Investigation of Mixed Mode Fracture in Marble by Using Two Different Test Configurations

M.R. Ayatollahi[1a], J. Akbardoost[2b]

[1,2] Fatigue and Fracture Laboratory, School of Mechanical Engineering, Iran University of Science and Technology Narmak, Tehran, Iran.

[a] m.ayat@iust.ac.ir, [b] jakbardoost@iust.ac.ir

Keywords: Rock fracture mechanics, mixed mode loading, Marble, modified MTS criterion.

Abstract. The mixed mode fracture behavior of Neyriz marble was studied experimentally and theoretically by using two different test configurations: 1) the center cracked circular disk (CCCD) specimen subjected to diametral compression, and 2) the edge cracked triangular (ECT) samples with different crack inclination angels under three point loading. The experimental results showed that there is a noticeable difference between the fracture resistances obtained from these two test specimens. The difference between the fracture resistances were justified using a modified form of the maximum tangential stress called MMTS which takes into account the effects of higher order stress terms A_3 and B_3 in addition to the singular terms. It was also shown that the MMTS criterion which uses A_3 in calculations of the fracture process zone (FPZ) length could provide good estimates for the experimental data.

Introduction

A good understanding of the mechanical behaviors of rock materials is important in many practical applications of rock engineering such as construction of dams, hydraulic fracturing of gas and oil wells, tunneling, rock cutting, excavation, fragmentation and etc. Fracture toughness is an important mechanical property in rock mechanics which describes the rock resistance against the crack propagation. The cracked rock masses and structures are usually subjected to complex loading conditions. Because of arbitrary orientation of cracks relative to the loading directions, brittle fracture in rocks may occur due to a combination of two major fracture modes, i.e. crack opening mode (mode I) and crack sliding mode (mode II). Thus, it is important to investigate mixed mode I/II brittle fracture in rocks. Various test specimens have been used in the past for mixed mode fracture experiments. The edge cracked beam subjected to three or four point bend loading [1], the compact tension-shear specimen [2], the centrally cracked Brazilian disk specimen [3], the cracked semi-circular specimen under three point bending [4] and the edge cracked triangular specimen [5] under three point bending are some of the specimens used frequently for mixed mode fracture tests on rocks. The experimental results in previous studies indicated that the mixed mode fracture toughness obtained from these test configurations depends significantly on the type of specimen used. For example, Aliha et al. [6] showed that the fracture resistance obtained from the SCB specimen is less than the one calculated from the BD samples. Therefore, for investigating the fracture behavior of rock materials under mixed mode loading, the geometry effects should be considered. In this paper, the value of mode I fracture toughness measured from two different test configurations are compared. The first test configuration is the center cracked circular disk (CCCD) specimen subjected to diametral compression. The second one was the edge cracked triangular (ECT) sample under three-point loading which has been suggested recently by Ayatollahi et al. [5]. The significant difference between the fracture toughness obtained from these test configurations is interpreted by a modified form of maximum tangential stress (MTS) criterion. This criterion takes into account two higher order terms A_3 and B_3 in the Williams series expansion of the crack tip stresses in addition to the singular term. Moreover, the mixed mode fracture behavior of Neyriz marble obtained from the ECT specimens with different crack inclination angels is investigated by the

MMTS criterion. It is demonstrated that the MMTS criterion can predict the fracture behavior of the ECT specimens under different mode mixities only by using the fracture toughness value obtained from the CCCD sample. For this purpose, two formulations are used for calculating the fracture process zone (FPZ). It is shown that the MMTS criterion which uses A_3 in calculations of the FPZ length provides good estimates for the experimental data.

Fracture theory

Using the Williams series expansions [7], the general form of elastic stress field around the crack tip can be written from as:

