

# Regularities of fatigue fracture of metal materials

A. N. Romanov

Research Institute for Mashine Science of Russian Academy of Sciences, Maly  
Kharitonevsky per. 4, Moscow, 101990 Russia.

[alrom37@mail.ru](mailto:alrom37@mail.ru)

**Keywords:** unique criterion, unique fatigue curve, damage.

**Abstract:** *the unique criterion of cycle fracture of metal materials is obtained and on its basis the existence of unique fatigue curve in all range of cycles loading (low-cycle and high-cycle fatigue) for a stage of crack initiation and stage of crack propagation is shown. The criterion allows on the extreme saved damage to calculate durability of elements of constructions, and also levels of the accumulated damages for any stage of loading. This criterion is independent of the type, structural state (i.e., treatment), form of a loading, of a material (single-frequency, double-frequency, asymmetric, having time lags, and programmable), as well as, of the loading conditions (temperature, frequency, type loading).*

*Unique criterion and unique curve of fatigue are proved by numerous experimental data for different metal constructional materials and various conditions of loading.*

*New approaches that can be applied to both the calculation methods for the cycle strength and to the rules for the choice of structural materials are proposed.*

## Introduction

Over many decades, resistance properties of structural materials with respect to deformation and fracture under cycle loading have been studied. Investigations aimed at the development of fracture criteria necessary for estimating the strength of machines, equipment, and structures of various application have also been performed. In these studies, force, deformation, and energy approaches were widely used. Each of these approaches now has dozens of proposals related to various criteria [1]. It should be noted that, as a rule, the existing criteria are reduced to equations of a fatigue curve for a given material and given loading conditions and, therefore, cannot be extended to other materials and testing conditions. In addition, these criteria are related to a limited number of materials and loading conditions that, as before, require a large amount of experimental data.

Compared to the other approaches, the energy one is the most general. Numerous studies have demonstrated [1], however, that the energy of cycle fracture may greatly exceed the energy spent on the single fracture of a sample and, thus, cannot be accepted as a criterion. In the case of a cycle loading, a considerable part of the energy is dissipated in the form of heat and is spent on reversible deformation in the cycles. Only the lesser part of the energy is directly spent for fracturing the material. The experimental determination of this part is extremely difficult [ROM 88]. The employment of this quantity for estimating the endurance of structural elements is also hampered.

In [3], a suggestion based on the energy approach is made that in the case of a low-cycle loading, the ultimate work of microstresses  $p$  in the plastic-deformation path  $L$  is the fracture work:

$$\int \kappa p dL = 1. \quad (1)$$

Here,  $k$  is a constant determined from the experiment.

The specified representations were fixed in a basis of reception of unique criterion of cycle fracture.

## 1. Unique criterion of fracture on fatigue crack initiation and propagation

### 1.1. Cinetic criterion of fracture on fatigue crack initiation.

Based on the studies of the Bauschinger effect [ROM 88] and concepts concerning the role of microdamages in the plastic-deformation path, I succeeded in finding a fracture criterion for the case of cycle loading within the given loading range in a stress-controlled loading (i.e., in a cycle with a given stress amplitude). In this case, the criterion has the form:

$$\int_0^{N_f} \frac{\varepsilon_p^2}{\varepsilon^2} dN + \int_0^{N_f} \frac{(\varepsilon_p - \varepsilon'_p)(\varepsilon - \varepsilon_p)}{\varepsilon^2} dN = 1. \quad (2)$$

Under the condition that, basically,  $\varepsilon_p \ll \varepsilon$ , criterion (2) can be expressed in the form:

$$\int_0^{N_f} \frac{\varepsilon_p^2}{\varepsilon^2} dN + \int_0^{N_f} \frac{\Delta\varepsilon}{\varepsilon} dN = 1, \quad (3)$$

here,  $\varepsilon_p$  is the plastic deformation (the hysteresis-loop width) in the tension half-cycle;  $\varepsilon_p - \varepsilon'_p = \Delta\varepsilon$  is the accumulated deformation in the cycle under consideration;  $\varepsilon'_p$  is the plastic strain in the compression half-cycle;  $\varepsilon$  is the material plasticity in the case of the sample single-stage fracture (this quantity provides the material carrying capacity); and  $N_f$  is the number of cycles to failure.

