On Multiaxial Fatigue Models for Notched Specimens

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Abstract. Fatigue in a notched specimen is a common problem in life assessment. The stress state at the notch root is often multiaxial and varies with distance even under uniaxial loading conditions. In the present work four multi-axial fatigue models are investigated. Especially, the Cruse-Meyer Model is extended to multiaxial fatigue and studied in detail. By combining it with the critical distance method, the notched fatigue life can be predicted reasonably.

1. Introduction

In design of a gas turbine engine, notches cannot be avoided, for instance, in disks and blades. They cause local stress concentrations and lead to fatigue failure earlier. In industrial design the stress concentration factor, K_t , is often used to characterize the degree of stress concentration. Under cyclic loading, the fatigue notch factor, $K_{f,}$ is introduced to describe the change of fatigue strength [1]. Notched fatigue strength depends not only on K_t, but also on the stress gradient which describes the material volume with a high stress. Peterson [2] and Neuber [3] try to describe the difference between K_f with K_t through empirical relations. Siebel and Stieler [4] proposed using the stress gradient at a notch root for evaluation of the notch effect. Taylor developed the former works and proposed the theory of critical distance (TCD) [5]. The TCD proposed four different versions: (a) the Point Method (PM) in which the stress at a distance from the notch root is used for life prediction; (b) the Line Method(LM), in which the average stress over a distance is used; (c) the Imaginary Crack Method (ICM) in which an imaginary crack of length is placed at the notch root and LEFM conditions assumed and (d) Finite Fracture Mechanics (FFM) in which the facture conditions are derived by assuming a finite increment of crack growth. At beginning the TCD is applied to predict the fatigue limit and focus on the HCF regime. Later Susmel and Talyor [6,7,8] successfully extended the TCD to the LCF regime. By employing an elasto-plastic critical plane approach, the TCD is extended to consider multiaxial fatigue.

Many structural components are usually subjected to repeated multiaxial loadings, such as rotating, biaxial/triaxial loading, tension-torsion loading etc. Even under uniaxial cyclic loading, the presence of a notch will cause a local multiaxial stress fields near the notch root. Fatigue evaluation under multiaxial loading becomes one of major concerns in modern structure design and must be considered in notched fatigue life estimation.

In the present paper, four multiaxial fatigue models are reviewed and compared. Then the extended Cruse-Meyer model is applied to the notched fatigue life prediction based on the Point Method and Line Method of the TCD group methods.

2. Multiaxial Fatigue Criteria - Brief Review

(a) Findley criterion

The fatigue damage of Findley model [9] is defined as a linear combination of the shear stress amplitude and the normal stress.

$$(\frac{\Delta\tau}{2}+k\sigma_n)_{\max}=f \quad . \tag{1}$$

The critical plane is defined as one or more planes within a material subject to a maximum value of the damage indicator, *f*. The critical plane of Findley model is calculated based on the macroscopic stress state but not related to the slip direction on the microscopic level. Findley did not defined a mathematical formula for the damage indicator, *f*. Some researchers assume that it can be determined by the Basquin's equation from the shear-mode cracking.

$$\frac{\Delta\tau}{2}+k\sigma_n=\tau_f'(2N_f)^b. \tag{2}$$

(b) Socie model

Based on crack growth observations, Socie [10] found that the normal strain in the plane of maximum shear strain will accelerate the fatigue damage process during crack opening. The mean normal stress $\sigma_{n,m}$ was included to take account of the friction force reduction between slip planes. This results in

$$\frac{\Delta\gamma_{\max}}{2}+\frac{\Delta\varepsilon_n}{2}+\frac{\sigma_{n,m}}{2}=f. \tag{3}$$

The equation was subsequently modified to correlate with total strain torsional data

$$\frac{\Delta\gamma_{\max}}{2}+\frac{\Delta\varepsilon_n}{2}+\frac{\sigma_{n,m}}{2}=\frac{\tau_f'}{G}(2N_f)^b+\gamma_f'(2N_f)^c \quad . \tag{4}$$

