Quasi-Brittle Fracture Diagrams under Low-Cycle Fatigue (Double-Frequency and Unsteady Loadings)

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Abstract. Stepwise crack propagation in quasi-brittle materials under cyclic loading is considered. Both stationary and no stationary loads under pulsating loading are studied, special attention being given to double-frequency loading. For analysis of this process, diagrams of quasi-brittle fracture of solids under cyclic loading conditions are proposed. One of curves of the proposed diagram bears resemblance to the Kitagawa-Takahashi diagram. Estimates of dimensionless mean velocity of stepwise crack propagation per loading cycle have been obtained in an explicit form. The relations derived for the mean crack growth rate can be considered as structural expressions for plotting Paris' curves.

Introduction

Double-frequency loading regime for a compressor disk is under consideration. Let one of frequencies of the double-frequency loading regime correspond to the "takeoff-landing" conditions, and the second one describe forced stationary vibrations of compressor blades. In the vicinity of the site where blades are fasten to the disk, the double-frequency loading regime for material loading is implemented. The simplest version of double-frequency loading is shown in Fig.1, where P is the total loading under double-frequency conditions, P_1 is loading corresponding to the "takeoff-landing" conditions, P_2 is loading due to vibration of blades. In the Fig. 1 at the right, transformation of original loading regime to two pulsating regimes is given.

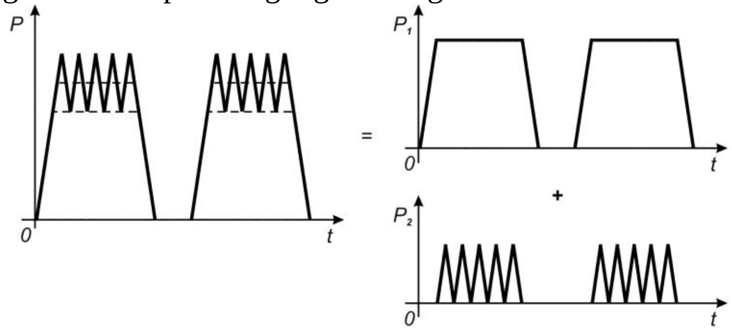

Fig. 1. Double-frequency loading regime for disk corresponding to rotating disk in the presence of vibration of blades.

Assume that there is internal crack-like defect in the vicinity of the site where blades fasten to the disk. Let the level of a compressor disk loading corresponds to low-cycle fatigue. Derive estimates of the number of cycles under double-cycle loading conditions. In order to obtain estimates for the number of cycles, the diagram of quasi-brittle fracture under low-cycle fatigue is used [1].

Diagrams of quasi-brittle fracture under single loading of specimen of finite width

Apply the simplest approximation of the $\sigma-\varepsilon$ diagram for elastic-plastic material when the diagram is approximated by a double-link straight line. The parameters of this approximation are as follows: E is the modulus of elasticity, σ_Y is the yield strength of material, and σ_Y is constant stress acting in accordance with the modified Leonov-Panasyuk-Dugdale model [2, 3], ε_0 is the maximum elastic material elongation ($\sigma_Y = E\varepsilon_0$), ε_1 is the maximum material elongation. Let r be the grain diameter for a granulated material with the regular structure. The Neuber-Novozhilov approach [4, 5] makes it possible to use solutions having a function of singularity for structured media.

Now consider an internal I mode crack. Diagrams of quasi-brittle fracture of solids with edge cracks have been obtained in [6]. Let an internal plane I mode crack extends rectilinearly. In addition to the internal rectilinear crack-cut of length $2l_0$, introduce into consideration model crack-cuts of length $2l = 2l_0 + 2\Delta$, each of pre-fracture zones Δ being located on the continuation of a real crack ($2l$ and Δ are lengths of model cracks and pre-fracture zones). The problem of fatigue fracture has two linear scales: if a grain diameter is defined by a material structure, then the second linear size is governed by the system itself. Under low-cycle fatigue conditions, the second linear scales serve as pre-fracture zone lengths Δ, which change in accordance with changing 1. the length of a real stepwise extending crack, and 2. the intensity of loading under double-frequency conditions. Emphasize that under single loading conditions, the critical pre-fracture zone length Δ^* is a completely definite parameter [7] and $2l^* = 2l_0 + 2\Delta^*$ is the critical macrocrack length, that is $\Delta^* / l_0 \ll 1$ for quasi-brittle materials.