$$\begin{Bmatrix}\sigma_r\\ \sigma_\theta\\ \tau_{r\theta}\end{Bmatrix}=\sum_{n=1}^{\infty}\frac{n}{2}A_n r^{\left(\frac{n}{2}-1\right)}\begin{Bmatrix}\left(3-\frac{n}{2}\right)\cos\left(\frac{n}{2}-1\right)\theta+\left(\frac{n}{2}+(-1)^n\right)\cos\left(\frac{n}{2}+1\right)\theta\\ \left(\frac{n}{2}+1\right)\cos\left(\frac{n}{2}-1\right)\theta-\left(\frac{n}{2}+(-1)^n\right)\cos\left(\frac{n}{2}+1\right)\theta\\ \left(\frac{n}{2}-1\right)\sin\left(\frac{n}{2}-1\right)\theta-\left(\frac{n}{2}+(-1)^n\right)\sin\left(\frac{n}{2}+1\right)\theta\end{Bmatrix}+\sum_{n=1}^{\infty}-\frac{n}{2}B_n r^{\frac{n}{2}-1}\begin{Bmatrix}\left(3-\frac{n}{2}\right)\sin\left(\frac{n}{2}-1\right)\theta+\left(\frac{n}{2}-(-1)^n\right)\sin\left(\frac{n}{2}+1\right)\theta\\ \left(\frac{n}{2}+1\right)\sin\left(\frac{n}{2}-1\right)\theta-\left(\frac{n}{2}-(-1)^n\right)\sin\left(\frac{n}{2}+1\right)\theta\\ \left(1-\frac{n}{2}\right)\cos\left(\frac{n}{2}-1\right)\theta+\left(\frac{n}{2}-(-1)^n\right)\cos\left(\frac{n}{2}+1\right)\theta\end{Bmatrix} \quad (1)$$

where r and θ are the polar coordinates as illustrated in Fig. 1, n is the order of term in the series expansion and A_n and B_n are the constant coefficients of the terms in the series expansion. The constant coefficients A_n and B_n in Eq. (1) depend on the specimen geometry and loading conditions and can be generally written in terms of dimensionless parameters A_n^* and B_n^* as:

$$A_n=\frac{P}{Rt}R^{(1-n/2)}\ A_n^* \quad (2)$$

$$B_n=\frac{P}{Rt}R^{(1-n/2)}\ B_n^* \quad (3)$$

where P is the applied load, t is the specimen thickness and R is a characteristic dimension like the radius of CCCD specimens or the width of ECT samples. Moreover, A_n^* and B_n^* only depend on the geometry and loading parameters like the crack length ratio (a/R), the loading span to width ratio (S/R) and etc. and are independent of the load and dimension of samples. These parameters can be obtained theoretically for simple geometries and numerically for more complicated cases.

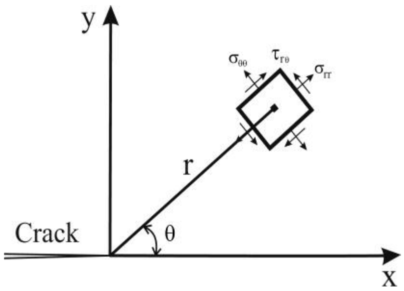


Fig. 1. Conventional crack tip co-ordinates.

According to the maximum tangential stress criterion [8], fracture commences from the crack tip along the direction of maximum tangential stress (θ_0). The MTS criterion also proposes that crack growth takes place when the tangential stress along θ_0 at a critical distance r_c from the crack tip reaches its critical value of $\sigma_{\theta\theta c}$. The critical stress $\sigma_{\theta\theta c}$ and the critical distance r_c are often taken to be the tensile strength f_t and the length of FPZ, respectively. The tangential stress $\sigma_{\theta\theta}$ along the fracture direction is obtained from Eq. (1) by setting $\theta=\theta_0$:

$$\begin{aligned}\sigma_{\theta\theta}(r,\theta_0)=&\left\{\frac{3}{4}r^{-0.5}A_1\left(\cos\frac{\theta_0}{2}+\frac{1}{3}\cos\frac{3\theta_0}{2}\right)+4A_2\left(\sin^2\theta_0\right)+\frac{15}{4}r^{0.5}A_3\left(\cos\frac{\theta_0}{2}-\frac{1}{5}\cos\frac{5\theta_0}{2}\right)\right\}+\ldots\\ &-\left\{-\frac{3}{4}r^{-0.5}B_1\left(\sin\frac{\theta_0}{2}+\sin\frac{3\theta_0}{2}\right)+B_2(0)+\frac{15}{4}r^{0.5}B_3\left(\sin\frac{\theta_0}{2}-\sin\frac{5\theta_0}{2}\right)\right\}+\ldots\end{aligned} \quad (4)$$