(c) Park-Nelson energy model

Under cyclic loading, the strain energy density varies as the path of stress and strain fluctuates. Strain energy under cyclic loading can be divided into two parts: variable and static (or mean) strain energy part. The lifetime is related to the strain energy in analogy to the Manson-Coffin equation, as

$$W=\Delta W+W_m^h=AN_f^{\alpha}+BN_f^{\beta} \quad . \tag{5}$$

The static strain energy is associated with the mean hydrostatic stress. The total variable strain energy per cycle in the above equation can be decomposed into deviatoric (plastic+elastic) and volumetric parts,

$$\Delta W=\Delta W_p^d+\Delta W_e^d+\Delta W^h \quad . \tag{6}$$

Further details of the Park-Nelson model are referred to literature [11].

(d) Extended Cruse-Meyer model

The Cruse-Meyer model [12] includes both cyclic strain range $\Delta\varepsilon$ and mean stress σ_m in an explicit power law form for the uniaxial loading condition:

$$N_f=A\Delta\varepsilon^B 10^{C\sigma_m}, \tag{7}$$

where *A*, *B*, *C* are model parameters, σ_m is the mean stress, N_f is the number of cycles to failure, $\Delta\varepsilon$ is the strain amplitude.

The model can be extended to multiaxial fatigue by combining with the critical plane concept. Taking the concept of the Cruse-Meyer model, material failure under stress multiaxial state is described by the maximum normal strain range and the mean normal stress acting on the maximum normal strain range plane. The extended Cruse-Meyer model is expressed as

$$N_f=A(\Delta\varepsilon_{\max})^B 10^{C\sigma_{n,mean}}. \tag{8}$$

Obviously, this equation is identical with the original Cruse-Meyer model.

3. Verification of the Fatigue Models

In the previous years, different multi-axial fatigue tests were done on Inconel 718 alloy, a typical high temperature Nickel-based alloy in the aerospace industry. In this study, four sets of multi-axial low cycle fatigue test results are collected from the public literatures [13,14,15,16]. The considered results included 100 data points which were tested at room temperature under uniaxial loading and proportional and non-proportional biaxial loadings.

Following the previous brief review, the fatigue failure criteria were used to calculate the fatigue lives. Various material constants were from uniaxial fatigue data by fitting method. Fig. 1 presents the comparison between the predicted fatigue lifetime with the experimental fatigue lifetime for each criterion. Most of data are within the scatter band with a factor of 2. In details the models show different life predictions.