In the case of loading within the given range of elastoplastic deformation in a cycle (rigid loading), the second term in relations (2) and (3) vanishes (the plastic-deformation accumulation does not occur). Then, criterion (3) for the case of the strain-controlled loading can be expressed as

$$\int_0^{N_f} \frac{\varepsilon_p^2}{\varepsilon^2} dN = 1, \quad (4)$$

At loading with the given scope elastoplastic of deformation in a cycle (strain-controlled loading) the determining accumulativ damages with growth of number of the cycles has determined as:

$$\eta = \int_0^N \frac{\varepsilon_p^2}{\varepsilon^2} dN, \quad (5)$$

at stress-controlled loading as:

$$\eta = \int_0^N \frac{\varepsilon_p^2}{\varepsilon^2} dN + \int_0^N \frac{\Delta\varepsilon}{\varepsilon} dN. \quad (6)$$

At loading with the given scope elastoplastic of deformation (rigid loading), when scope of plastic deformation in a cycle (width of a hysteresis-loop) changes insignificantly, in the field of small number of cycles to failure ( $N_f \leq 10^3$  cycles) with sufficient for practice accuracy the relationship is:

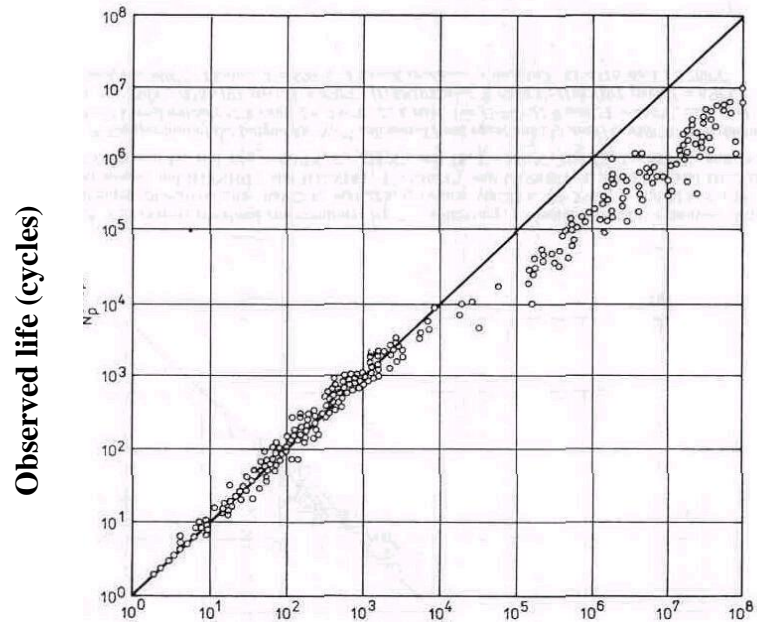
$$\varepsilon_p^2 N_f = \varepsilon^2 \quad \text{or} \quad N_f = \frac{\varepsilon^2}{\varepsilon_p^2}, \quad (7)$$

from which follows, that all metal materials in the specified interval of durabilities, have a unique curve of relationship  $\varepsilon_p/\varepsilon$  from  $N_f$ .

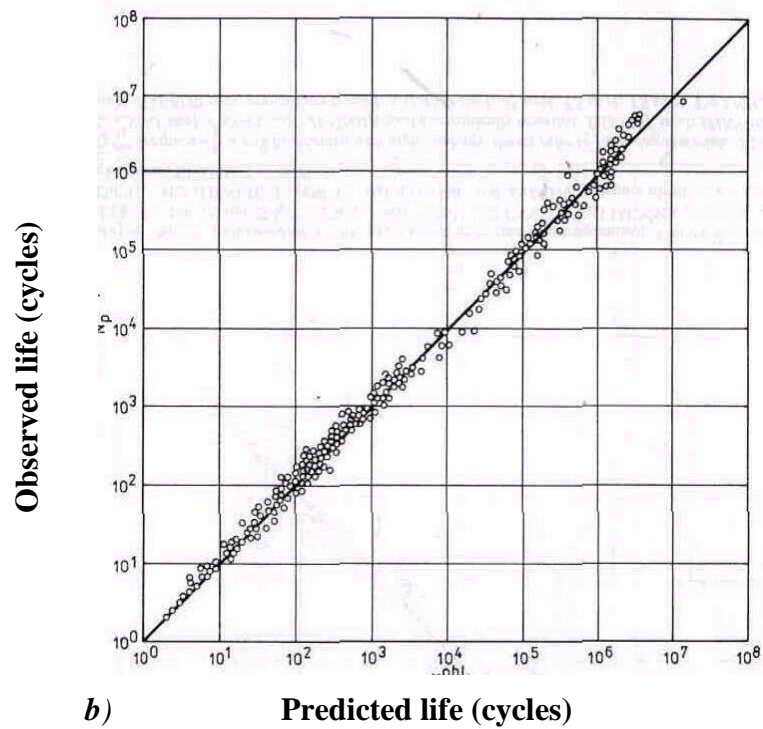
The criterions (2-4) satisfactorily describe fracture only at small number of cycles to failure (number of cycles to failure makes some thousand ones) - Fig. 1a and 2a. With increase of number of cycles to failure the amounting share of damages is brought by an elastic component of working stresses.