When diagrams of quasi-brittle fracture under conditions of low-cycle loading are plotted in [1], sufficient fracture criteria are used when the mode I cracks are considered

$$\frac{1}{r}\int_0^r \sigma_y(x,0)dx = \sigma_Y \,, \; 2v(x,0) = \delta^* . \tag{1}$$

Here $\sigma_y(x,0)$ are normal stresses on the crack continuation; Oxy are the rectangular coordinate systems oriented about right crack sides (the coordinate origin coincides with the model crack tip in the modified Leonov-Panasyuk-Dugdale model [1, 7]); $2v = 2v(x,0)$ is crack opening ($x < 0$), δ^* is the critical crack opening displacement.

The field of normal stresses $\sigma_y(x,0)$ on the model crack continuation $x > 0$ can be represented as a sum of two summands

$$\sigma_y(x,0) \cong K_{\mathrm{I}} / (2\pi x)^{1/2} + O(1) , \;\; K_{\mathrm{I}} = K_{\mathrm{I}\infty} + K_{\mathrm{I}\Delta} , \;\; K_{\mathrm{I}\infty} > 0 , \;\; K_{\mathrm{I}\Delta} < 0 , \tag{2}$$

where $K_{\mathrm{I}} = K_{\mathrm{I}}(l,\Delta)$ are total stress intensity factors (SIFs) at the tips of model cracks, $K_{\mathrm{I}\infty}$ are SIFs generated by stresses σ_∞ specified at infinity, $K_{\mathrm{I}\Delta}$ are SIFs generated by constant stresses σ_Y. The first and second summands in relation (2) are singular and smooth parts of solution, respectively. The first equality in criterion (1) controls stresses on the model crack continuation after averaging, these stresses being coincident with the yield strength σ_Y, and the second equality in criterion (1) describes real crack blunting at its tip.

The first equality of the sufficient criterion (1) is a typical strength fracture criterion [8], and the second equality of this criterion is a deformation criterion [8]. The sufficient fracture criterion (1) simultaneously takes into consideration both strength and deformation fracture criteria at specific points of a pre-fracture zone. In such an approach [1, 7] to description of fracture process, there is no need for discussing advantages and drawbacks of strength or deformation criteria [9-11]. According to Novozhilov [5], the proposed criterion (1) is sufficient one; it combines advantages of strength and deformation criteria and partly levels their drawbacks [1]. Selected specific points of the pre-fracture zone are well-adapted to description of stepwise crack tip advance and accumulation of damages in the pre-fracture zone material under fatigue conditions [1].

The simplest analytical representation of the field of stresses $\sigma_y(x,0)$ on the model crack continuation has the following form [7]

$$\sigma_y(x,0)=\frac{K_{\mathrm{I}\infty}+K_{\mathrm{I}\Delta}}{\sqrt{2\pi x}}+\sigma_\infty=\frac{\sigma_\infty}{\sqrt{x}}\sqrt{\frac{l}{2}}+\frac{2\sigma_Y\sqrt{\Delta}}{\pi\sqrt{x}}+\sigma_\infty. \tag{3}$$

Introduce into consideration critical stresses calculated via the necessary fracture criterion σ_∞^0 and sufficient one σ_∞^*. In the view of the study of low-cycle fatigue, the most interesting loading area is $\sigma_\infty^0 \le \sigma_\infty \le \sigma_\infty^*$. After system (1) is solved, critical fracture parameters for quasi-brittle materials [1, 7] are as follows

$$\sigma_\infty^*=\sigma_\infty^*\left[l^*,\sigma_Y,(\varepsilon_1-\varepsilon_0)/\varepsilon_0\right],\ \Delta^*=\Delta^*[l^*,\sigma_Y,(\varepsilon_1-\varepsilon_0)/\varepsilon_0];\ \sigma_\infty^0=\sigma_\infty^0(l_0,\sigma_Y),\Delta\equiv 0. \tag{4}$$