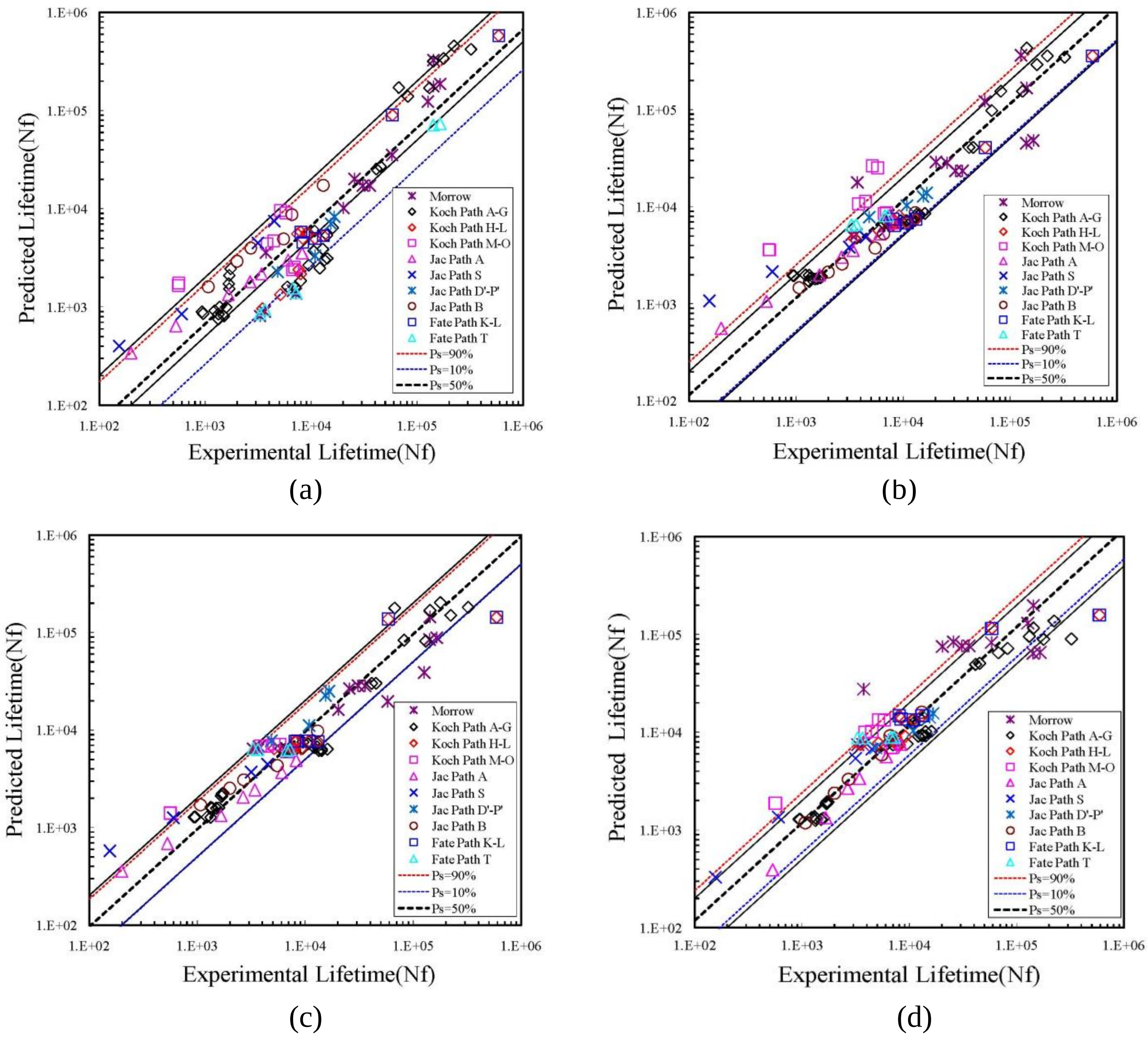


Figure 1. Comparison of the predicted fatigue lifetime and experimental data for Inconel 718 after (a) Findley model; (b) Socie model; (c) Park-Nelson model; (d) Extended Cruse-Meyer model.

To compare models quantitatively, the logarithm normal distribution analysis is presented. The calculated fatigue lifetime is compared with the experiments. Two error parameters are applied for the fatigue life verification:

(1) The mean value is defined as:

$$X = \frac{\sum[\log(N_p) - \log(N_e)]}{n}. \tag{9}$$

(2) The standard deviation is defined as:

$$S = \sqrt{\frac{1}{n-1}\sum\{[\log(N_f) - \log(N_e)] - X\}^2}, \tag{10}$$

where n is the total number of test data, N_f is predicted fatigue lifetime and N_e is experimental lifetime.

Obviously, the mean X predicts the conformity between the prediction and experiment. It indicates the degree of offset of the predicted fatigue lifetime from the experimental fatigue lifetime. If this value is negative, the predicted fatigue life will be on the conservative side. The standard deviation S reflects the scatter of the results. It provides a relative measure of the degree of scatter about the curve and the ability of failure criterion to correlate the stress/strain to the fatigue damage parameter.

Table 1. Comparison of mean values and standard deviations

Model	Mean value	Standard deviation
Findley Model	-0.17255	0.31927
Socie Model	0.05943	0.26505
Park-Nelson Model	-0.01941	0.2214
Extended Cruse-Meyer Model	0.0733	0.2387

A good model is one for which both the mean and standard deviation of the error are minimized. From the Table 1 we learn that the deviation of the stress based Findley Model is larger than the strain based models and the energy based models. The mean values are not so different, except the Findley model. Generally, Park-Nelson Model provides the best agreement with experiment and the smallest scattering. The extended Cruse-Meyer Model takes the second place. The latter is simple and practical for engineering application due to its explicit expression form. Both two models show good correlations with proportional and non-proportional loading cases.