The first term in relationships (2) and (3) determines the level of damage accumulated as a result of the action of the cycle plastic deformation. The second term corresponds to a damage caused by the accumulated plastic deformation for the number of the loading cycles under consideration. The fracture occurs when the damage level attains unity. The relations (5) and (6) make it possible to determine the level of the accumulated damage for an arbitrary number (which we are interested in) of loading cycles including programmable loading.

As experiments show, relationships (3) and (4) satisfactory describe fracture conditions in the region of low-cycle fatigue (Fig. 1a) but do not allow us to describe fracture within the region of a large number of cycles to failure (more than  $10^4$  cycles).



a)



b)

Fig. 1

The assumption about the damaging role of microstresses also in the path of the elastic deformation  $\varepsilon_e$  made it possible to find the fracture criterion in the form:

$$\int_0^{N_f} \frac{\varepsilon_p^2}{\varepsilon^2} dN + \int_0^{N_f} \frac{\Delta\varepsilon}{\varepsilon} dN + \int_0^{N_f} \frac{\varepsilon_p \varepsilon_e}{\varepsilon^2} dN = 1. \quad (8)$$

In this relationship the third term determines the level of the accumulated damage caused by the action of the elastic deformation in the half-cycle of the extension  $\varepsilon_e = \sigma/E$ .

The criterion (8) can be expressed as:

$$\int_0^{N_f} \frac{\varepsilon_p \varepsilon_{ep}}{\varepsilon^2} dN + \int_0^{N_f} \frac{\Delta\varepsilon}{\varepsilon} dN = 1, \quad (9)$$

where  $\varepsilon_{ep} = \varepsilon_p + \varepsilon_e$ .

In the case of strain-controlled loading, we can assume that  $\varepsilon_p = \text{const}$  and  $\varepsilon_{ep} = \text{const}$ . Then, relation (9) takes the form:

$$\frac{\varepsilon_p \varepsilon_{ep}}{\varepsilon^2} N_f = 1 \quad \text{or} \quad N_f = \frac{\varepsilon^2}{\varepsilon_p \varepsilon_{ep}}. \quad (10)$$

The kinetic criteria allowing to describe the levels of saved damages at any stage of cycle in fatigue of low-cycle and high-cycle fatigue, i.e. in all range of numbers of cycles to failure [1]:

at strain-controlled loading as:

$$\eta = \int_0^N \frac{\varepsilon_p^2}{\varepsilon^2} dN + \int_0^N \frac{\varepsilon_p \varepsilon_e}{\varepsilon^2} dN, \quad (11)$$

at stress-controlled loading as:

$$\eta = \int_0^N \frac{\varepsilon_p^2}{\varepsilon^2} dN + \int_0^N \frac{\Delta\varepsilon}{\varepsilon} dN + \int_0^N \frac{\varepsilon_p \varepsilon_e}{\varepsilon^2} dN \quad (12)$$

or

$$\eta = \int_0^N \frac{\varepsilon_p \varepsilon_{ep}}{\varepsilon^2} dN, \quad (13)$$

$$\eta = \int_0^N \frac{\varepsilon_p \varepsilon_{ep}}{\varepsilon^2} dN + \int_0^N \frac{\Delta\varepsilon}{\varepsilon} dN. \quad (14)$$

In relationship (8) first member determines a fatigue damage from action of cyclically convertible plastic deformation, the second member - damage from accumulated plastic deformation and third member - damage from action of elastic deformation.

The check of criteria (9) and (10) has shown their good conformity to experiment in all range of numbers of cycles to failure, i.e. both in low-cycle, and in high-cycle fatigue (Fig. 1b).