Fracture diagrams for quasi-brittle materials are pairs of curves $\sigma_\infty^0=\sigma_\infty^0(l_0,\sigma_Y)$, $\sigma_\infty^*=\sigma_\infty^*\left[l^*,\sigma_Y,(\varepsilon_1-\varepsilon_0)/\varepsilon_0,\sigma_\Delta/\sigma_Y\right]$. Analytical representations of critical fracture parameters for quasi-brittle materials with account for solution representation (4) has the form [1]

$$\frac{\sigma_\infty^*}{\sigma_Y}=\left[\left(1-\frac{5}{8\pi}\frac{\varepsilon_1-\varepsilon_0}{\varepsilon_0}\right)\sqrt{\frac{2l^*}{r}}+1\right]^{-1},\ \Delta^*=\frac{5^2}{2^9}\left(\frac{\varepsilon_1-\varepsilon_0}{\varepsilon_0}\right)^2\left(\frac{\sigma_\infty^*}{\sigma_Y}\right)^2 l^*;\ \frac{\sigma_\infty^0}{\sigma_Y}=\left(\sqrt{\frac{2l_0}{r}}+1\right)^{-1}. \tag{5}$$

Expressions (5) make sense if $1-(5/8\pi)\left[(\varepsilon_1-\varepsilon_0)/\varepsilon_0\right]>0$. For a plain specimen of the finite width, the critical fracture parameters can be written in the form [6]

$$\frac{\sigma_{\infty w}^*}{\sigma_Y}=\left[\left(1-\frac{5}{8\pi}\frac{\varepsilon_1-\varepsilon_0}{\varepsilon_0}\right)Y^*\left(\frac{l^*}{w}\right)\sqrt{\frac{2l^*}{r}}+\frac{1}{1-2l^*/w}\right]^{-1},\ \frac{\sigma_{\infty w}^0}{\sigma_Y}=\left(Y^0\left(\frac{l_0}{w}\right)\sqrt{\frac{2l_0}{r}}+\frac{1}{1-2l_0/w}\right)^{-1}. \tag{6}$$

Here $w<\infty$ is the plain specimen width, $Y^0=Y^0(l_0/w)$ and $Y^*=Y^*(l^*/w)$ are coefficients such that $K_{\mathrm{I}w}=YK_\mathrm{I}=Y\sigma_\infty\sqrt{\pi l}$. Values of the coefficient $Y^0=Y^0(l_0/w)$ for various ratios l_0/w and loading conditions can be found in handbooks [12, 13]. For calculations, the approximate relationship $Y^*\approx Y^0$ is used for quasi-brittle materials when $\Delta^*/l_0 \ll 1$. The calculation results on effect of the finite specimen length on critical fracture curves on the plane “crack length–internal load” are given in Fig. 2 in the log-log coordinates, when $w/r=500$.

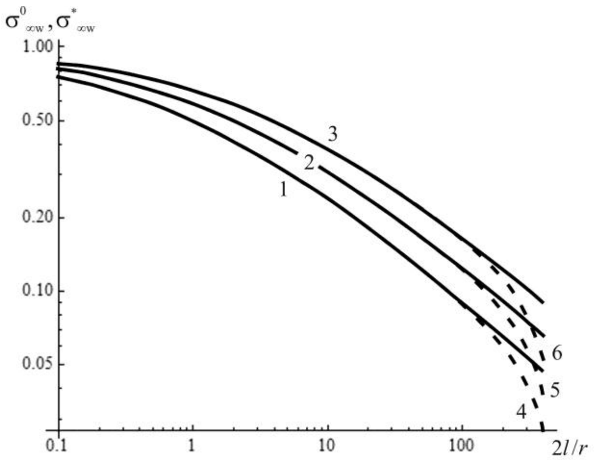

Fig. 2. Diagrams of quasi-brittle fracture for plain specimens of width $w=\infty$ and $w/r=500$, respectively.

In this Figure, the solid curve 1 and the dashed curve 4 are plotted via relation (5) for σ_∞^0 and via relation (6) для $\sigma_{\infty w}^0$, respectively; solid curves 2 and 3, and dashed lines 5 and 6 are plotted via relation (5) for σ_∞^* and via relation (6) for $\sigma_{\infty w}^*$, respectively, for $(\varepsilon_1-\varepsilon_0)/\varepsilon_0=1.5;2.5$. Pairs of curves 1-2, 1-3, and 4-5, 4-6 in Fig. 2 are diagrams of quasi-brittle fracture for plain specimens of infinite and finite width, respectively.

Diagram of quasi-brittle fracture under fatigue conditions (double-frequency regime)

The original diagram of quasi-brittle fracture under fatigue conditions coincides with the diagram of quasi-brittle fracture under single loading if a single-frequency pulsating loading regime with the constant amplitude [1] is considered. Below, as it is in work [1], a cyclically stable material is considered, i.e., the yield strength $\sigma_Y=const$ is independent of the cycle number. One of curves of quasi-brittle fracture diagrams is changed with regard to damage accumulation in the pre-fracture zone for subsequent loading cycles up to the first crack tip advance because the material of this zone is embrittled.

The Kitagawa-Takahashi diagram [14, 15] consists of one curve, its analytical representation can be found in Fig. 2, p.749 in [15]; this curve isolates the area of fatigue fracture from the area where there is no such fracture.

When the proposed diagram at low-cycle fatigue [1] is plotted, there is no need to use SIFs. It can be plotted depending on both elastic-plastic material properties and a crack length. This diagram consisting of a pair of curves divides the plane of "crack length vs. external load amplitude" $\left(2l/r,\sigma_\infty/\sigma_Y\right)$ into three subareas: 1. the subarea where fatigue fracture is not observed is located to the left and lower the curve σ_∞^0/σ_Y; 2. the subarea of fracture at single loading is located to the right and above the curve σ_∞^*/σ_Y; 3. the subarea of stepwise crack propagation due to embrittlement of material under cyclic loading conditions is located between curves σ_∞^0/σ_Y and σ_∞^*/σ_Y. When repeated loads are applied, the curve σ_∞^*/σ_Y is transformed, embrittlement of the pre-fracture zone material being considered. The diagram proposed in the log-log coordinates is confined by horizontal and inclined lines, see curves 1, 2 and 3 in Fig. 2 for plates of infinite width. Arrangement of the horizontal straight line is associated with the yield strength σ_Y and location of the inclined line is defined by the parameter $(\varepsilon_1-\varepsilon_0)/\varepsilon_0$ of inelastic material deformation. The proposed diagram has two threshold values σ_∞^0/σ_Y and σ_∞^*/σ_Y, see relation (6). The proposed modified Leonov-Panasyuk-Dugdale model applied to fatigue fracture closely resembles the model with hypothetic elements of fatigue [16, Fig. 1] at the real crack tip.

Consider the limitations

$$\sigma_\infty^0/\sigma_Y<\sigma_\infty^\Sigma/\sigma_Y<\sigma_\infty^*/\sigma_Y,\ \sigma_\infty^0/\sigma_Y<\sigma_\infty^+/\sigma_Y<\sigma_\infty^*/\sigma_Y,\ \sigma_\infty^\Sigma=\sigma_\infty^++2\sigma_{\infty 2}^+,\ \sigma_\infty^+=\sigma_{\infty 1}^+-\sigma_{\infty 2}^+. \quad (7)$$

Here σ_∞^Σ is the total amplitude of vibrations, $\sigma_{\infty 1}^+=const$ and $\sigma_{\infty 2}^+=const$ are amplitudes corresponding to the first and second stationary regimes, σ_∞^+ is the amplitude with regard to unloading from the second vibration regime. When limitations (7) are obeyed, we have the low-cycle fatigue just as in basic single loading regime with the amplitude σ_∞^+, so in double-frequency loading regime with the amplitude σ_∞^Σ. Obviously that when the ratios $\sigma_\infty^\Sigma/\sigma_Y\ge\sigma_\infty^*/\sigma_Y$ are valid, events can occur catastrophically: disk will be broken, for example, when a large bird is caught into an engine.