The parameters A_1 and B_1 are related to the stress intensity factors K_I and K_{II} as $A_1 = K_I / \sqrt{2\pi}$, $B_1 = -K_{II} / \sqrt{2\pi}$ and the parameter A_2 corresponds to the first non-singular term i.e. T-stress as $A_2=T/4$. The first or singular terms in Eq. (4) (i.e. the terms including the coefficients A_1 and B_1) are assumed to characterize the stress field near the crack tip, and the contribution of higher order terms is often neglected. For example, the conventional maximum tangential stress (MTS) criterion [8] uses only the singular term to predict brittle fracture in cracked specimens. It has been recently demonstrated that the higher order terms of crack tip asymptotic field can be of great effects on the fracture behavior of quasi-brittle materials like rocks, concretes, tough ceramics, etc. which have relatively large FPZ length (r_c) (see for example [1] , [3]). In fact, at the larger distances from the crack tip, the higher order terms in Eq. (4) are no longer negligible and are required to characterize the tangential stress more accurately. By taking into consideration the first three terms in Eq. (4), the maximum tangential stress component at the critical distance r_c can be obtained from:

$$\sigma_{\theta\theta}(r_c,\theta_0) = \left\{\frac{3}{4}\frac{K_{Ic}}{\sqrt{2\pi r_c}}\left(\cos\frac{\theta_0}{2}+\frac{1}{3}\cos\frac{3\theta_0}{2}\right)+4A_{2c}\left(\sin^2\theta_0\right)+\frac{15}{4}A_{3c}\sqrt{r_c}\left(\cos\frac{\theta_0}{2}-\frac{1}{5}\cos\frac{5\theta_0}{2}\right)\right\} - \left\{-\frac{3}{4}\frac{K_{IIc}}{\sqrt{2\pi r_c}}\left(\sin\frac{\theta_0}{2}+\sin\frac{3\theta_0}{2}\right)+B_{2c}(0)+\frac{15}{4}B_{3c}\sqrt{r_c}\left(\sin\frac{\theta_0}{2}-\sin\frac{5\theta_0}{2}\right)\right\} = f_t \tag{5}$$

where A_{2c}, A_{3c} and B_{3c} are the critical values of A_2, A_3 and B_3, respectively, and K_{Ic} and K_{IIc} are the mode I and II fracture toughness, f_t is the tensile strength of material and r_c is the critical distance or FPZ length. Eq. (5) is a modified form of MTS criterion (called MMTS) for predicting the fracture load of brittle and quasi-brittle materials. Substituting Eqs. (2) and (3) into Eq. (5) gives:

$$\sigma_{\theta\theta c} = f_t = \frac{K_{Ic}}{\sqrt{2\pi r_c}}\left\{\frac{3}{4}\left(\cos\frac{\theta_0}{2}+\frac{1}{3}\cos\frac{3\theta_0}{2}\right)+4\frac{A_2^*}{A_1^*}\sqrt{\frac{r_c}{R}}\left(\sin^2\theta_0\right)+\frac{15}{4}\frac{A_3^*}{A_1^*}\frac{r_c}{R}\left(\cos\frac{\theta_0}{2}-\frac{1}{5}\cos\frac{5\theta_0}{2}\right)\right\} - \frac{K_{IIc}}{\sqrt{2\pi r_c}}\left\{-\frac{3}{4}\left(\sin\frac{\theta_0}{2}+\sin\frac{3\theta_0}{2}\right)+\frac{15}{4}\frac{B_3^*}{B_1^*}\frac{r_c}{R}\left(\sin\frac{\theta_0}{2}-\sin\frac{5\theta_0}{2}\right)\right\} \tag{6}$$