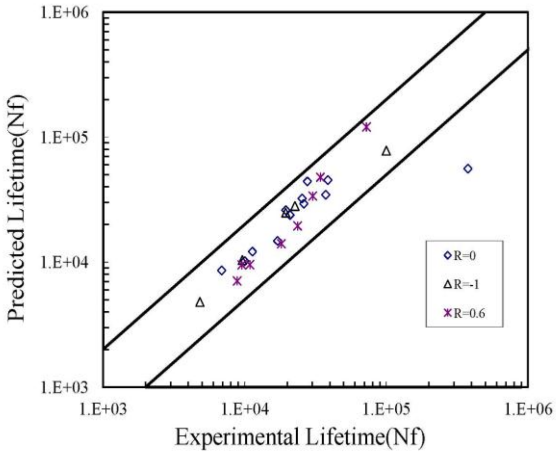
4. Smooth and Notched Fatigue Tests at Elevated Temperature

Strain controlled tests [17] were conducted in Karlsruhe on a series of tensile smooth and notched bar of Inconel 718 alloy at 300°C and 500°C. The stress concentration factor K_t for the notched bar is 2.1 and the notch radius is 3.255 mm. Three tensile loading paths (R=-1, 0, 0.6) were employed to identify the fatigue damage parameters. Extended Cruse-Meyer model was chosen for the following life prediction.

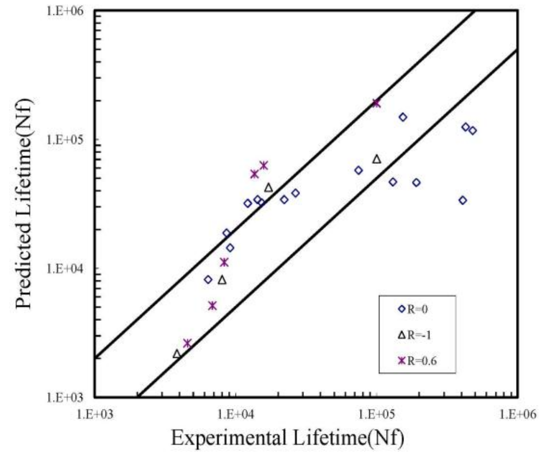
Table 2. Extended Cruse-Meyer Model Parameters

Temperature	Parameter A	Parameter B	Parameter C
$300^\circ C$	9.62E-6	-4.689	-0.0014
$500^\circ C$	1.81E-14	-9.11748	-0.00237

From the smooth fatigue test data, the least square fitting algorithm was used to obtain the parameters A,B,C of the extended Cruse-Meyer Model. The life comparison between the predicted results and the experimental results are plotted in Fig. 2. For 300°C the Cruse-Meyer model yields satisfied results and for 500°C, however, some data are fall outside the scatter band.



(a)



(b)

Figure 2. Fatigue life comparison results based on Cruse-Meyer model at (a) 300°C and (b) 500°C.

4. FEM Simulation for the Notched Tensile Bar

To study fatigue failure around the notch ABAQUS/Standard had been used for the FEM analysis (Fig.3). The stress-strain relationships for 300°C and 500°C are approximated by the Ramberg-Osgood model, as shown in Fig. 4. The elastic moduli are 188000 MPa and 174000 MPa for 300°C and 500°C, respectively. Computations are for cyclic loadings with isotropic hardening assumed.

