Harmonic testing of wells with a vertical fracture

Morozov P.E.

Institute of Mechanics and Engineering, Kazan Science Center, Russian Academy of Sciences, Russia

morozov@mail.knc.ru

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Abstract. This paper investigates the use of pressure waves for harmonic test of a fractured well in homogeneous or double-porosity reservoirs. New analytical expressions are developed to study the amplitude- and phase-frequency characteristics of a well intercepted by a finite-conductivity vertical fracture. The asymptotic solutions for analyzing harmonic test data influenced by wellbore storage and fracture storage effects are presented. The proposed solutions may be used to obtain the formation and fracture properties by harmonic testing of fractured wells.

Introduction

Hydraulic fracturing is an effective technique for increasing productivity of damaged wells and wells producing in low permeability formation. Various methods have been proposed to estimate reservoir and fracture properties from transient pressure and flow rate data [1-7]. The purpose of this study is to present the analytical solutions for harmonic test analysis of fractured wells.

The method of filtration (harmonic) pressure waves for obtaining reservoir parameters was first proposed by Chekalyuk [8]. Later a number of analytical models have been derived to represent harmonic pressure behavior for different well-reservoir configurations: line source [9-12], composite reservoirs [13, 14], dual porosity reservoirs [15-18], vertical wells with wellbore storage and skin effect [16], fractured wells [19, 20]. Method of harmonic well testing can be applied to determine reservoir parameters such as skin effect, damaged zone depth, formation permeability and compressibility. The interpretation of harmonic test requires the comparison of the experimental and theoretical transfer functions, defined as ratio of the pressure amplitude to the flow rate amplitude. Although harmonic tests are less affected by measurement noises, ones are significantly longer than conventional well tests. As shown by Hollaender et. al. [21], the characteristic derivative shape of the pressure modulus in the frequency domain had a behavior similar to time-pressure derivative curves in conventional well test data analysis. Therefore the interpretation methods developed for conventional tests may be applied to harmonic tests.

Harmonic testing. The basic concept of harmonic testing is the use of a sinusoidal flow rate variation instead of a step change as in conventional well testing. When a pseudo-steady flow regime is achieved after a few periods, both flow rate and wellbore pressure exhibit a sinusoidal behavior. It is then possible to identify the modulus of the response and phase shift between the two signals. These data are used to evaluate the reservoir parameters.

The periodic flow rate can be represented by the following complex form:

$$q(t) = Q_A e^{i\omega t}, \tag{1}$$

where Q_A is the amplitude of flow-rate, $\omega = 2\pi/T$ is the frequency and T is the period. Using Duhamel's superposition principle, the harmonic pressure response can be expressed as [21]:

$$P(\omega,t) = Q_A e^{i\omega t}\int_0^t g(t-\tau)e^{i\omega(\tau-t)}d\tau = Q_A e^{i\omega t}\int_0^t g(x)e^{-i\omega(x)}dx\,, \tag{2}$$

where g(t) is the derivative of the pressure drop corresponding to a unit-rate drawdown. The aim of harmonic testing is to evaluate the modulus and argument of the function, defined as ratio between pressure drop and flow rate:

$$H(i\omega,t) = \int_0^t g(x)e^{-i\omega(x)}dx\,. \tag{3}$$

Hollaender et. al. [21] indicates that for large values of time, the function H converges to the conjugate of the Fourier transform of the impulse response g ($g(t)=0$, if $t<0$):

$$H(i\omega) = \int_{-\infty}^{\infty} g(x)e^{-i\omega x}dx\,. \tag{4}$$

Using relationship between the Fourier transform and bilateral Laplace transform, the transfer function can be expressed as

$$H(i\omega) = [\bar{g}(u)]_{u=i\omega} = \lfloor u\bar{p}\,(u)\rfloor_{u=i\omega}\,, \tag{5}$$

where u is Laplace transform variable, $\bar{p}(u)$ is pressure drop corresponding to a unit-rate drawdown in Laplace space. The solutions in term of Laplace-transform variable are widely used to obtain pressure distribution and well responses for a wide variety of wellbore and reservoir configuration [22]. Based on these solutions and equation (5), the transfer functions for different well-reservoir configurations can be readily modeled and analyzed.

