q_R N_R - Q_F N_F \quad (12)$$

Thus provided that the initial condition of bone and the varying multi-field loadings are specified, both the damage factor D and the porosity p can be solved from this theoretical model. The above equations constitute a closed system describing the dynamic and non-equilibrium process of damage evolution. Especially, the constitutive model established in this model is based on the coupled first-order nonlinear differential equations in (2) and (12). The environmental stimulus Φ defined in equation (8) is regarded as the loading function determined by the externally applied mechanical, electrical, and magnetic loads. Thus the bone damage and remodeling with the effects of mechanical, electrical, and magnetic field loadings can be described by the following evolutionary equations of the damage factor D and the porosity p:

$$\begin{cases} \dot{D} = \Im[☎(\varepsilon, E_i, H_i), D, p] \\ \dot{p} = \Re[☎(\varepsilon, E_i, H_i), D, p] \end{cases} \quad (13)$$

The Damage and Healing of Bone in the Disuse State

It is shown by the above research that the influence of disuse status on bone density is very great, but its effect on bone injury is positive. We know that the bone quality, bone density and bone injury quantity presently more cared by people are related. Electromagnetic stimulation undoubtedly has effect on reconstruction of bone, we know from the that reconstruction of bone and bone injury are closely linked, electromagnetic stimulation has stimulation effect on reconstruction of bone, it would certainly have considerable influence on bone injury, what is the influence of electromagnetic field on bone injury? It the influence positive or negative? How shall we utilize the influence to improve the quality of rehabilitation process of osteoporosis? Those are problems that need to be solved . At present, the research in that aspect is sparse. Therefore, we need to establish the following model: The model for bone injury under effect of multiple physical fields that takes into consideration the repair effect was preliminarily established in authors paper [1,2,3], some injury behavior of bone under the circumstance of overload was investigated on that basis, and the influence of multiple field coupled load on the reconstruction behavior of bone injury was approximately understood. We know from the Ref.[1,2,3] that there are three induction factors for

reconstruction of bone: overload, disuse and hormone influence. Refer to the previous paper 0f authors [1,2] for the definitions and values of variables in each of the above expressions.

$$k_f = \begin{cases} c_0 & \Phi_L \le \Phi \le \Phi_U \\ c_{\min} + (c_0 - c_{\min})\left(\dfrac{\Phi}{\Phi_L}\right)^n & \Phi < \Phi_L \\ c_0 + (c_{\max} - c_0)(1 - e^{k_s \Phi}) & \Phi > \Phi_U \end{cases} \tag{14}$$

k_f is still the relationship coefficient between bone absorption and osteogenesis, but compared with the definition in the previous chapter, in order to research the reconstruction of bone and its injury under disuse status, we have additionally introduced an item, i.e., the value when $\Phi < \Phi_L$, where Φ is the external load stimulation factor,

Influence of estrin and bisphosphonate on bone injury and the improvement effect of electromagnetic field

We shall analyze the reconstruction of bone caused by hormonoprivia in this paper, which mainly includes osteoporosis. Estrin has extremely important effect for maintaining the balance between bone absorption and bone formation. Estrin can limit bone conversion through the estrin receptor of osteoblast and osteoclast. That kind of limitation begins to lose when estrin is deficient, then the entire bone conversion is increased. we can get the equation for injury quantity D and porosity p changing with time by the same means:

$$\begin{cases} \dfrac{dp}{dt} = Q_R N_R - Q_F N_F \\ \dfrac{dD}{dt} = \dfrac{dD_f}{dt} - \dfrac{dD_h}{dt} \end{cases} \tag{15}$$

$$\frac{dD_f}{dt} = k\varepsilon^q R_L \tag{16}$$

$$\frac{dD_h}{dt} = k_h N_F \tag{17}$$

$$N_R = k_b N_{R(\max)}(1 - e^{\frac{k_r D_s}{1 - D_s}}) \tag{18}$$

where the expression of N_R is different from that in the paper [2], a coefficient k_b was added,

$0 \le k_b \le 1$representing the inhibition coefficient of bisphosphonate on osteoclast, when bone absorption is fully inhibited, $k_b = 0$, when there is no bisphosphonate inhibitor, $k_b = 1$. The relationship between k_b and the dosage of bisphosphonate is as follows:

$$k_b = e^{C_b \Delta} \tag{19}$$

where Δ represents the dosage of bisphosphonate that is used, C_b represents the shape definition parameter. A coefficient is added here, since the influence of bisphosphonate on reconstruction of bone mainly comes from its inhibition on bone absorption. The influence of estrin on the entire reconstruction of bone is embodied in the following expression

$$N_F = k_e k_f N_R \tag{20}$$

where N_F is different from the definition in the previous chapters, a parameter k_e is added. k_e is the dimensionless representation of number difference between BMU in status of bone absorption caused by estrin reduction and BMU in status of osteogenesis. The relationship between k_e and estrin reduction is shown as

$$k_e = 1 - k_{\max}^e \left(1 - e^{\frac{C_e \delta}{1-\delta}} \right) \tag{21}$$