Now consider double-frequency loading at which $N_{1,j}$ is cycle number under "takeoff-landing" conditions, $N_{2,j}$ is cycle number under conditions of stationary blade vibrations. At the stepwise fatigue crack growth, the crack length changes, therefore it is necessary to introduce additional index j in $N_{1,j}$ and $N_{2,j}$ cycle numbers such that $j=1,2,...,j^*$, where j^* is the critical number of crack tip advances when a specimen falls to pieces, the specimen having an initial crack of length l_0 prior the first loading cycle. Changes in the subarea of a diagram of quasi-brittle fracture for cyclic loading conditions corresponding to the regime of stationary vibrations of blades with regard to damage accumulation are described by inequalities

$$\frac{\sigma_\infty^0}{\sigma_Y} < \frac{\sigma_\infty^\Sigma}{\sigma_Y} < \frac{\sigma_\infty^{*(s)}}{\sigma_Y}, 1 \le s_j \le N_{2,j}-1,\ j=1,2,...,j^*-1,\ \sigma_\infty^0 < \sigma_\infty^{*(s)} < ... < \sigma_\infty^{*(2)} < \sigma_\infty^{*(1)} = \sigma_\infty^*,$$
$$\sigma_\infty^\Sigma / \sigma_Y \ge \sigma_\infty^{*(s)} / \sigma_Y,\ s_j = N_{2j},\ j=1,2,...,j^*,\ \sigma_\infty^0 < \sigma_\infty^{*(s)}, \tag{8}$$

when the critical number of loading cycles (blade vibrations) N_2^* is calculated in such a way

$$N_2^* = 1 + \sum_1^{j^*-1} N_{2,j},\ N_{2,j} \ge 2,\ j=1,2,...,j^*-1,\ N_{2,j^*}=1. \tag{9}$$

Here $\sigma_\infty^{*(s)}$ is the critical load obtained via the sufficient quasi-brittle fracture criterion in the sth loading cycle till $j=1,2,...,j^*-1$ advance, s_j is the cycle number between $j-1$ and j advances, $s_j=1$ corresponds to material in the original state after each crack tip advance, $N_{2,j}$ the number (group) of blade vibration cycles between $(j-1)$ and j crack tip advances, N_2^* is the critical number of loading cycles (9). The first line of relation (8) describes damage accumulation in the pre-fracture zone. In the second line of relation (8) the condition is imposed under which crack tip advance takes place. Relations (8) describe transformation of quasi-brittle fracture diagrams for a composite structure with variable properties under cyclic loading conditions. The cumulative effect of damage accumulation in the pre-fracture zone is observed. The initial sharp crack of length $2l_j$ propagates after the jth advance by Δ_j only in material after its embrittlement. Lengths of model $2l_j$ cracks differ from those of real $2l_{j-1}$ cracks by two pre-fracture zone lengths $2\Delta_j$ after each crack tip advance.

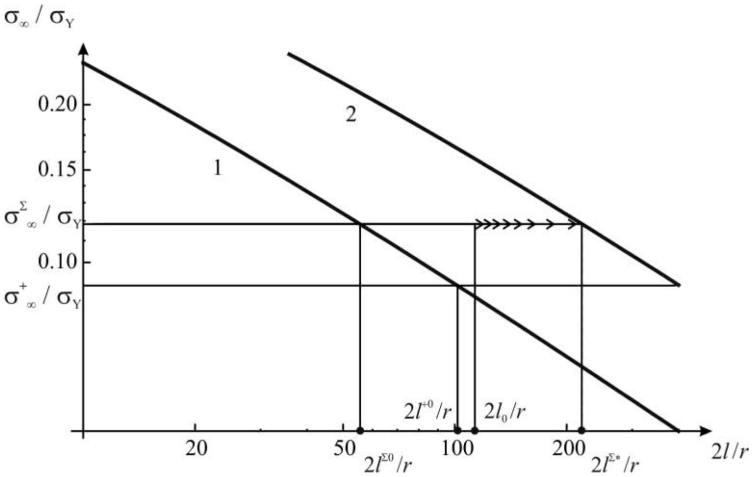

Fig. 3. Stepwise fatigue crack growth.

The scheme shown in Fig. 3 elucidates the stepwise fatigue crack growth during double-frequency loading (see Fig.1 and relations (8) and (9)). The original quasi-brittle fracture diagram is plotted in the log-log coordinates: curve 1 corresponds to stresses σ_∞^0/σ_Y from (5), curve 2 corresponds to stresses σ_∞^*/σ_Y from (5), and the horizontal straight line $\sigma_\infty^\Sigma/\sigma_Y = const$ with arrows reflects advances of real crack tips. Points $\left(2l^{\Sigma 0}/r, \sigma_\infty^\Sigma/\sigma_Y\right)$ and $\left(2l^{+0}/r, \sigma_\infty^+/\sigma_Y\right)$ corresponding to lengths of cracks $2l^{\Sigma 0}$ and $2l^{+0}$ on curve (1) are specified for given loading levels $\sigma_\infty^\Sigma/\sigma_Y$ and σ_∞^+/σ_Y, respectively. On curve 2, the critical point $\left(2l^{\Sigma *}/r, \sigma_\infty^\Sigma/\sigma_Y\right)$ is specified at which the specimen is divided into two parts $2l_{j^*} = 2l^{\Sigma*}$. In order to obey both limitations from (7), the initial length $2l_0$ of a real crack should be governed by the limitations $2l^{+0}/r < 2l_0/r < 2l^{\Sigma*}/r$. Multiple arrows from the point $\left(2l_0/r, \sigma_\infty^\Sigma/\sigma_Y\right)$ to the point $\left(2l^{\Sigma*}/r, \sigma_\infty^\Sigma/\sigma_Y\right)$ on the horizontal straight line $\sigma_\infty^\Sigma/\sigma_Y$ reflect stepwise advance of the real crack tip.

For the loading level $\sigma_\infty^\Sigma/\sigma_Y$, we have

$$\frac{\sigma_\infty^\Sigma}{\sigma_Y} = \left[\left(1 - \frac{5}{8\pi}\frac{\varepsilon_{j-1}^\Sigma - \varepsilon_0}{\varepsilon_0}\right)\sqrt{\frac{2l_{j-1}}{r}} + 1\right]^{-1}, \Delta_j^\Sigma = \frac{5^2}{2^9}\left(\frac{\varepsilon_{j-1}^\Sigma - \varepsilon_0}{\varepsilon_0}\right)^2 \left(\frac{\sigma_\infty^\Sigma}{\sigma_Y}\right)^2 l_{j-1},\ j = 1, 2, ..., j^*. \tag{10}$$

For the loading level σ_∞^+/σ_Y, the same relations as (10) are obtained, if the superscript Σ in (1) is replaced by the sign $+$. It is obvious that $\Delta_j^\Sigma > \Delta_j^+$, therefore, the length of the crack tip advance is $\Delta_j = \Delta_j^\Sigma$: fracture occurs in material after its embrittlement.

The difference between inelastic elongations $\left(\varepsilon_{j-1}^\Sigma - \varepsilon_{j-1}^+\right)$ provokes to damage accumulation in pre-fracture zone material. Coffin [17, 18] and his followers found that there exists a relationship between inelastic material deformation and the number of cycles. The number of cycles of blade vibrations $N_{2,j}$ between the $j-1$ and j crack tip advances are calculated as follows

$$\left(N_{2,j}\right)^C = \left(\varepsilon_1 - \varepsilon_0\right)/\left(\varepsilon_{j-1}^\Sigma - \varepsilon_{j-1}^+\right),\ j = 1, 2, ..., j^* - 1. \tag{11}$$

Here $0,2 \le C \le 1$ are Coffin's constants, numerical values of which depend on material properties [17, pp.76-77]. Parameters of original material $\left(\varepsilon_1 - \varepsilon_0\right)$ and inelastic elongations $\left(\varepsilon_{j-1}^\Sigma - \varepsilon_{j-1}^+\right)$ in two loading regimes enter the Coffin equation.

