ON THE CHARACTERISTIC LENGTH USED IN NOTCH FRACTURE MECHANICS

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Abstract

In this paper, we examine relationships between characteristic length and half of the notch radius, grain size, plastic zone and the minimum of the relative stress gradient.
Each method to determine characteristic length is discussed at the end of each section. The paper ends with the author's opinion about the different methods.

Introduction

Local fracture criteria are based on an accurate description of stress, strain or strain energy density in a close volume to the defect promoting fracture. They are based on two local parameters in term of stress, strain or strain energy density and a characteristic length [1]. However, it has been proved that fracture process is not governs by the maximum stress (or strain or strain energy density) but by a lower stress (or lower strain or strain energy density) called the effective stress. The first idea of a local stress fracture criterion has been suggested by Orowan [2].
Effective stress based on average stress over a characteristic length have been earlier considered by Neuber [3], Novozhilov [4]. The failure criterion proposed by Novozhilov [4] and expanded by Seweryn [5], suggests toconsider the average normal stress along the anticipated path of the failure. So failure occurs when the average stress equals a material dependent value, denoted by $\sigma*_c$, which is the stress at failure. The average stress is considered along the effective distance. This method is called sometimes the line method [6].

The fracture criterion can be written in the following generalized form:

$$\frac{1}{X_c}\max_{-\pi<\theta\leq\pi}\int_0^{X_{ef}}\sigma_{\theta\theta(r)}dr = \sigma_c^* \quad (1)$$

where X_{ef} is the effective distance , $\sigma*_c$ the failure stress, $\sigma_{\theta\theta}$ the circumferential stress, r and θ polar coordinates.

Local fracture criterion based on a characteristic stress corresponding to a characteristic length on the stress distribution has been introduced by Peterson [7]|. Whitney and Nuismer [8] have proposed this criterion with the following form:

$$\max_{-\pi<\theta\leq\pi}\left[\min_{0\leq r\leq X_{ef}}\sigma_{\theta\theta}(r)\right] = \sigma_c^{**} \tag{2}$$

where σ^{**}_c another characteristic stress. This approach is called the point method (PM)[6]. Pluvinage [1] has proposed to averaging the stress distribution over the entire process volume V_{ef}. Then the fracture criterion has the following form :

$$\frac{1}{V_{ef}}\iiint_{\Omega}\sigma_{yy}(x,y,z)\times\phi(x,y,z,\chi)\,dv = \sigma_c^{*} \tag{3}$$

where, $\chi(x)$ and $\sigma_{yy}(x)$ are relative stress gradient and opening stress or maximum principal stress along notch tip, respectively. This method is called Volumetric Method (VM). In this paper, we examine relationships between characteristic length and half of the notch radius, grain size, plastic zone and the minimum of the relative stress gradient.

Characteristic length related to notch geometry

<u>Relationship between characteristic length and notch radius</u>

The characteristic length was associated with the notch radius firstly in the Creager and Paris [9] analysis of the stress distribution at notch tip. For rounded V-notches, analytical expression of notch tip stress distribution for elastic material was developed by Filippi et al [10]. They introduce in this analytical expression, the distance between the origin of the polar coordinates system and the notch tip r_0. This distance r_0 depends on notch radius and notch angle. For the particular case of a zero notch angle, one finds:

$$r_0 = {}^{\rho}/_{2} \tag{7}$$

This leads to the following value of the characteristic length called here effective distance

$$X_{ef} = \frac{\rho}{2} \tag{8}$$

<u>Validity of the value of effective distance equal to half the notch radius</u>

Experimentals results obtained by Ayatollahi et Al (11) indicate that the relationship between effective distance and half of the notch radius is satisfied for a brittle material PMMA at 60°C (figure 1).

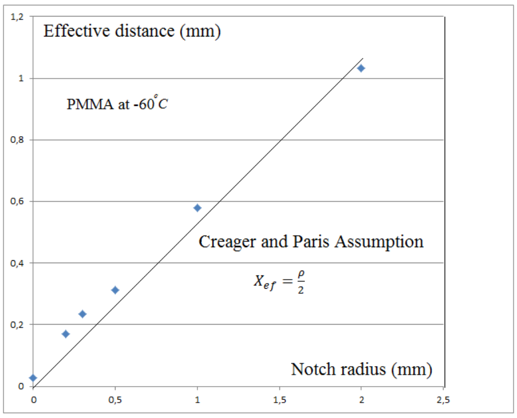

Figure 1 : Effective distance versus notch radius for PMMA at -60°C [11].

For a low strength steel, Akkouri et Al [12] have found a linear relationship between effective distance determined by Volumetric method [1] and notch radius of the following type

$$X_{ef}^{c} = D.\rho + B \quad (9)$$

(Often D coefficient is not strictly equal to 2).