$0 \le \delta \le 1$ is also a dimensionless quantity, representing the extent of estrin reduction; C_b represents shape definition parameter; $k_{\max}^e$ represents maximum quantity of reduction in k_e. δ can be represented as the function of t:

$$\delta = 1 - e^{C_y t} \tag{22}$$

where C_y is estrin attenuation coefficient, which defines the rate of estrin attenuating with time. We know that the influence of estrin reduction on reconstruction of bone comes from two aspects: weakening the inhibition to bone absorption and lowering stimulation effect to osteogenesis. In fact, that would produce the same effect, therefore, we have an attenuation coefficient for the relationship coefficient between osteogenesis and bone absorption. k_f is still the relationship coefficient between bone absorption and osteogenesis, and it reflects the

relationship between reconstruction of bone and external load. Since the simulation at that time needs to use two circumstance, including overload and stimulation insufficiency, we select and use those used in the previous section

$$k_f = \begin{cases} c_0 & \Phi_L \le \Phi \le \Phi_U \\ c_{\min} + (c_0 - c_{\min})\left(\dfrac{p}{p_0}\right)^n & \Phi < \Phi_L \\ c_0 + (c_{\max} - c_0)\left(1 - e^{k_s \Phi}\right) & \Phi > \Phi_U \end{cases} \tag{23}$$

we can get the equation for injury quantity D and porosity p changing with time by the same means:

Numerical examples

To illustrate the application of the above model in analyzing the damage and remodeling process of bone, we will present several representative examples under different loading conditions in what follows. The model has been implemented using a forward Euler scheme to integrate equation (13) with respect to time. For the sake of simplicity, the bone is considered as a cubic bone section subjected to a uni-axial compressive pressure P and pulsed electromagnetic loads. The side length of the cube, a, is 1.0 cm. it is assumed that the strain ε, the electric field E_i and the magnetic field H_i all return to zero at the end of each load cycle, their changing ranges and peaks are the same for all the load cycles. An initial porosity of 4.43% is specified, which allows the equilibrium between the Haversian canals removed and added by new BMUs [12]. A time increment of 0.5 day is set in each simulation step to integrate equations (2) and (12). In this study, the damage behavior of the bone in its remodeling and modeling state will be simulated andthe disuse-mode remodeling will be excluded. To show the influence of multi-field loading on bone damage the following loading cases are investigated: overloading and extreme overloading.

Variation of damage and porosity for extreme overload

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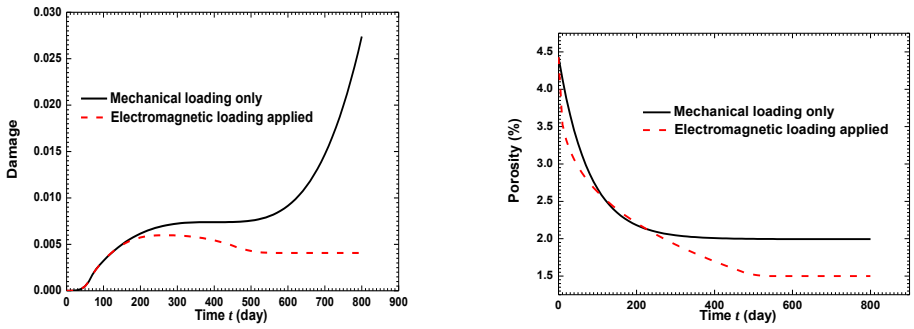

Fig. 1: Variation of damage and porosity for extreme overload ($P = 2.0\,\mathrm{KN}$)

Numerical results show that although the mechanical loadings can inevitably cause fatigue damage in bone materials, bone tissues can resist the damage accumulation and control the damage level to a sustainable value. This amazing ability ensure the bone a much longer life when it operates under conditions of high loading, subjected to stresses and strains that cause damage if not repaired.

Influence of electromagnetic field on bone injury behaviour under the disuse status and in the rehabilitation process

It was mentioned before that physiotherapy was another method for treating disuse osteoporosis, i.e., the treatment aim can be reached by means of electromagnetic stimulation. Then what is the influence of electromagnetic field on bone injury? We shall firstly research the influence of electromagnetic field on bone injury behaviour under the disuse status, an electric field of 5V/m and 10V/m is respectively exerted onto the bone tissue under disuse status, then compare with that without exertion of electric field, and the result is shown in Figure 2 and Figure 3.