Crack tip advance under low-cycle loading conditions (Paris' curves)

Obtain estimates of the mean dimensionless rate of crack tip advance for fatigue fracture during one loading cycle

$$V = \frac{\Delta_j^\Sigma/r}{N_{2,j}} = \frac{5^2}{2^9}\left(\frac{\varepsilon_{j-1}^\Sigma - \varepsilon_0}{\varepsilon_0}\right)^2 \left(\frac{\varepsilon_{j-1}^\Sigma - \varepsilon_{j-1}^+}{\varepsilon_1 - \varepsilon_0}\right)^{1/C} \left(\frac{\sigma_\infty^\Sigma}{\sigma_Y}\right)^2 \frac{l_{j-1}}{r}, \quad j = 1, 2, ..., j^* - 1. \tag{12}$$

The mean rate $V = \left(\Delta_j^\Sigma/r\right)/N_{2,j}$ can be measured sufficiently easily in experiments [19].

Taking into consideration relations (10), it is easy to obtain analytical expressions for description a stepwise fatigue crack growth in the form

$$dl/dN = f\left[l/r, \sigma_\infty^\Sigma/\sigma_Y, \sigma_\infty^+/\sigma_Y, \left(\varepsilon_1 - \varepsilon_0\right)/\varepsilon_0, C, w/r\right]. \tag{13}$$

The analytical expression for the crack propagation law (13) includes the current crack length l/r, the plain specimen width w/r, load parameters $\sigma_\infty^\Sigma/\sigma_Y, \sigma_\infty^+/\sigma_Y$, parameters of the $\sigma - \varepsilon$ diagram of quasi-brittle material $\left(\varepsilon_1 - \varepsilon_0\right)/\varepsilon_0$, and Coffin's constant C. Calculation results obtained via relations (13) are given in Figs. 4a and 4b for the linear $C = 1$ and nonlinear $C = 0.5$ summation of damages, when $\left(\varepsilon_1 - \varepsilon_0\right)/\varepsilon_0 = 2,5$, $\sigma_\infty^\Sigma/\sigma_Y = 0,12$, $\sigma_\infty^+/\sigma_Y = 0,1$. Curves 1 and 2 in each Figure correspond to plain specimens of the infinite ($w \to \infty$) and finite ($w = 4000$) width. The difference between curves in Figs. 4a and 4b is associated with different types of summation of damages in pre-fracture zone material: increase in the rate V in the case of nonlinear summation is much less than that in the case of linear summation. Curves 2 in each Figure are reversed S-shaped. "Explosive" rise of the rate V of the crack tip advance before a specimen is broken apart is associated with the finite width of a specimen. This increase in the rate V bears no relation to specimen material since it is conditioned by specimen geometry.