The fracture initiation angle θ_0 in Eq. (6) can be obtained from solving the following differential equation based on MMTs criterion:

$$\frac{\partial\sigma_{\theta\theta}}{\partial\theta} = \frac{\partial}{\partial\theta}\left[\frac{K_{Ic}}{\sqrt{2\pi r_c}}\left\{\frac{3}{4}\left(\cos\frac{\theta_0}{2}+\frac{1}{3}\cos\frac{3\theta_0}{2}\right)+4\frac{A_2^*}{A_1^*}\sqrt{\frac{r_c}{R}}\left(\sin^2\theta_0\right)+\frac{15}{4}\frac{A_3^*}{A_1^*}\frac{r_c}{R}\left(\cos\frac{\theta_0}{2}-\frac{1}{5}\cos\frac{5\theta_0}{2}\right)\right\} - \frac{K_{IIc}}{\sqrt{2\pi r_c}}\left\{-\frac{3}{4}\left(\sin\frac{\theta_0}{2}+\sin\frac{3\theta_0}{2}\right)+\frac{15}{4}\frac{B_3^*}{B_1^*}\frac{r_c}{R}\left(\sin\frac{\theta_0}{2}-\sin\frac{5\theta_0}{2}\right)\right\}\right] = 0 \tag{7}$$

If the fracture initiation along (θ_0) is obtained from Eq. (7) and is substituted into Eq. (6), the fracture load can be estimated for any mixed mode loading conditions. For example, for opening mode (or pure mode I), the constant coefficients B_n become zero. Due to symmetry in the geometry and loading conditions, θ_0 for a mode I crack is also zero. Thus, Eq. (6) can be simplified for pure mode I as:

$$\sigma_{\theta\theta c} = f_t = \frac{K_{Ic}}{\sqrt{2\pi r_c}}\left(1+3\frac{A_3^*}{A_1^*}\frac{r_c}{R}\right) \tag{8}$$

In order to use MMTS criterion (Eq. (6) the FPZ length or r_c should be known. There are several formulations in the literature for calculating the FPZ length [9]-[12]. In this paper, two formulas are considered: the conventional equation proposed by Schmidt [12] and a new equation where the term A_3 is included the Schmidt's model. Schmidt [12] has suggested an equation based on the maximum principal stress theory for evaluating the size of FPZ. According to this model, the size of FPZ can be calculated from:

$$r_c = \frac{1}{2\pi}\left(\frac{K_{Ic}}{f_t}\right)^2 \qquad (9)$$

If the higher order terms are employed for characterizing the stress field in the Schmidt's model, a new equation can be obtained for calculating r_c as:

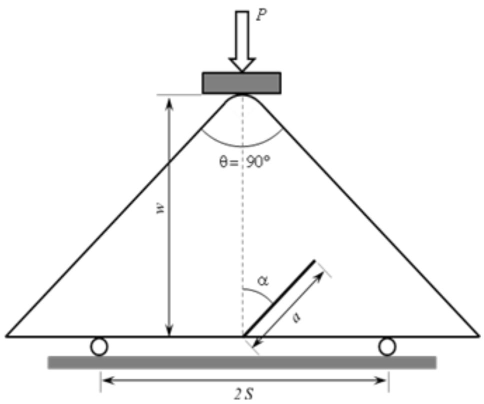
$$r_c = \left[\frac{f_t\sqrt{2\pi} \pm \sqrt{2\pi f_t^2 - 12\frac{A_3^*}{A_1^*}\frac{K_{Ic}}{w}}}{6\frac{A_3^*}{A_1^*}\frac{K_{Ic}}{w}}\right]^2 \qquad (10)$$

It should be noted that only the lowest positive value of Eq. (10) is physically acceptable.
By replacing Eq. (10) into Eq. (6), the MMTS criterion can be used for predicting the fracture resistance of brittle and quasi-brittle materials under mixed mode loading. In the next section, the fracture behavior of two test configurations will be investigated in detail by using Eq. (6) and it will be demonstrated how the MMTS criterion can consider the geometry effect.

Experiments

In the previous section, a modified form of MTS criterion was described for investigating the geometry effect on mixed mode fracture behavior of brittle and quasi-brittle materials like rocks, concretes, etc. In the present section, the fracture behavior of two different test configurations is assessed by this criterion. The first test configuration is the center cracked circular disk (CCCD) specimen subjected to diametral compression. The second one is the edge cracked triangular (ECT) sample under three-point loading which was proposed recently by Ayatollai et al. [5]. These two test configurations are shown schematically in Fig. 2.
The mode I fracture toughness was first obtained from the center cracked circular disk (CCCD) specimen subjected to diametral compression as shown in Fig. 3Fig. 3. The specimens were made of Nyriz marble which is excavated from Fars province mines in Iran. The dimensions of tested specimens were *R*=75 mm, *t*=16 mm and *a*=22.5 mm, in which *R* and *t* are the specimen radius and thickness, respectively, and *a* is the semi-crack length. For preparing each specimen and introducing an initial notch of length 20 mm, the water jet machine was used. The width of crack generated by water jet machine was 1.5 mm, thus the initial center notches were sharpened by a fret saw with a thin saw blade of 0.3 mm thickness to make the final crack length 22.5 mm. The prepared samples were tested by a universal ball-screw test machine of capacity 50 kN under diametral compressive loading condition as shown in Fig. 3. The tests were carried out under displacement control conditions with a constant cross head speed of 0.08 mm/min. The average fracture load obtained from four experiments was P_f =9.93 kN.