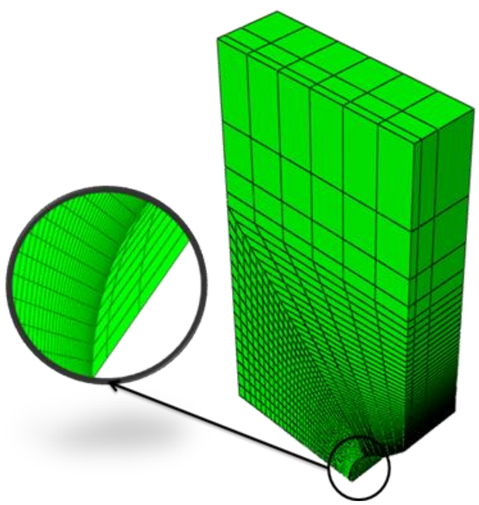


Figure 3. FEM model

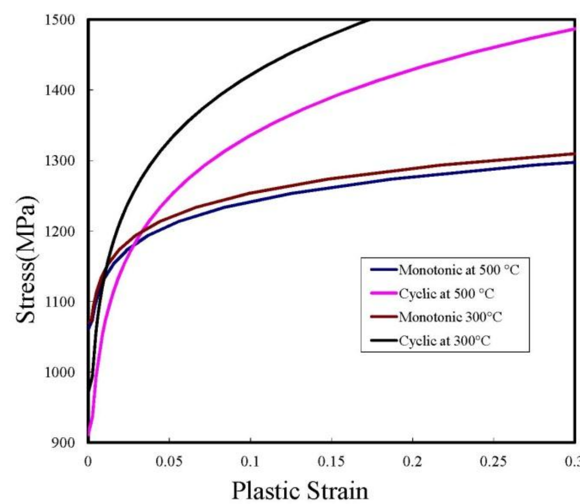
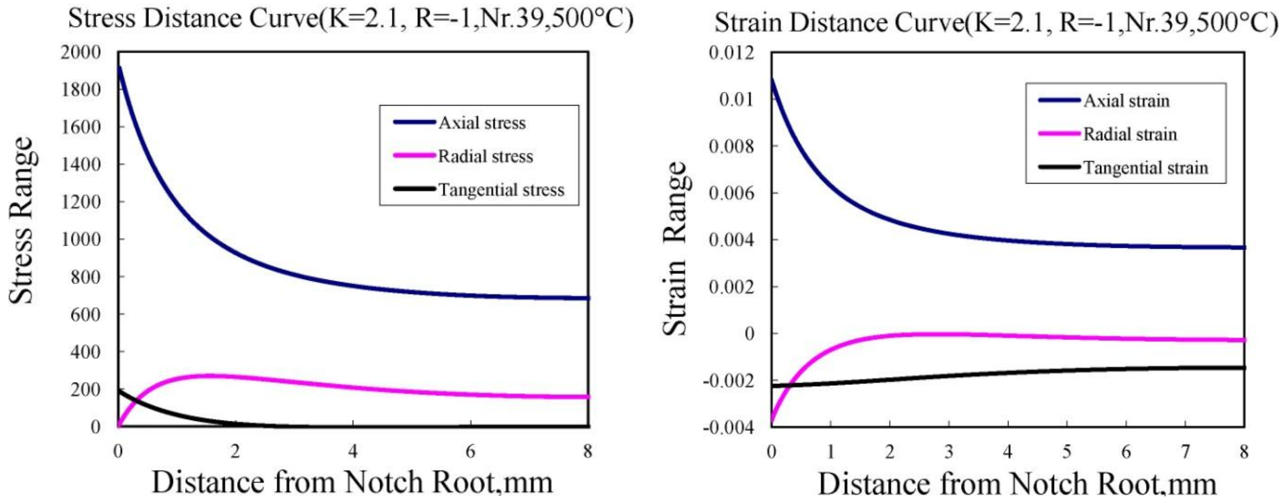


Figure 4. Stress-strain curves for Inconel 718.