Discussion on relationship between effective distance and notch radius

For brittle materials, relationship given by equation (8) is satisfied if notch radius is greater than a critical value. If not, the physical nature of the characteristic length prevails and is related to grain size. However, when the material becomes more ductile, deviation from equation (8) arises. For very ductile material, the half of the notch radius is a distance less than the distance where the maximum stress occurs. It loses its definition of characteristic length because this doesn't take into account the most stressed region at crack tip.

Characteristic length related to material properties

Characteristic length connected to grain size

Characteristic length connected with grain size is the basis of the Ritchie, Knott and Rice [13] local stress fracture criterion (RKR). This criterion belongs to Point Method (PM) and the characteristic length is called the characteristic distance X_c and is equal to grain size. This criterion is devoted not to notch but to crack but can be used for very brittle material where the characteristic length is very small and taken as equal to grain size for physical reasons.

Characteristic length connected to plastic zone size

Taylor et al [14] in the theory of critical distance (TCD) hve suggest to use a definition of the characteristic length L close to the value of the "plane stress plastic zone size" :

$$L = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_c^*}\right)^2 \quad (10)$$

In Taylor et al definition, the effective distance is equal to 2L in line method and L/2 in point method. In the above definition, K_{Ic} is the plane strain material toughness, whereas the inherent material strength under static loading. $\sigma*_c$ is upper but close to ultimate strength. In this procedure, it is assumes that characteristic distance is intrinsic to material then it is the same for a sharp and a blunt notch. This assumption is also valid for the effective stress at failure. Lazzarin and Zambardi [15] have postulate that for fracture strain energy density (SED W*) is independent of the opening angle ψ then for ψ = 0 (a crack) and for $\psi = \pi$ (a flat specimen), then the effective distance is given by :

$$X_{ef} = \frac{(1+\upsilon)(5-8\upsilon)}{4\pi}\left(\frac{K_{Ic}}{\sigma_c^*}\right)^2 \qquad (11)$$

Definition of effective distance can be more generally written as

$$X_{ef} = \frac{\zeta}{\pi}\left(\frac{K_{Ic}}{\sigma_c^*}\right)^2 \qquad (12)$$

with ζ = 1 for Taylor et al [14] and ζ= 0845 for Lazzarin et al [15]. Using The implicit gradient method (IGM) applied along with the maximum principal stress criterion, Tovo and Livieri [16] have proposed that the constant ζ is equal to 0.545.

<u>Evolution of notch plastic zone with notch radius</u>

One note that the "plastic zone size" defined in Taylor et al [17] and Lazzarin et al [19] approaches are relative to crack plastic zone size. It has been shown [1] that the notch plastic zone depends on notch radius. It decreases when the notch radius decreases. The size of the notch plastic zone R_ρ may also express by the following relationship:

$$R_\rho = A\left(\frac{K_{\rho,c}}{\sigma_y}\right)^2 \qquad (13)$$

where A is a constant, $K_{\rho,c}$ the notch fracture toughness and σ_y the yield stress..

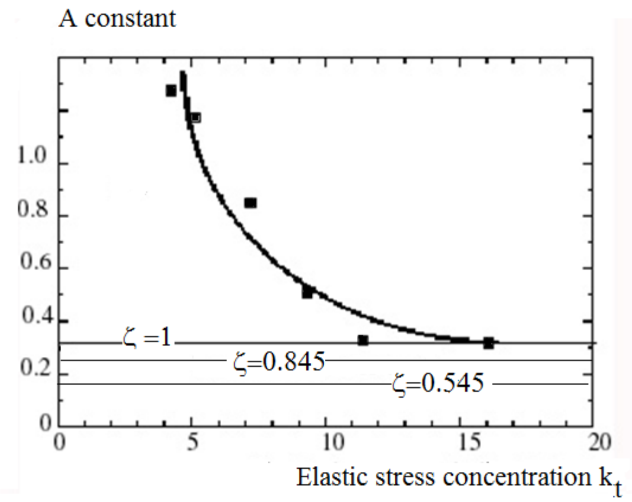

Figure 2 : Evolution of A constant versus elastic stress concentration. Low strength steel [1].

One notes that in this case, the yield stress is introduced in equation (13) but one assumes to have a similar evolution than characteristic length given in equation (12). On figure 2, evolution of A constant is reported not versus the notch radius but versus theelastic stress concentration factor k_t. One notes that a negative power function describes A evolution. The notch plastic zone has been computed by FE method using the Von Mises plasticity criterion. material is a low strength steel. On figure 2, values proposed by Taylor et al [17] (ζ=1), by Lazzarin et al [19] (ζ=0.845) or by Tovo and Livieri [20] ζ= 0.545) are also reported. The A constant trends to reach for a crack the value (ζ=1). The other values (ζ= A= 0.845 or ζ=A= 0.545) are lower than the asymptotic value. Increasing of notch fracture toughness with notch radius has been taken into account. Figure 2 indicates clearly the difficulty to use a material constant (the plastic zone) as effective distance. The choice of the crack plastic zone size is also questionable when the problem concerns a notch with a non-zero radius.

<u>Discussion</u>

For brittle materials, the characteristic length is related to the grain size. In some other cases instead, such as composites, polymers, wood, etc., the material is not formed by conventional grains, so that the effective distance value is related to other internal microstructural barriers.

For more ductile material, the grain size is a distance too small to be greater than the maximum stress distance and to use it as characteristic length loses its physical meaning.

Generally in equation (12), constant ζ is often equal to unity, whereas but its value changes as the definition used to calculate the equivalent stress field varies: for instance, if the IGM is applied along with the maximum principal stress criterion, as it will be done below, constant ζ is equal to 0.545 [17]. This not clarify the choice of the constant ζ. In procedure describe by [6], it is assumes that characteristic distance is intrinsic to material then it is the same for a sharp and a blunt notch. This assumption is also valid for the effective stress at failure. One sees in the next section, this assumption is discussed in details and the conclusion cannot support it.

The second relevant difference is that the IGM uses the ultimate tensile stress, σ_u, as the reference failure stress, whereas the TCD calculates the inherent material strength according to the procedure sketched in Figure 3. σ_c* is generally much larger than σ_u. The choice of the reference failure stress is then not easy.

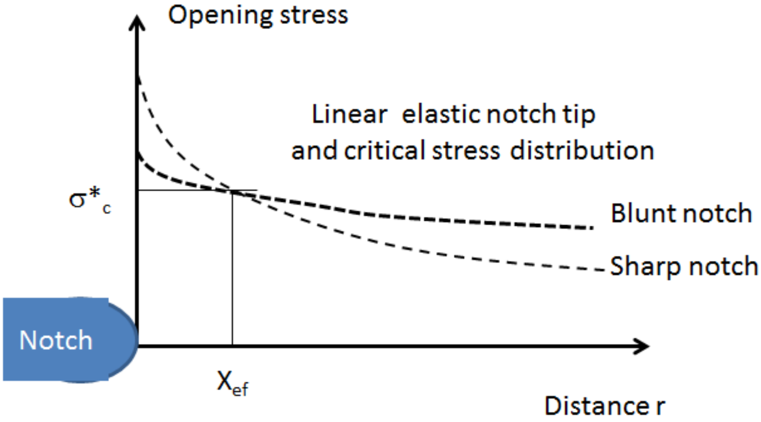

Figure 3 : Procedure to determine the characteristic length and the inherent material strength according to Taylor (6).One notes that this procedure is only valid for brittle materials.

Finally, it is assumed that the characteristic length is intrinsic to material. This assumption can be seen through the different types of stress distribution on the same material with different notch geometry, loading mode etc. which are very different. For physical reasons, it is necessary that in any case the maximum stress distance is less that the characteristic length. For particular situations, this condition is not fulfilled.

Characteristic length related to stress distribution

<u>Effective distance defined as the minimum of the relative stress gradient</u>

In the Volumetric Method (VM) [1], the effective distance is considered as a characteristic of the stress distribution. The Volumetric Method is a local fracture criterion, which assumed that the fracture process requires a certain volume. This volume is assumed as a cylindrical volume with effective distance as its diameter and thickness as its height. Physical meaning of this fracture process volume is “the high stressed region” where the necessary fracture energy is stored. The difficulty is to find the limit of this “high stressed region”. This limit is a priori not a material constant but depends on loading mode, structure geometry and load level. The size of the fracture process volume reduces to the effective distance according to the above mentioned assumptions and is obtained by analysis of the stress distribution.

A graphical method based on the relative stress gradient χ associated the effective distance to the minimum of χ. This inflexion point is given easily as the minimum of relative stress gradient χ. χ is then a tool for determining the effective distance and not its definition.

The effective stress for fracture is then considered as the average volume of the stress distribution over the effective distance. However stresses are multiply by a weight function in order to take into account stress gradient due to geometry and loading mode.