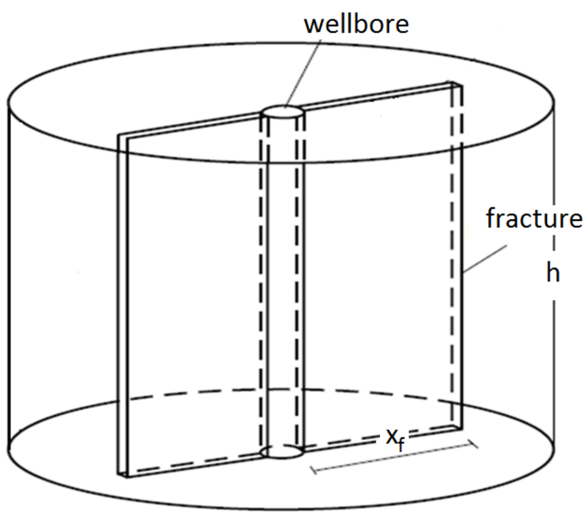

Fig.1. Vertical well with fracture.

Finite conductivity fracture without fracture storage effect. Consider a vertically fractured well (fig. 1) in an infinite, homogeneous reservoir that contains a slightly compressible fluid of constant viscosity μ. The reservoir has a permeability k, thickness h and total compressibility β. Let us assume that the well is intercepted by fully penetrating vertical fracture of half-length x_f, width w and permeability k_f. First consider a model in which the total compressibility of the fracture is neglected.

The transient pressure behavior of a fractured well produced at a constant flow rate Q is described by the system of integral equations in Laplace space [6, 7]:

$$\overline{p}_d(u) = \frac{1}{2}\int_{-1}^{1}\overline{q}(x',u)K_0\left(\sqrt{u}|x_d - x'|\right)dx' - \frac{\pi}{F_{CD}}\int_0^{x_d}\int_0^{x'}\overline{q}(x'',u)dx''dx' + \frac{\pi x_d}{F_{CD}u}, \tag{6}$$

$$\int_{-1}^{1}\overline{q}(x',u)dx' = \frac{1}{u}, \tag{7}$$

where $p_d = \dfrac{2\pi kh(p_k - p)}{Q\mu}$, $t_d = \dfrac{kt}{\mu\beta x_f^2}$, $x_d = \dfrac{x}{x_f}$, $F_{CD} = \dfrac{k_f w}{kx_f}$ is dimensionless fracture conductivity, p_k is initial pressure, q is flux density and $K_0(z)$ is modified Bessel function of the second kind of order 0. In case of infinite-conductivity fracture ($F_{CD} = \infty$) the system of integral equations (6)-(7) reduces to the result obtained by Barenblatt et. al. [11]. The assumption of infinite fracture conductivity is valid whenever the dimensionless fracture conductivity $F_{CD} > 300$ [1, 2].

The transfer function for a well with a finite-conductivity fracture is expressed by the system of integral equations:

$$H(i\omega_d) = \frac{1}{2}\int_{-1}^{1}q(x',i\omega_d)K_0\left(\sqrt{i\omega_d}|x_d - x'|\right)dx' - \frac{\pi}{F_{CD}}\int_0^{x_d}\int_0^{x'}q(x'',i\omega_d)dx''dx' + \frac{\pi x_d}{F_{CD}}, \tag{8}$$

$$\int_{-1}^{1}q(x',i\omega_d)dx' = 1, \tag{9}$$

where $\omega_d = \omega\mu\beta x_f^2/k$ is dimensionless frequency. To solve the system of integral equations (8)-(9) the fracture is divided into n discrete, uniform flux elements [1, 5]. The resulting system of linear equations with complex coefficients is solved to determine the flux modulus distribution along the fracture $|q(x_d,i\omega_d)|$, the wellbore pressure modulus $A = |H(i\omega_d)|$ and phase shift $\varphi = \arg H(i\omega_d)$.

