Deformation and fracture in the layer of rock under shear base

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Keywords: 3D modeling, rock layer, deformation, localization, shear band, fault, dilatancy, gravity

Abstract. The paper presents the results of 3D modeling of the formation of damage and destruction zones in the rock layer with a crack in the base under longitudinal shift (strike slip), i.e., a tearing mode. We consider regularities of formation of shear bands and investigated changes of their forms with depth. We obtain and analyze the three-dimensional structure of localized deformation zones, which is formed at the horizontal fault of the rock layer.

Introduction

Deformation in geomaterials can proceed in various modes depending on the structure, porosity, and stress state of a medium. Shear deformation often involves volumetric changes and depends strongly on pressure. In this case, large cracks and fractures can appear for which the preceding processes are normally loosening, microcrack accumulation with an increase in volume, and strain localization.

Among tasks on deformation development in the upper layers of the Earth's crust a special place is taken by fracture formation processes in the sedimentary rock, which are induced by the horizontal shear rupture of the base (strike slip). The mentioned task is of much scientific and applied significance as this fracture mode is most typical. Shear deformation occurs everywhere in the Earth's crust. Under certain loading conditions and rock properties we can observe shear bands and formation of cracks that often have an intermittent or zigzag-like rather than rectilinear character.

Both the vertical surface of a localized shear that splits the zone in two parts and the spatial system of branched localization zones are found to form under given deformation conditions. These zones can have a propeller-like form. Due to the specific shape of deformation development such type fractures are termed flower structures of the strike slip zone. Thus, the violations have a complex structure and the analysis of natural and experimental data is a difficult task.

The performed computation demonstrates that the complete distribution pattern of the stress-strain state especially in terms of deformation and fracture localization in the geomedium can be frequently formed using the three-dimensional modeling.

The main method for the investigation of mechanisms of deformation structure formation in the Earth's crust is the observation of natural phenomena and experiments performed on equivalent materials, for example [1-5]. