The Calculation in whole Process Rate Realized with Two of Type Variable under symmetrical cycle for Elastic-Plastic Materials Behavior

Yu Yangui [1,a] , MaYanghui [2,b]

[1] Zhejiang GuangXin New Technology Application Academy of Electromechanical and Chemical Engineering, 13E east edifice 160 International garden Tiannushan Road

310007 Hangzhou, CHINA

[2] Zhejiang Science and Technology University, 1 Xiaoheshan Road，310023 Hangzhou CHINA

[a] gx_yyg@126.com, [b] anqingxufeng@126.com

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Abstract. This paper adopts two types of variables a and D among fatigue-damage-fracture disciplines, Put forward several calculation models of the crack growth rate and the damage evolving rate at each stage for elastic-plastic steels with cracks which are obtained to use the mathematical methods in mechanics and combining program calculation of computer. Provide some conversion methods between the variables, the equations and the dimensional units; Indicate their physical and the geometrical meanings for some key parameters; Communicate the cross relations between their computing models at each stage and between the disciplines each other. This will be having practical significance for promoting developing and applying of each discipline.

Introduction

Adopt the crack size a as a variable in the fracture mechanics to describe crack growth process, and adopt the damage variable D in the damage mechanics to describe a damage evolutive process. If can communicate and convert the relations each other for that between the damage variables, the equations and the dimensional units which can describe the material behavior between varied disciplines and provide some conversion methods, thus we are also able to adopt the same variable a_1 and a_2 or the variables D_1 and D_2 to compute the crack growth rate or damage evolving rate for their curves by divided two stages severally; Even can also adopt same variable a or D by means of computing program to describe its change rule in whole process. Such this will be having practical significance for stint manpower and fund in fatigue-damage-fracture test, for promoting developing and applying of each discipline.

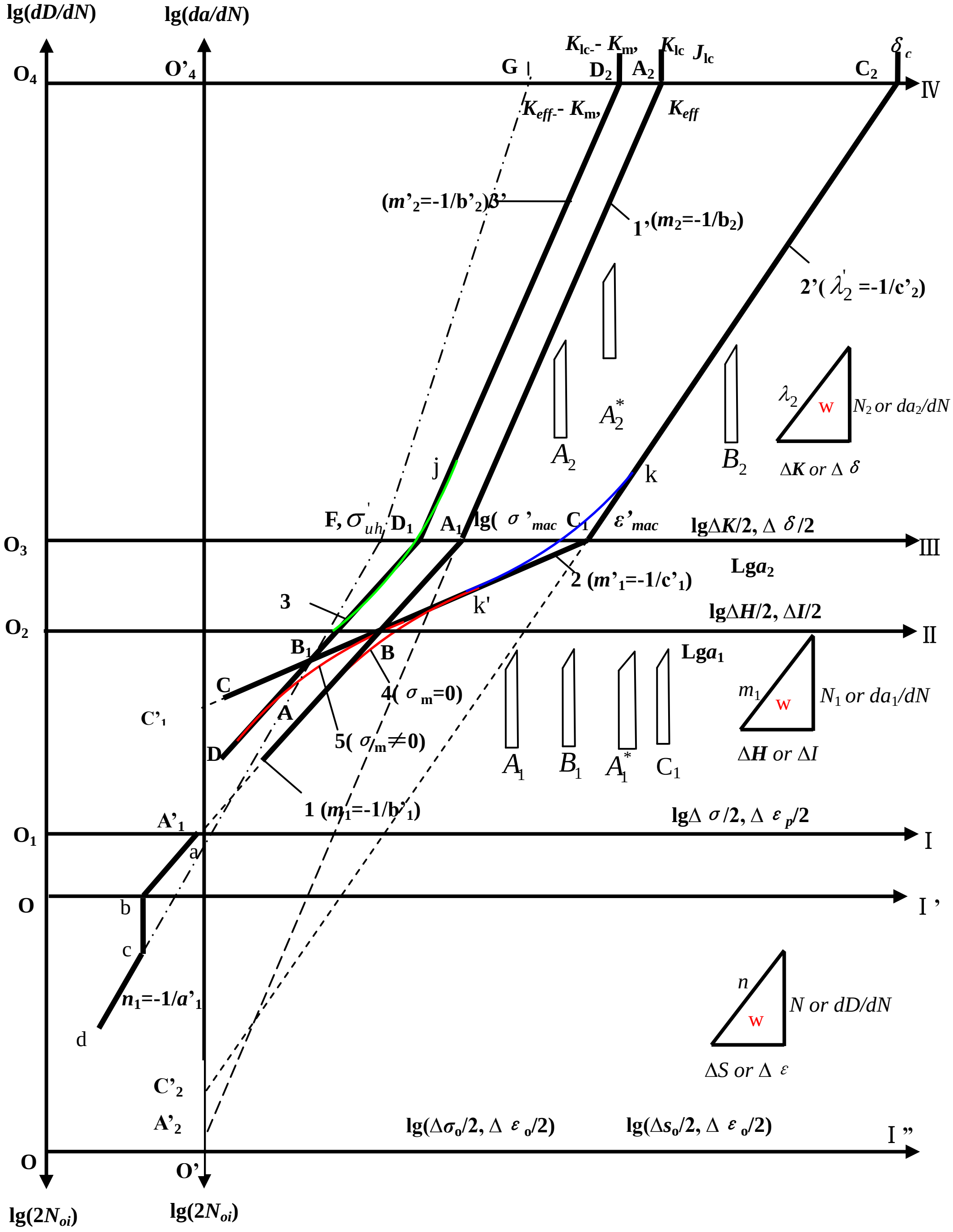

Figure. 1 Bidirectional combined coordinate system and bidirectional curves in the whole process (Combined cross figure of fatigue-damage-fracture

The calculations of the rate at each stage for elastic-plastic material with crack

For elastic-plastic material with short crack a_1, its growth rate can be calculated by means of the multiplication with plastic strain and stress parameters $\varepsilon \times \sigma$ at the crack forming stage; For long crack a_2, can adopt the stress intensity factor K_1 as well as the crack tip open displacement δ_t to calculate, but their calculations are restricted by the work stress $\sigma \le 0.5\sigma_s$ or $\sigma > 0.5\sigma_s$ (σ_s is yield stress) at the crack growth stage.

The computing of rate at crack forming stage. The growth rate curves of short crack a_1 for the elastic-plastic material at crack forming stage (the first stage) are just described with curve 4 ($\sigma_m = 0$) in the positive direction coordinate system in Fig1[1~3]. And its computing equation about short crack growth rate is,

$$da_1 / dN_1 = A_1^* \cdot Q^{\frac{m_1 m_1}{m_1 + m_1}} \tag{1}$$

$$A_1^* = 2[4(\sigma'_f \varepsilon'_f)]^{-\frac{m_1 m_1}{m_1 + m_1}} \frac{(\ln a_{fc} - \ln a_{01})}{(N_{fc} - N_{01})}, \ (\sigma_m = 0\ ) \tag{2}$$

$$Q = (\varepsilon_p \sigma) \cdot a_1^{1/(\frac{m_1 m'_1}{m_1 + m'_1})}, \ (MPa \cdot \sqrt[\frac{m_1 m'_1}{m_1 + m'_1}]{m}) \tag{3}$$

Where the parameter Q is defined as the short-crack strain-stress factor, it is driving force of short crack growth at first stage. Its unit is $MPa \cdot \sqrt[\frac{m_1 m'_1}{m_1 + m'_1}]{m}$. m_1 or m'_1 is material constant under high cycle or low cycle fatigue, respectively. $m_1 = -1/b_1'$, b_1' is fatigue strength exponent under high cycle fatigue; $m'_1 = -1/c_1'$, c_1' is fatigue ductility exponent under low cycle fatigue. The parameter A_1^* is a comprehensive material constant, it should obtain from experiment. Author think, its physical meaning of the A_1^* is a concept of power, is a maximal increment value of used energy to make fracture in a cycle; Its geometrical meaning is a maximal area of micro-trapezia to approximate to beeline, is an intercept between $O_1 - O_3$ on the y-axis which is a projection of corresponding to curve 4 (Fig.1). Its slope of micro-trapezia area just is corresponding to the exponent $\frac{m_1 m_1'}{m_1 + m_1'}$ of rate equation (1). The comprehensive material constant A_1^* is having function relation with among other each parameter in equation (2) [1~3]. a_{01} is micro-crack initial size at the first stage, a_{fc} is crack size corresponding to the stress amplitude σ'_{fc} or strain amplitude ε'_{fc} to make achieving to fracture in one cycle; N_{01} is an initial life, $N_{01}=0$; N_{fc} is to make fracture life in a cycle, So,

$$\frac{(\ln a_{fc}-\ln a_{01})}{(N_{fc}-N_{01})}=1 \qquad (4)$$

Come to light, there are obvious difference between long crack and short crack behavioral characteristics, conform to their own characteristics, should provide a transitional size a_{tr} (a_{mac}) from short crack to macro-long crack, so the equation (2) become as form below

$$A^*_{1tr}=2[4(\sigma'_f\,\varepsilon'_f)]^{-\frac{m_1m_1}{m_1+m_1}}\frac{(\ln a_{mac}-\ln a_{01})}{(N_{mac}-N_{01})},\sigma_m=0 \qquad (5)$$

Here, physical meaning of A^*_{1tr} is also a concept of power that is the maximal increment value used out energy from short-crack growth to long-crack size a_{mac} in one cycle (for example steel 16MnR, suppose to take its transitional size $a_{tr}=a_{mac}=1mm$, so that computing equation of short crack growth rate connected with long crack growth rate is as follow

$$da_1/dN_1=A^*_{1\mathrm{tr}}\cdot Q^{\frac{m_1m_1}{m_1+m_1}} \qquad (6)$$

Its expansion equation should be

$$da_1/dN_1=2[4(\sigma'_f\,\varepsilon'_f)]^{-\frac{m_1m_1}{m_1+m_1}}\times(\Delta\varepsilon_p\Delta\sigma)^{\frac{m_1m_1}{m_1+m_1}}\times a_1, \qquad (7)$$

If use the damage variable D_1 to express that is equivalent with above mentioned eq. (7), then its equation of damage evolving rate is

$$dD_1/dN_1=A^*_{1\mathrm{tr}}\cdot Q'^{\frac{m_1m_1}{m_1+m_1}}=A^*_{1\mathrm{tr}}\cdot(\Delta\varepsilon_p\Delta\sigma)^{\frac{m_1m_1}{m_1+m_1}}\times D_1 \qquad (8)$$

Where the parameter Q' in eq (8) is converted from corresponding to Q, the parameters Q' is defined as the damage strain-stress factor that are also driving force of damage evolving process of material, its unit is $MPa\cdot D^{1\big/\frac{m_1m'_1}{m_1+m'_1}}$ or MPa. Here it must be defined in 1m (meter) equivalent to 1000 of damage-units, 1mm (millimeter) equivalent to one of damage-unite and must put up conversion of dimension and units. It should point that unit of the da_1/dN_1 in eq. (6) and (7) is $m/cycle$, and the unit of the dD_1/dN_1 in eq. (8) is unit number of damage per cycle ($D/cycle$).

The computing of rate at crack growth stage. In Fig.1, the beeline C_1C_2 via logarithm to predigest transacting can be represented as growth behavior for long crack. Come to light that the crack tip open displacement δ_t can describe its behavior; its equation is as discussed Dover [4]

$$\frac{da_2}{dN_2} = B_2 \times \Delta\delta_t^{\lambda_2} \tag{9}$$

Where the parameter λ_2 is material constants, B_2 is a material constant and should obtain from experimentation at crack growth stage (the second stage). Authors repeated research and think that its physical meaning is also a concept of power that is the maximal increment value used energy $(\Delta\delta_c)^{-\lambda_2}/cycle$ to make fracture in one cycle, its geometrical meaning is a micro-trapezia area to approximate to beeline, also is an intercept between $O_3 - O_4$ on the y-axis (Fig.1), its slope of micro-trapezia area just is corresponding to exponent λ_2 of rate equation. The parameter B_2 is a comprehensive material constant which is to have a function relation among with other parameter as follow,

$$B_2 = 2(2\delta_c)^{-\lambda_2} \times (a_{c2} - a_{02})/(N_2 - N_{02}),\ (\sigma_m = 0\ ) \tag{10}$$

$$v_{tr} = (a_{c2} - a_{02})/(N_2 - N_{02}) \approx 6\times10^{-8} \sim 1\times10^{-7}, (m/cycle) \tag{11}$$

Author research finding that the parameter v_{tr} is a constant of rate, is defined as a transitional rate from short crack to long crack, it is existing rate in name only caused by made for prefabricated crack of specimen, v_{tr} must be confirmed from experiment. a_{c2} and a_{02} are critical size and initial size at the second stage. N_2 and N_{02} are critical life and initial life. For the sake of safety, the comprehensive constant B_2 should be computed as follows

$$B_{2eff} = 2(2\delta_{2eff})^{-\lambda_2} \times (a_{2eff} - a_{02})/(N_2 - N_{02}),\ (\sigma_m = 0\ ) \tag{12}$$

Where the δ_{2eff} is the effective value of the crack tip open displacement during steady growth course for long crack which it must also be obtained from experiment.
Thus, equation of rate at second stage is become as following form

$$\frac{da_2}{dN_2} = 2(2\delta_{2eff})^{-\lambda_2} \times (\Delta\delta_t)^{\lambda_2} \tag{13}$$

Computing of rate to arise small plastic strain zone. For computing of long crack growth rate under the condition at rack tip only to arise small plastic strain zone, which it should be used by the D-M' mathematical model to compute crack tip open displacement range $\Delta\delta_t$ as discussed by [5],

$$\Delta\delta_t = \left(\frac{8\sigma_s a_2}{\pi \cdot E} \ln\sec\frac{\pi \times \Delta\sigma}{2\sigma_s} \right) \tag{14}$$

Where $\Delta\delta_t$ can be educed and obtained from fracture mechanics as following form

$$\Delta\delta_t = \frac{K_{1\max}^2 - K_{1\min}^2}{E \cdot \sigma_s} \tag{15}$$

And the parameter δ_{eff} in eq. (12) could be deduced as following forms, suppose that

$$\delta_{eff} = \left((K_{1c} / \sigma_s)^2 \times \frac{\sigma_s}{E} \right)_{2eff} = (0.15 \sim 0.5)\delta_c; \quad or \delta_{eff} = (0.15 \sim 0.5)\delta_c \tag{16}$$

$$\delta_c = \varepsilon_s \left(\frac{K_{1c}}{\sigma_s} \right)^2 = \left(\frac{K_{1c}}{\sigma_s} \right)^2 \times \frac{\sigma_s}{E} \tag{17}$$

Here K_{1c} is the critical value of stress intensity factor. So the growth rate of long crack at the second stage is expressed as following form,

$$\frac{da_2}{dN_2} = 2\left(2(K_{1c} / \sigma_s)^2 \times \frac{\sigma_s}{E} \right)_{2eff}^{\lambda_2} \times v_{tr} \left(\frac{8\sigma_s a_2}{\pi \cdot E} \ln\sec\frac{\pi \times \Delta\sigma}{2\sigma_s} \right)^{\lambda_2} \ (m / cycle) \tag{18}$$

If put up conversion for variable, in equation (13) adopt $\Delta\delta_t^{'}$ replace the $\Delta\delta_t$, and $\Delta\delta_t^{'}$ is defined to be "crack tip damage open displacement range", then the $\Delta\delta_t^{'}$ is just the driving force of material damage at crack growth stage under $\sigma \le 0.5\sigma_s$ condition, so the damage evolving rate is

$$\frac{dD_2}{dN_2} = 2\left(2(K'_{1c} / \sigma_s)^2 \times \frac{\sigma_s}{E} \right)_{2eff}^{-\lambda_2} \times v_{tr} \left(\frac{8\sigma_s D_2}{\pi \cdot E} \ln\sec\frac{\pi \times \Delta\sigma}{2\sigma_s} \right)^{\lambda_2} \ (D / cycle) \tag{19}$$

Where the material constant K'_{1c} is defined as the critical damage stress intensity factor of equivalent to K_{1c} , its unit is $MPa\sqrt{D}$. The unit of other parameter in equation is invariant. It must be point that the unit of crack growth rate is the m/cycle in eq.(18); but the unit of damage evolving rate is the unit number of damage per cycle (damage-unit number /cycle) in eq.(19), that is equivalent to crack growth size per cycle.

Computing of rate to arise bigger plastic strain zone. On the other hand, under the stress condition $\sigma > 0.5\sigma_s$, according to the model as discussed by reference [6] to compute the crack tip open displacement, it can educe by same method as follow

$$\frac{da_2}{dN_2} = 2(2\delta_{2eff})^{-\lambda_2} v_{tr} \times \left(\pi a_2 \sigma_s [(\varepsilon_{\max} - \varepsilon_{\min}) / \sigma_s)]^2 / E \right)^{\lambda_2} \tag{20}$$

Where δ_{2eff} notionally take $(0.3 \sim 0.7)\delta_c$, but it must be confirmed from experiment. E is a modulus of elasticity. $\varepsilon_{\max}$, $\varepsilon_{\min}$ is severally maximal and minimal strain value. On the other hand, the damage evolving rate equation become below,

$$\frac{dD_2}{dN_2} = 2(2\delta'_{2eff})^{-\lambda_2} v_{tr} \times \left(\pi D_2 \sigma_s [(\varepsilon_{\max} - \varepsilon_{\min})/\sigma_s)]^2 / E\right)^{\lambda_2} \qquad (21)$$

Computing of rate in whole process

It should also point that positive curve $aBk'C_1kC_2$ ($\sigma_m = 0$) in the positive direction coordinate system in Fig 1 is just describing the changing law of the rate in whole process for elastic-plastic material behavior. Thus we are able to adopt the same variable a_1 and a_2 or the variables D_1 and D_2 to compute the evolving rate for the curves by divided two stages severally; Even can also adopt same variable a or D by means of computing program to describe its change rule in whole process.