Fig. 2 presents the pressure and flux modulus distribution along a low conductivity fracture ($F_{CD} = 1$). As shown in Fig. 2b, for high dimensionless frequencies the flux density is high at the portions of the fracture near the wellbore. The amplitude- and phase-frequency characteristics of a well intercepted by a finite- and infinite -conductivity fracture are shown in Fig. 3. The derivative of pressure modulus in the frequency domain has a behavior similar to conventional time-pressure derivative curves [4]. In case of finite-conductivity fracture the derivative of pressure modulus displays characteristic flow-regimes: (1) bilinear flow with a straight line of slope 1/4, (2) linear flow with a straight line of slope 1/2 and (3) pseudo-radial flow with a straight line of zero slope.

Asymptotic analysis. Using equation (5) and asymptotic solution [3, 6] for bilinear flow period (a linear incompressible flow in fracture and linear compressible flow in formation), the transfer function can be expressed as

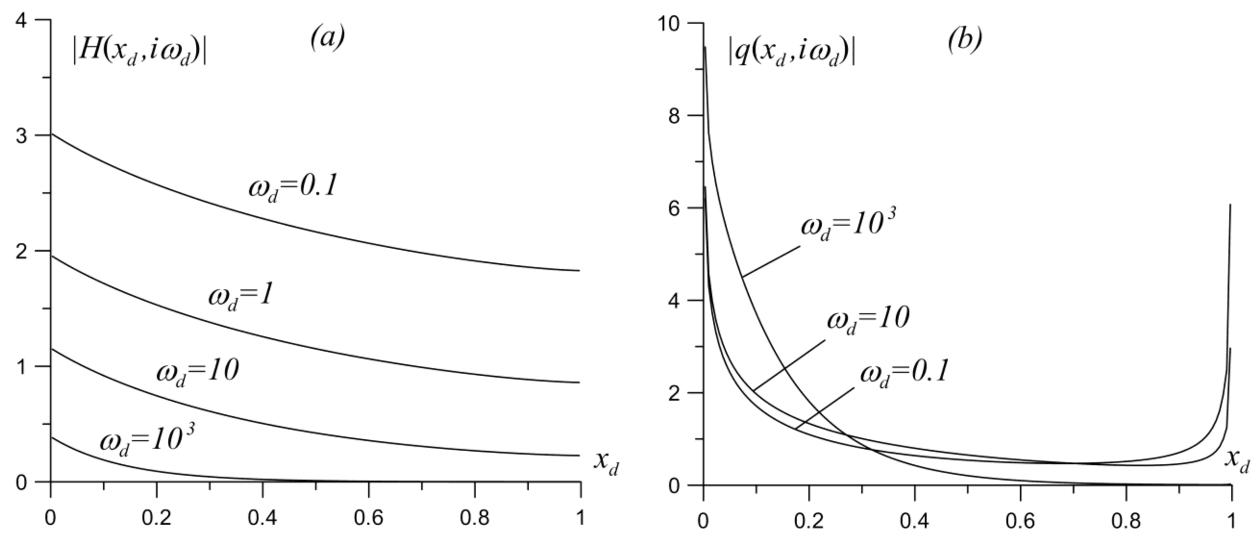

Fig.2. Pressure modulus *(a)* and flux modulus *(b)* distribution along a finite-conductivity fracture at various dimensionless frequencies.