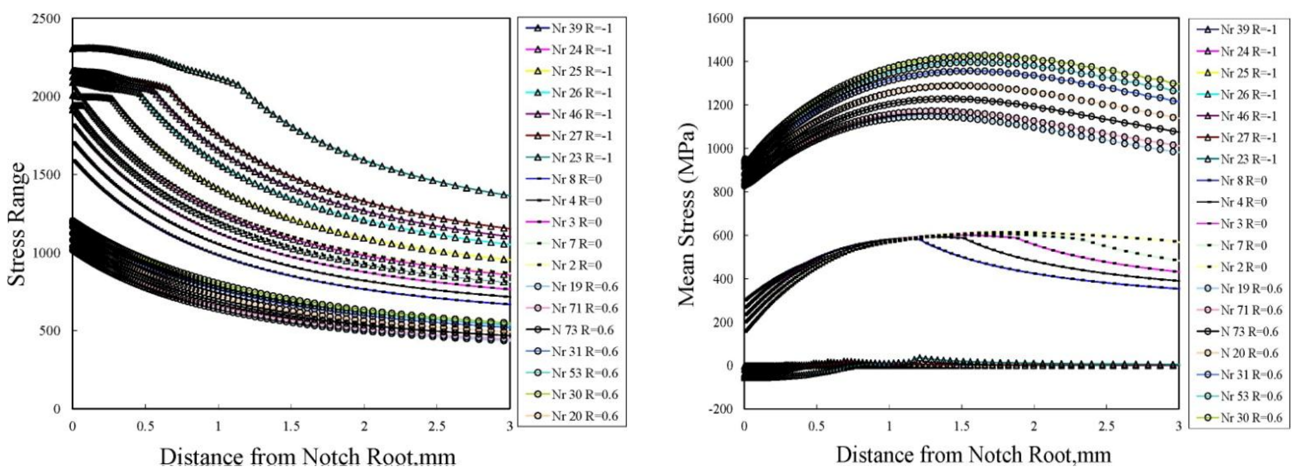
In fatigue analysis the alternating stresses and strains are of importance which are half of the stress and strain range. Fig. 5 shows stress and strain range distributions at the maximal loading along the radial direction from the notch root for R=-1. Due to the small peak stress at the notch root there is no significant yielding around the notch root. Below the root surface the stress state is general multiaxial loading mode, although the axial tensile stress dominates. Additionally, the stresses and strains vary with the distance substantially. Hence, the multiaxial fatigue models find application to analyze the problem.

As indicated in the Fig. 6(a), the stress concentration and plastic deformations happen near the notch root when the mean stress is high enough and may cause strain hardening and cyclic hardening. Due to the local plastic deformations, the peak stress range is leveled off. Then the stress range decreases along the radial direction, however, the mean stress shows in a parabolic form, the lowest point is at the notch root and the highest at the distance ca. 1.5 mm (Fig. 6(b)).



(a) (b)

Figure 5. (a) Stress range and (b) strain range distributions along the radial direction from the notch root at the maximal loading.



(a) (b)

Figure 6. The stress distribution curves of all simulated results along the radial direction from the notch root of (a) axial stress and (b) mean stress at $500^{\circ}C$.

5. Notched Fatigue Prediction

The prediction from the extended Cruse-Meyer model is displayed in Fig. 7, based on the local stresses and strains at the notch root. The parameters *ABC* are taken from the smooth fatigue tests listed in Table 2. Obviously, the prediction contains a wide scattering band and is too conservative, in comparing with experiments.

Combining the extended Cruse-Meyer model with the critical distance method should improve the prediction. Two criteria, i.e. PM based and LM based, can be defined as:

$$N_{p,d} = [A(\Delta\varepsilon_{\max})^{B} 10^{C\sigma_{n,mean}}]_{d}, \qquad (10)$$

$$N_{p,l} = \frac{1}{d}\int_{0}^{d} A(\Delta\varepsilon_{\max})^{B} 10^{C\sigma_{n,mean}}, \qquad (11)$$