(*a*)



(*b*)

Fig. 2 Test specimens: (*a*) center cracked circular disk (CCCD) specimen subjected to mode I loading condition , (*b*) Edge crack triangular (ECT) specimen subjected to three-point bend loading [5].

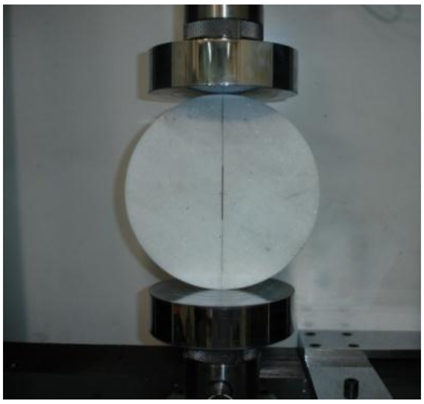


Fig. 3 The CCCD specimen inside the test machine.

Therefore, the average fracture toughness can be obtained by setting n=1in Eq. (2) as follows:

$$K_{Ic} = A_{1c}\sqrt{2\pi} = \frac{P_f}{Rt}\sqrt{2\pi R}A_1^* \qquad (11)$$

where P_f is the fracture load, A_1^* is a dimensionless parameter (sometimes called geometry factors) which depends on geometry and loading conditions. The dimensionless factor A_1^* was calculated numerically for CCCD specimen as A_1^* = 0.14. Thus, the fracture toughness of Neyriz marble was obtained from the CCCD tests as K_{Ic}= 0.742 MPa.m$^{0.5}$.

In the next stage, the edge cracked triangular (ECT) samples were used for investigating mixed mode fracture in Neyriz marble. In this test configuration, a triangular plate of width *w* that contains an edge crack of length *a* is loaded by three-point bending. When this specimen is subjected to a compressive load *P*, by changing the crack orientation angle α, different mode mixities from pure mode I to pure mode II are achieved. When the crack line is along the direction of applied load (α=0), the ECT specimen is subjected to pure mode I (or pure opening mode). But for non-zero crack angles, mode II also appears in crack deformation. Ayatollahi et al. [5] carried out a number of experiments on ECT specimens under mixed mode loading. Their samples were manufactured from a sheet of Neyriz marble having the thickness of 16 mm. Other dimensions of the specimens were: *a*=22.5mm, *w*=75mm. Thus the ratio of *a*/*w* was equal to 0.3 like the CCCD specimens described earlier. The span to width ratio for all samples was also *S*/*w*= 0.4. In order to cover the full range of mixed mode I/II cases, the following crack angles were considered by Ayatollahi et al. [5] for experiments: α={0° (pure mode I), 10°, 20°, 30°, 40°,52.5° (pure mode II)}. The average of fracture loads for each mode mixities reported in [5] are listed in Table 1.

In order to calculate the fracture parameters of ECT specimens such as the values of K_{Ic}, T_c, A_{3c}, K_{IIc} and B_{3c}, Eqs. (2) and (3) were used as follows:

$$K_{Ic} = A_{1c}\sqrt{2\pi} = \frac{P_f}{wt}\sqrt{2\pi w}A_1^* \qquad (12)$$

$$T_c = 4A_{2c} = 4\frac{P_f}{wt}A_2^* \qquad (13)$$

$$A_{3c} = \frac{P_f}{wt}\frac{1}{\sqrt{w}}A_3^* \qquad (14)$$

$$K_{IIc} = B_{1c}\sqrt{2\pi} = \frac{P_f}{wt}\sqrt{2\pi w}B_1^* \qquad (15)$$

$$B_{3c} = \frac{P_f}{wt}\frac{1}{\sqrt{w}}B_3^* \qquad (16)$$

where *w* is the width of ECT specimen. It should be noted that the characteristic dimension of ECT sample is taken to be its width. The dimensionless factors A_1^*, A_2^*, A_3^*, B_1^*, B_3^* in Eqs.

(12) to (16) can be determined numerically for the ECT samples by using the finite element over-deterministic (FEOD) method. This method has been proposed recently by Ayatollai and Nejati [13] and is able to determine the constants coefficients of Williams series expansion. In the FEOD method, the finite element analysis is used for calculating the displacement components of a large number of nodes around the crack tip. Then, an over-determined set of simultaneous linear equations is obtained by using the Williams series expansion for displacement field around the crack tip. Finally, the least-square method is employed to solve the over-determined equations and to calculate the crack parameters. Fig. 4 shows the variations of the dimensionless parameters relative to the crack inclination angle (α) obtained using the FEOD method for the ECT specimen with a/w=0.3 and S/w=0.4. Table 1 shows the fracture parameters of the tested ECT samples under mixed mode loading calculated from Eqs. (12) to (16) and using the results presented in Fig. 4.

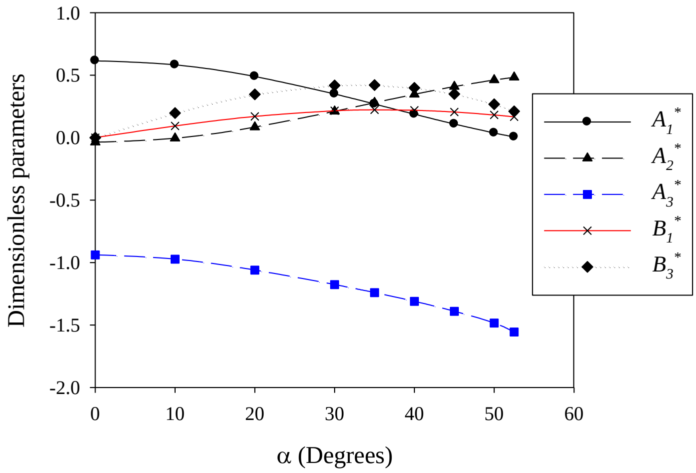


Fig. 4 Variations of dimensionless parameters versus crack inclination angle (α) for ECT specimen.

Table 1. The fracture parameters of tested ECT samples.

α (Degrees)	P_f (kN) [5]	K_{Ic} (MPa.m$^{0.5}$)	T_c MPa	A_{3c} (MPa.m$^{-0.5}$)	K_{IIc} (MPa.m$^{0.5}$)	B_{3c} (MPa.m$^{-0.5}$)
0	3.49	1.229	-0.430	-9.984	0	0
10	3.575	1.191	-0.059	-10.589	0.191	2.136
20	3.653	1.023	1.023	-11.794	0.355	3.845
30	3.81	0.762	2.674	-13.649	0.468	4.849
40	4.25	0.456	4.906	-16.942	0.532	5.145
52.5	5.39	0.016	8.665	-25.525	0.513	3.446

Theoretical investigations of fracture

Mode I fracture analysis. According to Table 1, the mode I fracture toughness of Neyriz marble obtained from the ECT specimens is K_{Ic}= 1.229 MPa.m$^{0.5}$. However, this value has a significant discrepancy with the fracture toughness value obtained from the CCCD specimens (K_{Ic}= 0.742 MPa.m$^{0.5}$). Therefore, the geometry of specimen exhibits a noticeable effect on fracture toughness.

Here we show that the MMTS criterion is able to justify the geometry effects on fracture toughness. Eq. (8) was derived for any test configurations subjected to pure mode I loading condition. By assuming that the tensile strength is a constant material property independent of specimen geometry, the left hand side of Eq. (8)**Ошибка! Источник ссылки не найден.** is also independent of specimen size and geometry and is a constant value for any given material.