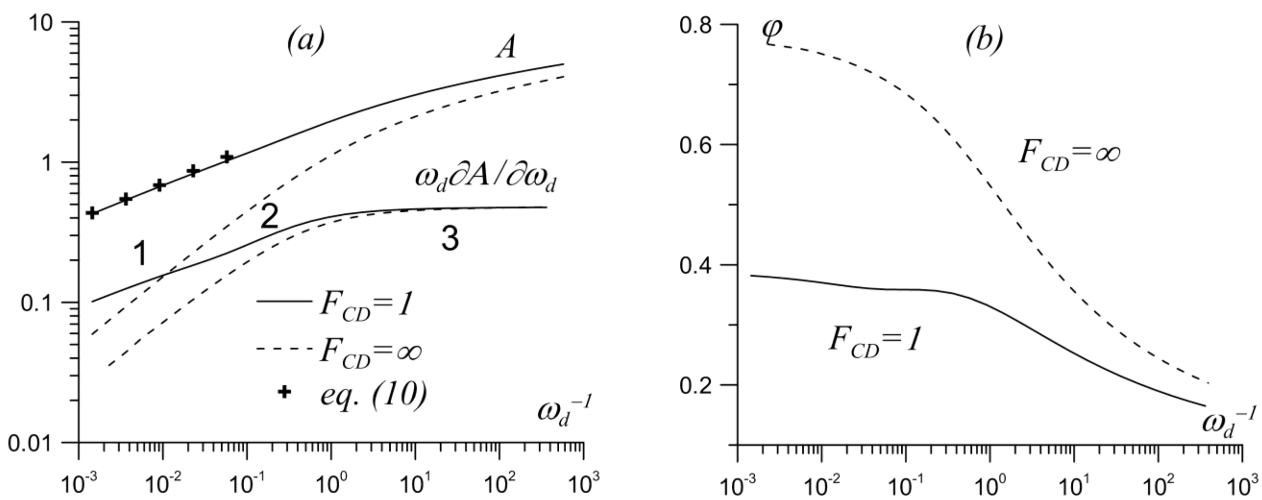

Fig.3. Pressure modulus and its derivatives curves *(a)*, phase shift curves *(b)* for a well with finite- and infinite-conductivity fracture.

$$H(i\omega_d) = \frac{\pi}{\sqrt{2F_{CD}\sqrt[4]{i\omega_d}}}, \ (\omega_d >> 1). \tag{10}$$

By using the equation (5) and asymptotic solutions for linear and pseudo-radial flow period [1, 11], the transfer function for a well with infinite-conductivity fracture at high and low dimensionless frequencies can be expressed as:

$$H(i\omega_d) = \frac{\pi}{2\sqrt{i\omega_d}}, \ (\omega_d >> 1), \tag{11}$$

$$H(i\omega_d) = -\ln\left(\sqrt{i\omega_d}\right) + \ln 4 - \gamma, \ (\omega_d << 1), \tag{12}$$

where γ is Euler's constant. The equations (10) and (11) indicate that phase shift is equal to π/8 for bilinear flow period and π/4 for linear flow period [21].
It should be noted that asymptotic relations similar to (11) and (12) are used in the method of periodic heating of a planar probe placed on an anisotropic sample [23]. According to this method, the thermal properties of the sample substrate can be determined by the amplitude and phase of the temperature oscillations of the probe.

Wellbore storage effect. Theoretical and experimental studies of harmonic well tests have shown that at high frequencies one must take into account the influence of wellbore storage effect [16, 24]. The transfer function for fractured well with wellbore storage effect is given by

$$H(i\omega_d)=\frac{i\omega_d \bar{p}_d(i\omega_d)}{1-C_d\omega_d^2\bar{p}_d(i\omega_d)}, \tag{13}$$

where $C_d=\dfrac{C}{2\pi\beta h x_f^2}$, C is the wellbore storage coefficient, $\bar{p}_d(u)$ is Laplace transform of pressure corresponding to a unit-rate drawdown.
The asymptotic solution in the Laplace space for bilinear flow period with wellbore storage effect is given by Wong et. al. [4]. In this case, the transfer function can be expressed as

$$H(i\omega_d)=\frac{\pi}{\sqrt{2F_{CD}}\sqrt[4]{i\omega_d}+\pi C_d i\omega_d},\ (\omega_d>>1). \tag{14}$$

As shown in Fig. 4, the wellbore storage is influence on phase shift, pressure modulus and its derivatives in case of high dimensionless frequencies. Pressure modulus behavior dominated by wellbore storage has a unit slope (Fig. 4a). The equations (13) and (14) indicate that at high dimensionless frequencies the phase shift tends toward π/2 (Fig. 4b).

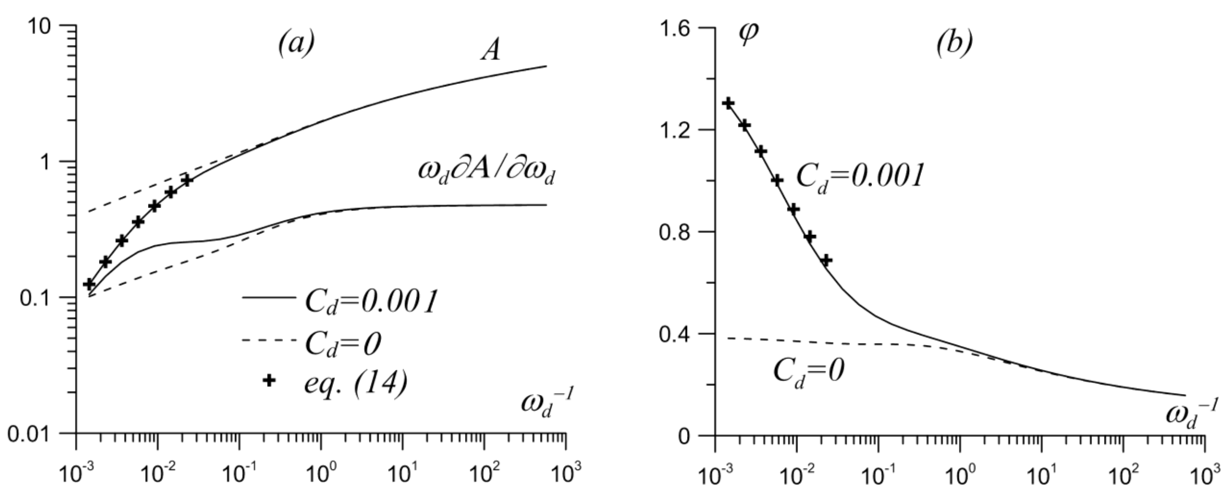

Fig.4. Pressure modulus and its derivatives curves *(a)*, phase shift curves *(b)* for a fractured well with and without wellbore storage effect.

Finite conductivity fracture with fracture storage effect. The general Laplace domain solution for a well with a finite-conductivity fracture including the fracture storage was developed by

Kruysdijk [5, 7]. The transfer function for fractured well with fracture storage effect is expressed by the system of integral equations:

$$H(x_d, i\omega_d) = \frac{1}{2}\int_{-1}^{1} q(x', i\omega_d) K_0\left(\sqrt{i\omega_d}\,|x_d - x'|\right) dx', \tag{15}$$

$$\int_{-1}^{1} q(x', i\omega_d)\left[K_0\left(\sqrt{i\omega_d}\,|x_d - x'|\right) + \frac{\pi}{F_{CD}\sigma}\left(\frac{\cosh(|x_d - x'|\sigma)}{\tanh(\sigma)} - \sinh(|x_d - x'|\sigma)\right)\right] dx' =$$
$$= \frac{\pi}{F_{CD}\sigma}\left(\frac{\cosh(x_d\sigma)}{\tanh(\sigma)} - \sinh(|x_d|\sigma)\right), \tag{16}$$

where $\sigma = \sqrt{\frac{i\omega_d}{\eta_{fd}}}$, $\eta_{fd} = \frac{k_f \beta}{k\beta_f}$ is dimensionless hydraulic diffusivity of the fracture, β_f is fracture storage coefficient. Using asymptotic solution for short-time pressure behavior [2, 3] and equation (5), the transfer function can be expressed as

$$H(i\omega_d) = \frac{\pi}{F_{CD}}\left[\frac{i\omega_d}{\eta_f} + \frac{2\sqrt{i\omega_d}}{F_{CD}}\right]^{-0.5}, \quad (\omega_d >> 1). \tag{17}$$

Note that the expression similar to (17) is given by Despax et. al. [19]. The comparison of amplitude- and phase-frequency characteristics of a fractured well with and without the fracture storage effect is presented in Fig. 5.