Under work stress $\sigma \le 0.5\sigma_s$, if adopt crack size a as variable, computing expression of crack growth rate in whole process is as following form,

$$\begin{aligned} da/dN &= 2[4(\sigma_f \times \varepsilon_f)]^{-\frac{m_1 m_1}{m_1+m_1}} \times (\Delta\varepsilon_p \Delta\sigma)^{\frac{m_1 m_1}{m_1+m_1}} \times a \\ &+ 2\left(2(K_{1c}/\sigma_s)^2 \times \frac{\sigma_s}{E}\right)^{-\lambda_2} \times v_{tr} \left(\frac{8\sigma_s a}{\pi \cdot E} \ln\sec\frac{\pi \times \Delta\sigma}{2\sigma_s}\right)^{\lambda_2} \end{aligned} \quad (m/cycle) \qquad (22)$$

If adopt the damage variable D, computing expression of damage evolving rate in whole process is as following form,

$$\begin{aligned} dD/dN &= 2[4(\sigma_f \times \varepsilon_f)]^{-\frac{m_1 m_1}{m_1+m_1}} \times (\Delta\varepsilon_p \Delta\sigma)^{\frac{m_1 m_1}{m_1+m_1}} \times D \\ &+ 2\left(2(K'_{1c}/\sigma_s)^2 \times \frac{\sigma_s}{E}\right)_{eff}^{-\lambda_2} \times v_{tr} \left(\frac{8\sigma_s D}{\pi \cdot E} \ln\sec\frac{\pi \times \Delta\sigma}{2\sigma_s}\right)^{\lambda_2}, \end{aligned} \quad (D/cycle) \qquad (23)$$

Under stress $\sigma > 0.5\sigma_s$ condition, if adopt crack size a as variable, the expression of crack growth rate in whole process is

$$da/dN = 2[4(\sigma_f \times \varepsilon_f)]^{-\frac{m_1 m_1}{m_1+m_1}} \times (\Delta\varepsilon\Delta\sigma)^{\frac{m_1 m_1}{m_1+m_1}} \times a + 2(2\delta_{eff})^{-\lambda_2} \times v_{tr} \left(\pi a \sigma_s [(\varepsilon_{\max} - \varepsilon_{\min})/\sigma_s)]^2 / E\right)^{\lambda_2} \quad (24)$$

If adopt the damage variable D, the expression of damage evolving rate in whole process is as following form

$$dD/dN = 2[4(\sigma_f \times \varepsilon_f)]^{-\frac{m_1 m_1}{m_1+m_1}} \times (\Delta\varepsilon_p \Delta\sigma)^{\frac{m_1 m_1}{m_1+m_1}} \times D + 2(2\delta'_{eff})^{-\lambda_2} \times v_{tr} \left(\pi D \sigma_s [(\varepsilon_{\max} - \varepsilon_{\min})/\sigma_s)]^2 / E\right)^{\lambda_2}, \quad (25)$$

Summary

1. For elastic-plastic materials, the short-crack stress-strain factor Q and the damage stress-strain factor Q' expressed with multiplication should be and could be defined severally as driving force of the short-crack growth in fracture mechanics and as driving force of damage evolving in damage mechanics at first stage.

2. Want to the computation expressions for the crack growth rate to make conversion to the expressions of the damage evolving rate, its key consist in to make the crack size a convert to be the damage variable D. Only define the "1mm of crack size" equivalent to "1 unit of damage value"；The dimension and unit of other parameters in equation are all to keep invariability, again via mathematical derivation can convert their relations and must define their relative dimensions and units.

3. The parameters A^{*}_{1tr} and B_{2eff} are respectively comprehensive material constants at the first and the second stage, they are all with other parameters to have functional relations, their physical meanings are a concept of power, are a maximal increment value of used energy in one cycle; their geometrical meanings are a micro-trapezia area to approximate to beeline.

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