Thus, the following relation can be written for two different test configurations of CCCD and ECT made of the same material:

$$f_t = \frac{K_{Ic}}{\sqrt{2\pi r_c}}(1+3\frac{A_3^*}{A_1^*}\frac{r_c}{R})\bigg|_{CCCD} = \frac{K_{Ic}}{\sqrt{2\pi r_c}}(1+3\frac{A_3^*}{A_1^*}\frac{r_c}{w})\bigg|_{ECT} \tag{17}$$

Eq. (17) can be rewritten in order to derive a relation between the values of apparent fracture toughness in the two different test configurations CCCD and ECT:

$$\frac{K_{Ic}|_{CCCD}}{K_{Ic}|_{ECT}} = \frac{\frac{1}{\sqrt{2\pi r_c}}(1+3\frac{A_3^*}{A_1^*}\frac{r_c}{w})\bigg|_{ECT}}{\frac{1}{\sqrt{2\pi r_c}}(1+3\frac{A_3^*}{A_1^*}\frac{r_c}{R})\bigg|_{CCCD}} \tag{18}$$

It is recalled that R is the radius of the CCCS samples and w is the width of the ECT specimens. The dimensionless parameters A_1^* and A_3^* for ECT samples under pure mode I (α=0) can be extracted from Fig. 4 as A_1^*=0.615 and A_3^* = -0.94. These dimensionless parameters were obtained for the CCCD specimens under mode I loading from FEOD method as A_1^*=0.14 and A_3^* =0.11. The other important parameter in Eq. (18) is the FPZ length or critical distance r_c. As mentioned earlier, two equations were used for calculating r_c: the equation proposed by Schmidt (i.e. Eq. 9) [12] and a modified Schmidt's model (i. e. Eq. 10). In order to use these equations, tensile strength (f_t) and mode I fracture toughness (K_{Ic}) should be known. The tensile strength of Neyriz marble was obtained from indirect tensile test using the un-cracked Brazilian disk (BD) under diametral compression as f_t=5.125 MPa. The reference mode I fracture toughness (K_{Ic}) was also assumed to be K_{Ic}= 0.742 MPa.m$^{0.5}$ from the CCCD specimen. Accordingly, if Eq. (9) is used, the value of r_c would be obtained as r_c=3.34 mm. The value of FPZ length was calculated as r'_c=4.76 mm using Eq. (10). Hereafter, the prime symbol (') is used for parameters that are related to the critical distance obtained from modified form (Eq. 10) such as r'_c.
By replacing the values of A_3^*/A_1^* corresponding to the CCCD and ECT specimens in Eq. (18) and using the value of r_c = 3.34 mm, the fracture toughness for ECT specimen is estimated to be 1.06 MPa.m$^{0.5}$ which shows 14% of error with respect to the experimental results (K_c=1.23 MPa.m$^{0.5}$). If the modified model of Eq. (10) is used for calculating the FPZ length, r'_c should be replaced in Eq. (18) instead of r_c. By using r'_c=4.76 mm, Eq. (18) gives the fracture toughness of ECT specimen as K_c=1.21 MPa.m$^{0.5}$ which has a noticeably reduced discrepancy of 2% relative to K_c=1.23 MPa.m$^{0.5}$ [5]. Consequently, the MMTS criterion can justify the dependency of fracture toughness on specimen geometry by taking into consideration the higher order term A_3.

Mixed mode fracture analysis. To investigate the mixed mode fracture resistance of Neyriz marble obtained from the ECT samples with different inclination angles, the MMTS criterion or Eq. (6) was employed. For this purpose, first the crack initiation angles (θ_0) were determined from Eq. (7) for each mode mixity. In Eq. (7), the value of FPZ length was obtained from both Eq. (9) and Eq. (10) i.e. r_c=3.34 mm and r'_c=4.76 mm as described in the previous section. In addition, the same tensile strength of Neyriz f_t=5.125 MPa was used. The values of dimensionless parameters A_1^*, A_2^*, A_3^*, B_1^*, B_3^* can also be extracted from Fig. 4. By substituting these parameters into Eq. (7), the crack initiation angle (θ_0) was determined for each mode mixity. After calculating θ_0, the fracture curve for the ECT samples can be obtained from Eq. (6). Fig. 5 shows the fracture behavior of ECT specimen under mixed mode loading based on the MMTS criterion (Eq. 6) and the conventional MTS criterion(8). It is seen in this figure that the MMTS criterion using r'_c is more accurate than that using r_c. It is again seen that A_3 has a significant role in the accurate prediction of the FPZ length. Fig. 5 also shows that the fracture curve of the conventional MTS criterion has a significant discrepancy with the test results in comparison with the MMTS criterion especially in pure mode I and pure mode II loading conditions. This can be mainly because the MMTS criterion considers the influence of the higher order terms A_2, A_3 and B_3.