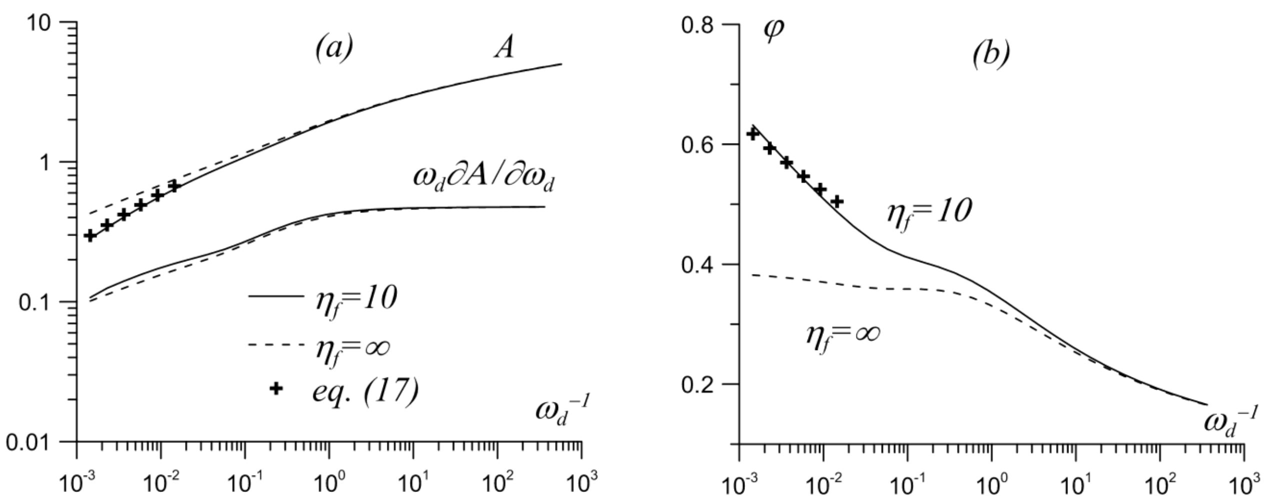

Fig.5. Pressure modulus and its derivatives curves *(a)*, phase shift curves *(b)* for a fractured well with and without fracture storage effect.

Fractured well in a double-porosity reservoir. Representation of naturally fractured reservoir as the double-porosity medium was first introduced by Barenblatt, Zheltov and Kochina [11]. A double-porosity reservoir consists of two distinct porous media of separate porosity and permeability: the matrix medium (with high storativity but a low permeability) and fissures (high permeability and limit storativity). Several models to study the pressure behavior of wells intersected by a vertical fracture in double-porosity reservoirs were developed [5, 6].

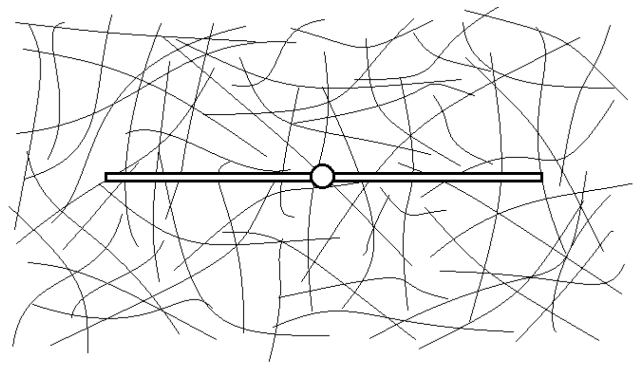

Fig.6. Vertically fractured well in a double-porosity reservoir.