The assumptions indicated on the damaging role of microstresses in the path of the elastic and plastic deformations made it possible to describe fracture conditions in the entire range of fracturing cycle loading (i.e., for a different number of fracturing cycles) in both the region of low-cycle and high-cycle fatigue fracture (Fig. 1b) by unique relation (9).

The criterion obtained testifies to the fact that, in the case of a cycle loading, the damage-accumulation process and fracture conditions obey the unique rule (law) independently of loading conditions.

Experimental verification of criteria (9) and (10) confirmed the existence of a unique criterion for the fracture of any given metallic material in any given structural state (i.e., independently of the of treatment form) with arbitrary conditions of cycle loading (temperature, loading frequency, cycle asymmetry, etc.). In this case, loading conditions affect the characteristics of resistance with respect to deformation, which enter into relationships (9) and (10). However, in accordance with these criteria, the loading conditions do not change the rule (law) of summing damages, including the programmable loading (e.g., two-frequency, steplike, overloaded, having time lags in loading half-cycles, etc.).

Criteria (9) and (10) describe the kinetics of damage accumulation, the extreme case of which corresponds to the appearance of a macrocrack.

## 1.2. Criterion of fracture on fatigue crack propagation.

At a stage of crack propagation the rate of accumulation of damages is estimated:

$$\eta = \int_0^N \frac{v_{ep} v_p}{v_c^2} dN + \int_0^N \frac{\Delta v}{v_c} dN, \quad (15)$$

In a limiting case (fracture) of relationship (15) will be:  
at strain-controlled loading as:

$$\int_0^{N_f} \frac{v_{ep} v_p}{v_c^2} dN = 1, \quad (16)$$

at stress-controlled loading as:

$$\int_0^{N_f} \frac{v_{ep} v_p}{v_c^2} dN + \int_0^{N_f} \frac{\Delta v}{v_c} dN = 1, \quad (17)$$

here,  $v_p$ ,  $v_e$ ,  $v_{ep}$ , and  $\Delta v$  are the plastic (residual reversible), elastic, and elastoplastic (reversible) of the crack borders opening displacement in the extension half-cycle and the accumulated crack borders opening displacement in the cycle, respectively, and  $v_c$  is the

ultimate of the crack borders opening displacement in the case of a single-stage fracture of a sample with a crack.

An experimental verification of criterions (16) and (17) has confirmed their validity.

## 2. Unique curve of fatigue of the metal materials

The received criteria testify that at cycle loading process of accumulation of damages and the conditions of fracture is submit to a uniform rule (law), irrespective of the conditions of loading. It gives the bases to assume, that there is also unique curve of fatigue.

It is valid, if in relationships (9) and (10) to enter accordingly for strain-controlled and stress-controlled loading the following designations:

$$\alpha_e = \frac{\sqrt{\varepsilon_p \varepsilon_{ep}}}{\varepsilon}, \quad \alpha_p = \frac{\sqrt{\varepsilon_p \varepsilon_{ep} + \Delta\varepsilon \cdot \varepsilon}}{\varepsilon}, \quad (18)$$

That criterion (12) and (13) can be copied as:

$$\alpha_e^2 \cdot N_f = 1, \quad \alpha_p^2 \cdot N_f = 1. \quad (19)$$

Where  $\alpha_p$  is determined on middle characteristic  $\varepsilon_p$  and  $\Delta\varepsilon$ .

The experimental check of criteria (9) and (10) has confirmed existence of the unique curve of fatigue for low-cycle and many-cycle loading (Fig. 2). Moreover, the generalized curve of fatigue is unique for any metal materials (and them any structural state (i.e., treatment) and any conditions cycle loading (temperature, frequency loading, asymmetry of a cycle). The conditions loading influence the characteristics of resistance to the deformation, included in relationships (9) and (17), but do not change a rule (law) of summation of damages according to these criteria, including at program loading (two-frequency, step, with overloads, with temporary having time lags in half-cycle loading, etc.).