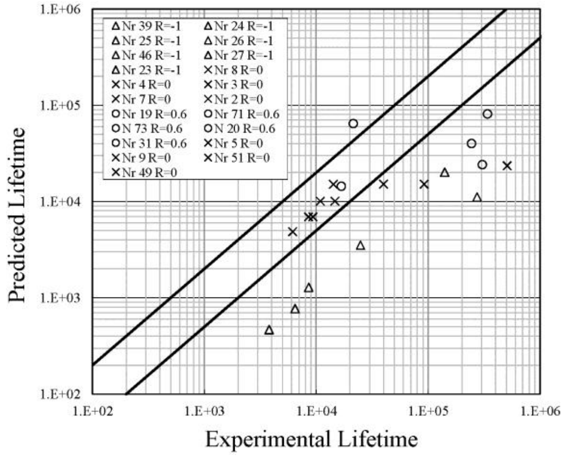
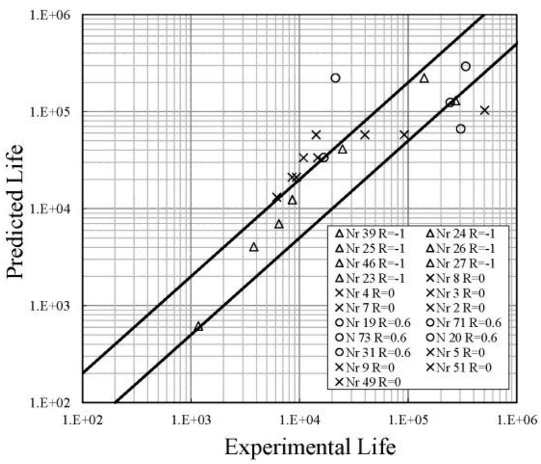
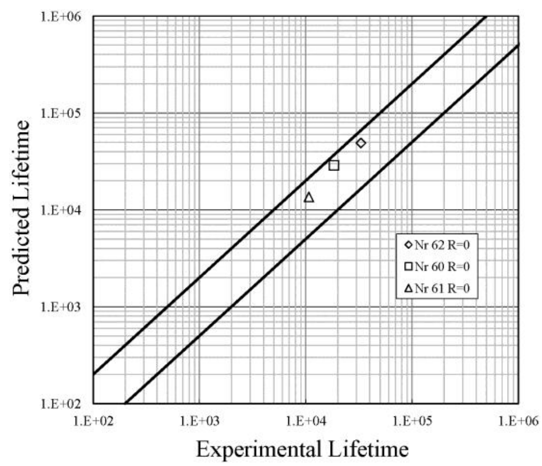


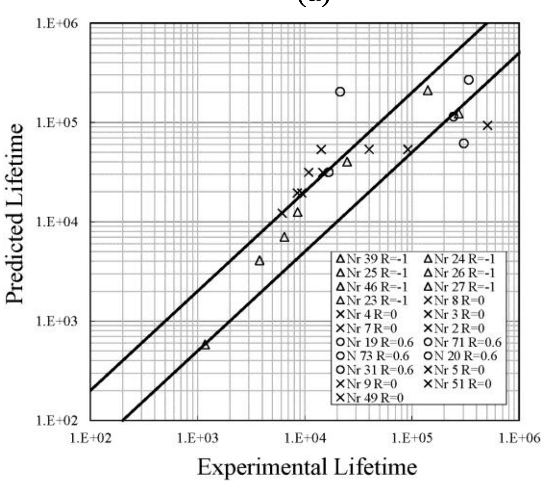
Figure 7. The comparison between predicted and experimental results at $500^{\circ}C$ on surface.



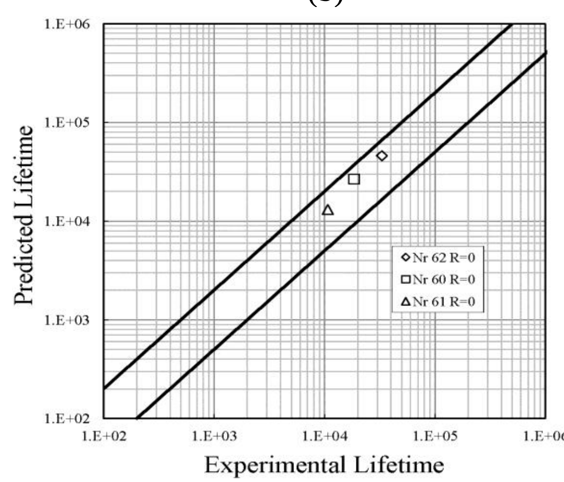
(a)



(b)



(c)



(d)

Figure 8. The comparison between predicted and experimental lifetime at (a) $500^{\circ}C$, (b) $300^{\circ}C$ using PM, d=0.394 and (c) $500^{\circ}C$, (d) $300^{\circ}C$ using LM, d=0.596.