The transfer function for a well with an infinite-conductivity fracture in a double-porosity reservoir (Fig. 6) is expressed by the system of integral equations:

$$H(i\omega_d)=\frac{1}{2}\int_{-1}^{1}q(x',i\omega_d)K_0\left(\sqrt{i\omega_d f(i\omega_d)}\,|x_d-x'|\right)dx',\quad \int_{-1}^{1}q(x',i\omega_d)dx'=1, \qquad (18)$$

where $f(i\omega_d)=\dfrac{\lambda+i\omega_d\psi(1-\psi)}{\lambda+i\omega_d(1-\psi)}$, $\omega_d=\dfrac{\omega\mu(\beta_1+\beta)x_f^2}{k}$, $\psi=\dfrac{\beta_1}{\beta_1+\beta}$, $\lambda=\dfrac{\alpha x_f^2}{k}$, β_1 is a fissures storativity coefficient, k is a fissures permeability, α is interporosity flow coefficient.

As shown in Fig. 7, the presence of interporosity flow is clearly indicated by a distinct hump on the pressure modulus derivative and phase shift plots.

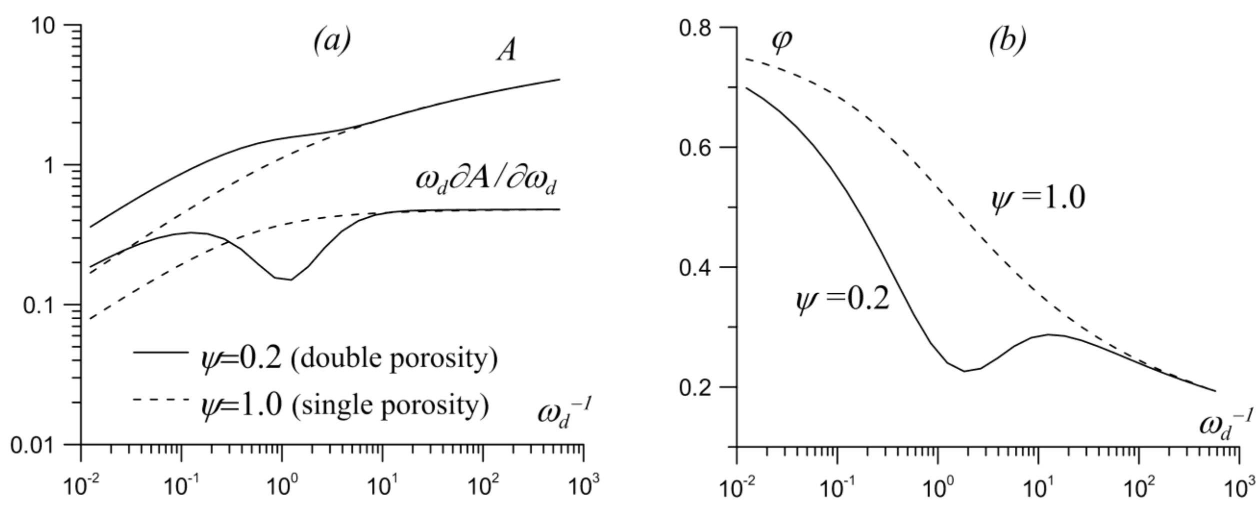

Fig.7. Pressure modulus and its derivatives curves *(a)*, phase shift curves *(b)* for a well with infinite-conductivity fracture in a double-porosity reservoir.

Summary

In this study, new analytical solutions are presented for analyzing amplitude- and phase-frequency characteristics of fractured wells in homogeneous or double-porosity reservoirs. The influence of wellbore storage effect, fracture storage and conductivity on the pressure modulus and phase shift is investigated. In case of high dimensionless frequencies a set of asymptotic solution is derived. These solutions can be used to solve the inverse problems for obtaining the formation and fracture properties.

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References

[1] A.C. Gringarten, H.J. Ramey Jr. and R. Raghavan: SPE J. Vol. 14, № 4 (1974) p. 347–360.