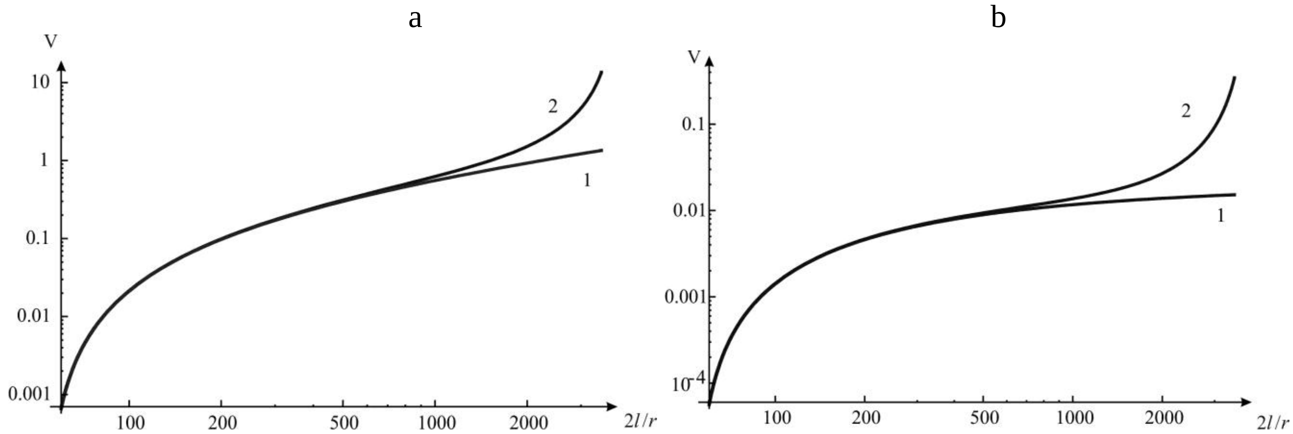

Fig. 4. Curves of increase in crack lengths under linear (a) and nonlinear (b) summation of damages in pre-fracture zone material.

Stepwise fatigue crack growth under cyclic loading conditions is discussed in detail in monograph [20, pp 122-132], where kinetics of the plasticity zone at crack tips is directly related to features of fatigue crack growth. Much attention is given in the same monograph as to the self-similarity of the process of crack tip advance. The work by Ciavarella et al. [21] is natural continuation and generalization of the Barenblatt-Botvina approach [22]. In Conclusions of this work [21], the attention is given to other approaches in which crack propagation laws different from classical Paris' curves [23-25] have been proposed.

The considered law (12) generalizes the linear (43) and nonlinear (44) representations from the seventh section of work [21]. The effect of a finite specimen width on fracture process taking into consideration the correction to SIFs has been evaluated approximately. The author succeeded in plotting a curve that looked like classical Paris' curve only after transformations of the law (12). The similar statement is also valid in accordance with the results of work [1]. It seems to be not feasible to guess beforehand what is a form of the law to be chosen for describing crack growth [21], since scenarios of fatigue loads may be very various and conditions of self-similarity in stepwise crack growth, generally speaking, are not fulfilled. In addition, it is necessary to consider transient processes at the stage of onset of crack advance at the final stage before a specimen of the finite width is broken apart, and not just the second stage of the process corresponding to steady-state process of crack tip advance. In representation of the rate of change in the length of crack at its stepwise growth in structured materials, preference should be given to the law in the form (12). It would be desirable to find these laws be guided by using more or less plausible models of material deformation and damage accumulation in them near crack tips under cyclic loading conditions. In Murakami and Millers' opinion "fatigue damage requires to be expressed in terms of a crack" and "…models which ignore the reality of fatigue damage as expressed in terms of cracks should not be used for fatigue life predictions" [23, p. 991]. Under fatigue failure of metals "the problem of nonlinearity of damage accumulation …, which can be realized at various structural levels must be taken as fundamental" [26, p. 14]. Under nonlinear strain of materials, self-organization of a system occurs in pre-fracture zones at the mesolevel [27].

Conclusion

The proposed diagram of quasi-brittle fatigue fracture describes subcritical stepwise crack growth and predicts lifetime of a specimen under fatigue conditions using approximations of the standard $\sigma-\varepsilon$ diagrams of quasi-brittle material and characteristics of damage accumulations in pre-fracture zone material. The modified Leonov-Pansyuk-Dugdale model allows fatigue damages of material in the pre-fracture zone to be expressed in terms of cracks just as in the case of linear, so in the case of nonlinear summation of damages. The lifetime of a specimen depends on a loading program. For considered loading regimes, the derived relations (laws) describing crack growth do not resolve into simple laws of fatigue crack propagation. These laws, in general case, are not self-similar processes. The work was financially supported by Russian Foundation for Basic Research (Grant 10-08-00220).

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