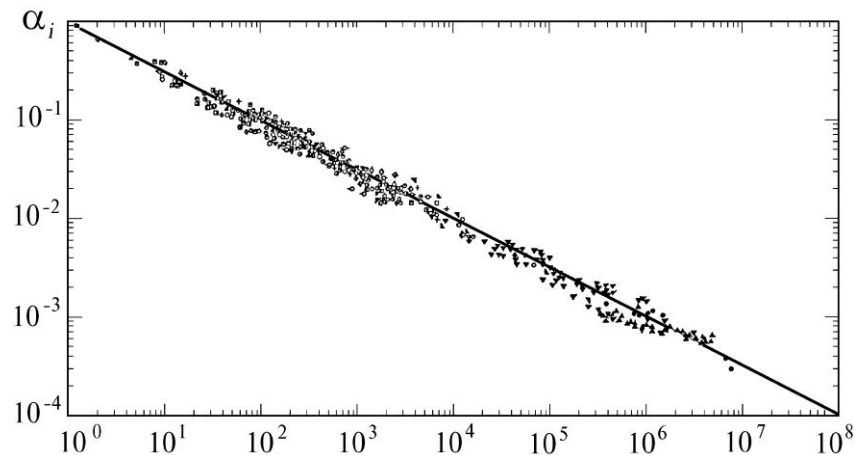


Fig.2. A unique (generalized) curve of the fatigue of metal materials

If by analogy with previous we shall enter designations

$$\alpha_{me} = \frac{\sqrt{V_{yn}V_p}}{v_c}, \quad \alpha_{mp} = \frac{\sqrt{(V_{yn}V_p + \Delta v \cdot v_c)}}{v_c}. \quad (20)$$

These criterions (16) and (17) will receive the following kind:

$$\alpha_{me}^2 \cdot N_p = 1, \quad \alpha_{mp}^2 \cdot N_p = 1. \quad (21)$$

The limiting cases of accumulation of damages and condition of fracture submit to the same law, as at stage of crack initiation, with that only by difference that for each of stages of fracture are inherent of the characteristics of deformation.

The experimental check of criterions (16) and (17) has confirmed their validity (light points on a curve, Fig. 2).

The results of tests of following materials given in Fig. 1 and 2: 22k steel ( $r_\sigma = -0,3; -0,7; -0,9$ ; specimens with notch;  $T = 150^\circ\text{C}; 270^\circ\text{C}; 350^\circ\text{C}; 450^\circ\text{C}$ ); 16GNMA - steel; H18N10T - steel ( $T = 450^\circ\text{C}; 550^\circ\text{C}; 650^\circ\text{C}$ , programmed loading; loading with two frequencies;); 0,22 % C steel; 1H13 steel; 45 steel (two-steps loading, programmed loading); 21H13 steel (two-steps loading, ); CSN steel; SAE4340 steel, treated; SAE4340 steel; Inconel 713C-LC; Inconel 713C-SG; Vaspaloy BW; Vaspaloy BK; Vaspaloy A; TS steel ( $r_\sigma = -0,7; -0,3; -0,9$ ;  $T = 270^\circ\text{C}; 350^\circ\text{C}; 450^\circ\text{C}$ ; specimens with notch); AD-33 ( $r_\sigma = 0; -1; +0,5$ ); Ni-Mo- steel.

The unique curve of fatigue is unique for: all metal constructional materials; low-cycle and high-cycle fatigue (i.e. for the all numbers of cycles before fracture; fracture on fatigue crack initiation and crack propagation; any kinds loading (cycle stretching - compression, torsion, bend); any temperatures of the tests, at which the plastic properties are shown; any frequencies of loading; anyone asymmetry of a cycle of loading;- stress-controlled and strain-controlled loading.

### 3. Summary

Thus, we can affirm the existence of a unique criterion of fracture of metallic materials in the entire possible time range of loading (low-cycle and many-cycle fatigue) on fatigue crack initiation and crack propagation. This criterion is independent of the type, structural state (i.e., treatment), form of a loading, of a material (single-frequency, double-frequency, asymmetric, having time lags, and programmable), as well as of the loading conditions (temperature and frequency). New approaches that can be applied to both the calculation methods for the cycle strength and to the rules for the choice of structural materials are proposed.

### References

- [1] Romanov A. N., *Fracture at low-cycle fatigue*, Moscow, Nauka 1988, 280 p.
- [2] Romanov A. N., "A unique criterion for the fatigue fracture of metallic materials", *Doklady Physics, Nauka*, vol.46, № 9, 2001, p. 672-674.
- [3] Novozhilov V.V., Rybakina O.G. in Proceedings of the III Conference on mechanical problems of fatigue, IMASH, Moscow, 1966, p. 71.