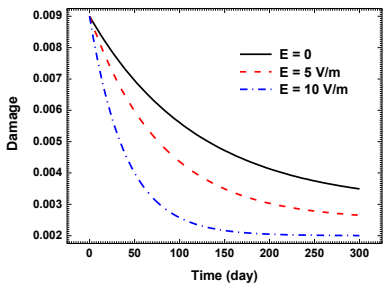

Figure 2 Diagram of bone injury variable D changing with time under the disuse status with different electric field

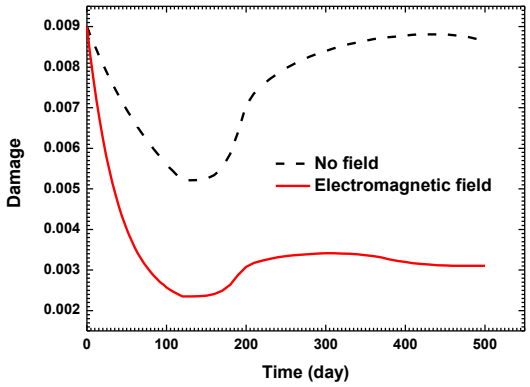

Figure 3 Effect of electromagnetic field in the rehabilitation process of osteoporosis

Influence of bisphosphonate and hormone replacement therapy on treatment of osteoporosis and the improvement effect of electromagnetic field

The variables and parameters newly used in the model can be shown in Table 1

Table 1 The variables and parameters newly used in the model

Exsion	System variable	
Δ	Dosage of bisphosphonate	
δ	Reduction quantity of estrin	
	System constant	**Constant value**
$k^e_{\max}$	Maximum difference between bone absorption and osteogenesis caused by estrin reduction	0.5
C_b	Action coefficient of bisphosphonate	-0.16
C_e	Action coefficient of estrin	-0.2
C_y	Attenuation coefficient of estrin	-0.15

Besides the cause in physiological aspect, such as hormone, reduction in amount of exercise is also a very important cause for senile osteoporosis. In order to show that influence more directly, we select equivalent modulus of elasticity $E = E_0(1-D)$ for reflecting bone

quality, and the result is shown in the diagram.

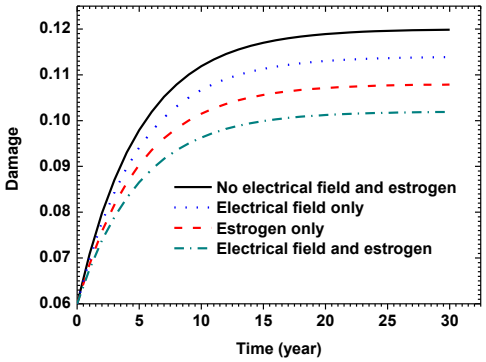

Figure 4 Influence of electromagnetic field on the bone injury of senile osteoporosis and hormone replacement therapy

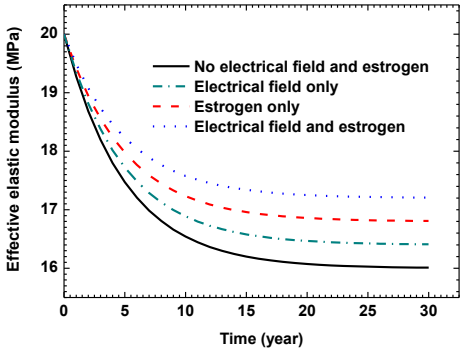

Figure 5 Influence of electromagnetic field on the bone quality of senile osteoporosis and hormone replacement therapy

Results

The results show that those mechanical and non-mechanical factors can really have an effect on the bone damage and healing. Although the mechanical loadings can inevitably cause fatigue damage in bone materials, bone tissues can resist the damage accumulation and control the damage level to a sustainable value. This amazing ability ensure the bone a much longer life when it operates under conditions of high loading, subjected to stresses and strains that cause damage if not repaired. On the other hand, electromagnetic treatment is a more effective way to reduce the bone damage and strengthen the bone to resist the damage accumulation caused by more extreme loadings. It can stimulate the bone tissues to accelerate the repairing rate and to create more new bone to make the bone stronger. The mechanical loadings can do the same in some cases, but it produces more damage at the same time. While the electromagnetic field can repair more damage rather than create it.

Improved understanding of bone cell biology is a key issue in the prevention of osteoporosis

and in the improvement of physical and pharmaceutical treatment methods, as well as in prosthetic design. These features may be utilized in controlling healing process of injured bones. But it should be mentioned here that all the results are obtained on the basis of the numerical model which may be different from damage behaviour of actual bone materials. So further experimental verification is obviously required before the proposed model can be applied to the medical clinical practice.

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