[2] H. Cinco-Ley, F. Samaniego-V. and N. Dominguez-A.: SPE J. Vol. 18, № 4 (1978) p. 253–264.

[3] H. Cinco-Ley and F. Samaniego-V.: JPT Vol. 33 № 9 (1981) p.1749–1766.

[4] D.W. Wong, A.G. Harrington and H. Cinco-Ley: SPE Formation and Evaluation Vol. 1 № 5 (1986) p.470–480.

[5] C.P.J.W. van Kruysdijk: paper SPE18169 presented at the SPE Annual Technical Conference and Exhibition, 2-5 October, Houston, Texas (1988)

[6] H. Cinco-Ley and H.-Z. Meng: paper SPE18172 presented at the SPE Annual Technical Conference and Exhibition, 2-5 October, Houston, Texas (1988)

[7] B.D. Poe, P.C. Shah and J.L. Elbel: paper SPE24707 presented at the SPE Annual Technical Conference and Exhibition, 4-7 October, Washington (1992)

[8] E.B. Chekalyuk: Oil Industry № 11 (1947) p. 29-32, (in Russian).

[9] E.B. Chekalyuk: *Basics of oil and gas deposits piezometry* (State Publishing House of Technical Literature UkrSSR, Kiev 1961), (in Russian).

[10] S.N. Buzinov and I.D. Umrikhin: *Study of reservoirs and wells by the elastic mode of filtration* (Nedra Publishing House, Moscow 1964), (in Russian).

[11] G.I. Barenblatt, V.M. Entov and V.M. Ryzhik: *Theory of unsteady filtration of fluids and gases* (Nedra Publishing House, Moscow 1972), (in Russian).

[12] C.H. Kuo: SPE Journal Vol.12, №6 (1972) p. 499–507.

[13] Yu.M. Molokovich and A.S. Shkuro: *The issues of improving the development of Tatarstan oil fields* (KSU Publishing House, Kazan 1962), p. 17-38. (in Russian).

[14] A.J. Rosa and R.N. Horne: paper SPE22698 presented at the SPE Annual Technical Conference and Exhibition, Dallas, Oct. 6-9 (1991).

[15] Yu.P. Zheltov and V.S. Kutlyarov: J. Appl. Mech. and Tech. Phys. № 6 (1965) p. 45–49.

[16] M.N. Kazi-Aoual, G. Bonnet and P. Jouanna: Eur. J. Mech. B/Fluids Vol. 10, № 1 (1991) p.51–72.

[17] Yu.M. Molokovich, A.I. Markov et. al.: *The development of fractured-porous reservoir by unsteady drainage* («Regent» Publishing House, Kazan 2000), (in Russian).

[18] M.N. Ovchinnikov: *Interpretation of the reservoirs study results by filtration waves of pressure* («Novoe Znanie» Publishing House, Kazan 2003), (in Russian).

[19] D. Despax, R. Dovis, J.-M. Fedele and J.-P. Martin, U.S. Patent 6,801,857. (2004)

[20] C. Jeong, C. Huh and L.F. Kallivokas: J. Petr. Sc. and Eng. Vol. 76 (2011) p. 116–123.

[21] F. Hollaender, P.S. Hammond and A.C. Gringarten: paper SPE77692 presented at the SPE Annual Technical Conference and Exhibition, San Antonio, Texas, 29 Sept.–2 Oct. (2002).

[22] E. Ozkan and R.Raghavan: SPE Formation and Evaluation Vol. 6, № 3 (1991) p. 359–368.

[23] S.N. Kravchun, S.T. Davitadze, N.S. Mizina and B.A. Strukov: Physics of the Solid State Vol. 39, № 4. (1997) p. 675–680.

[24] A.G. Gavrilov, A.N. Mardanshin, M.N. Ovchinnikov and A.V. Shtanin: Oil and Gas Business, http://www.ogbus.ru/authors/Gavrilov/Gavrilov_1.pdf (2006) (in Russian).