It should be noted that the all the parameters in Eq. (6) or Eq. (7) except for r_c and f_t, were determined from numerical analysis and can be calculated for any other test configuration as well. The parameter r_c was also obtained from the fracture test conducted on the CCCD specimen under pure mode I loading. Therefore, it can be concluded that the MMTS criterion is able to predict the fracture resistance of the ECT samples under mixed mode loading, just by using the mode I fracture toughness value obtained from the CCCD test sample. This point can be considered as a noticeable and useful advantage for the MMTS criterion.

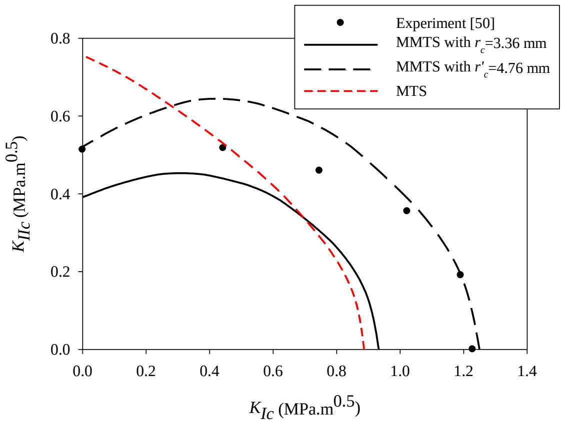


Fig. 5 fracture behavior of ECT specimen under mixed mode loading.

Conclusion

The mixed mode fracture behavior of Neyriz marble was studied theoretically and experimentally by using the center cracked circular disk (CCCD) specimen and the edge cracked triangular (ECT) sample. It was shown that the values of mode I fracture toughness obtained from these two test configurations were significantly different. A modified form of MTS criterion (MMTS criterion) was used for justifying the difference between the values of fracture toughness. It was also demonstrated that the MMTS criterion using the term A_3 in calculation of the FPZ length can predict the mixed mode fracture resistance of Neyriz marble obtained from the ECT samples only by using the mode I fracture toughness measured from the CCCD specimens.

References

[1] M. R. M. Aliha and M. R. Ayatollahi: Fatigue Fract. Engng. M. Vol. 32 (2009), p. 987.

[2] R.K. Zipf and Z.T. Bieniawski: in *Proceedings of 27nd US Symposium on Rock Mechanics*, 1986, pp. 16-23.

[3] M. R. M. Aliha, R. Ashtari and M. R. Ayatollahi: J. Appl. Mech. Mater. Vol. 5-6 (2006), p. 181.

[4] K. Khan and N.A. Al-Shayea: Rock Mech. Rock Engng. Vol. 33 (2000), p. 179.

[5] M. R. Ayatollahi, G. R. Hosseinpour and M. R. M. Aliha: in *Proceedings of ICEM2010 conference*, Malaysia, 2010.

[6] M. R. M. Aliha, M. R. Ayatollahi, D. J. Smith and M. J. Pavier: Engng. Fract. Mech. Vol. 77 (2010), p. 2200.

[7] M. L. Williams: J. Appl. Mech. Vol. 27 (1957), p.109.

[8] F. Erdogan and G. C. Sih: J. Basic Eng-T ASME. Vol. 85 (1963), p. 519.

[9] M. R. M. Aliha and M. R. Ayatollahi: J. Am. Ceram. Soc. Vol. 94 (2011), p. 561.

[10] Z. P. Bazant and J. Planas: *Fracture and size effect in concrete and other quasibrittle materials*, CRC Press in LLC, 1998.

[11] B. L. Karihaloo: Int. J. Fracture. Vol. 95 (1999), p. 379.

[12] R. A. Schmidt: in *Proc 21st US Symp On Rock Mech*, 1980, pp.581-90.

[13] M. R. Ayatollahi MR and M. Nejati: Fatigue Fract. Engng. M. Vol. 34 (2011), p. 159.