<u>Effective distance and constraint</u>

The elastic stress fields in a region surrounding the crack tip can be characterized by the following solution [17]

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) + T\delta_{xi}\delta_{xj} + A_3\sqrt{2\pi r} + 0(r) \ , \qquad (14)$$

where K_I is the stress intensity factor, $f_{ij}(\theta)$ is the angular function, δ_{ij} is the symbol of Kronecker's determinant. A polar coordinate system (r,θ) with an origin at the crack tip is used. The second term is called the *T*-stress. T-stress is a constant stress acting parallel to the crack line in the direction *xx* of the crack extension with a magnitude proportional to the gross stress. The non-singular term *T* may be a tensile or a compressive stress. Positive *T*-stress strengthens the level of crack tip stress triaxiality and leads to high crack tip constraint while negative *T*-stress leads to the loss of constraint. Several methods have been proposed in literature to determine the T-stress for cracked specimen. The stress difference method has been proposed by Yang et al. [18]. In this method, the T-stress is evaluated from stress distribution on the line of crack extension. Generally computed by finite element method, it is the difference between stress σ_{xx} parallel to crack plane and opening stress σ_{yy}.
For a notch, T stress is not constant along the ligament as we can see on figure 4. In this figure, T stress results have been obtained by finite element method and are relative to a pipe submitted to internal pressure. The geometry considered in this study is a pressurized cylinder with a V-shaped longitudinal surface notch.

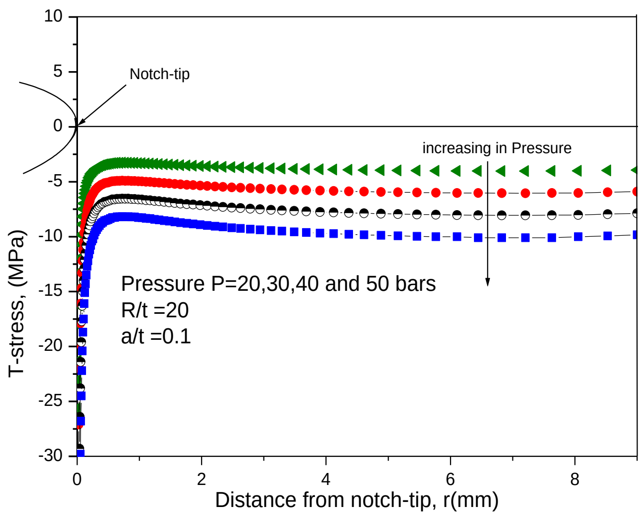

Figure 4: Evolution of T stress versus distance over ligament. Pipe submitted to internal pressure. Diameter 400 mm thickness 10 mm.

The wall thickness is 10 mm and the diameter of the pipe is 400 mm. Four different values of a/t were ranged from a/t = 0.1 to 0.75 as well as four different values of *p* ranging from pressure of 20 bars to 50 bars. Averaging the T-stress inside the effective distance (determined by Volumetric Method), the effective T-stress (T_{ef}) can be defined in the following form:

$$T_{ef} = \frac{1}{X_{ef}} \int_0^{X_{ef}} T_{xx}(r).\Phi(r).dr \ . \quad (15)$$

Here, $T = T_{xx} = (\sigma_{xx} - \sigma_{yy})_{\theta=0}$ is the T-stress distribution along of the ligament (r) in the xx direction and $\Phi(r)$ is the weight function.

One notes that constraint (defined by the effective T stress) plays a role on the effective distance value. It increases when the constraint increases from negative to positive values (figure 5).

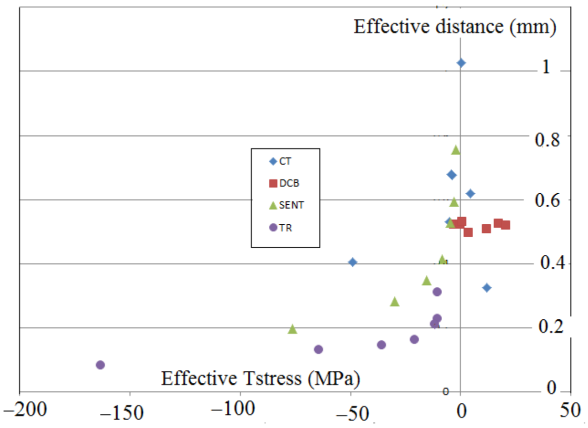

Figure 5 : Evolution of the effective distance versus effective T stress for a X52 steel pipe. Values obtained from 4 specimen types ((CT, SENT, DCB and RT).

Conclusion

Notch fracture is based on local stress fracture criteria where the inherent fracture stress is obtained by generally by averaging the stress distribution over a characteristic length. In the present paper, the three ways to obtained this characteristic length are discussed. One is related to notch geometry, the second is intrinsic to material and the third is related to the stress distribution. It is well known that fracture toughness, including notch fracture toughness is sensitive to constraint. Effective distance obtained from the minimum of the relative stress gradient is actually the only way to described evolution of notch fracture toughness versus constraint by the way of stress triaxiality, Q factor or T stress.

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