One key issue in the critical distance method is determining the critical distance, *d*. Until now there is no effective method except searching to determine the critical distance for elasto-plastic problem. The most popular method is to determine the distance by minimizing the deviations between prediction and experiments. For instance, one may use the least square method to minimize $||N_{p,d}-N_f||$ or $||N_{p,l}-N_f||$ and to find the most appropriate distance based on the R=-1 at 500°C. It follows that the critical distance for the PM is 0.394 mm and 0.596mm for the LM. Generally, one may assume that the distance should be independent of loading ratio and temperature.

Figures 8(a) and 8(b) show the results for Cruse-Meyer PM model at 500°C and 300°C. The prediction of the Cruse-Meyer LM is documented in Fig. 8(c) and 8(d). Results confirm significant improvement in predictions, in comparing with Fig. 7. The critical distance concept introduces a new variable into the model and makes it more flexible. However, applicability of it needs more detailed experimental and computational efforts. From the results we can observe LM is less scattered than PM.

6. Conclusion

The present work shows the simple Cruse-Meyer model can be extended to multiaxial fatigue directly and confirms the same quality in prediction as other established multiaxial models. Combining it with the critical distance concept provides a reasonable prediction for notched specimen fatigue. However, determining the critical distance needs suitable elastoplastic FEM analysis. The basic assumption here is that the critical distance should be constant for the specific material and fatigue model which has to be verified further. Empirical determination of the critical distance needs to be refined.

References

[1] Bannantine JA, Comer JJ, Handrock JL. Fundamentals of metal fatigue analysis. Upper Saddle River, New Jersey: Prentice Hall, 1990.

[2] Peterson, R. E., Analytical approach to stress concentration effects in aircraft materials. In: Proceedings of the WADC Symposium on Fatigue of Metals. U.S. Air Force, Dayton, OH, USA, 1959.

[3] Neuber, H., Theoretical calculation of strength at stress concentration, *Czech. J. Phys*., **19,** 400, 1969.

[4] E. Siebel and M. Stieler. Dissimilar stress distributions and cyclic loading(in german). Z.VDI,97.

[5] David Taylor. The Theory of Critical Distances. Elsevier, 2007.

[6] Susmel L, Taylor D. A novel formulation of the theory of critical distances to estimate lifetime of notched components in the medium-cycle fatigue regime. Fatigue Fract Eng Mater Struct 2007, 30(7):567–81.

[7] Susmel L, Taylor D. On the use of the theory of critical distances to estimate fatigue strength of notched components in the medium-cycle fatigue regime. In: Proceedings of FATIGUE 2006 Atlanta, USA.

[8] Susmel L., Modified Wöhler curve method, theory of critical distances and Eurocode 3: A novel engineering procedure to predict the lifetime of steel welded joints subjected to both uniaxial and multiaxial fatigue loading, International Journal of Fatigue, Volume 30, Issue 5, May 2008, Pages 888-907,

[9] Findley W.N., A theory for the effect of mean stress on fatigue of metals under combined torsion and Axial loading or bending, Journal of Engineering for Industry, 1959, pp. 301-406

[10] Socie DF, Morquis GB., Multiaxial Fatigue. Society of Automotive Engineers, Inc. Warrendale, Pa, 1999.

[11] Drew Nelson, Jinsoo Park. Evaluation of an energy-based approach and a critical plane approach for predicting constant amplitude multiaxial fatigue life. International Journal of Fatigue, 22:23–39, 2000.

[12] Cruse T, Mahadevan S, Tryon R., Fatigue reliability of gas turbine engine structures, NASA Contractor Report NASA/CR-97-206215,1997.
[13] Koch J., Pro-portional and non-proportional biaxial fatigue of Inconel 718, University of Illinois, 1985.
[14] Fatemi A, Kurath P, Multiaxial fatigue life predictions under the influence of mean-stresses. Journal of Engineering Materials and Technology 1988, 110:380.
[15] Jacquelin B, Hourlier F, Pineau A. Crack initiation under low-cycle multiaxial fatigue. Multiaxial Fatigue 1982, 285:313.
[16] Morrow D.L. Biaxial-tension fatigue of Inconel 718. University of Illinois at Urbana-Champaign 1988, 49(6):128.
[17] J.Hofmeister. Research project report for lufo-3. University of Karlsruhe,Karlsrhe, 2008.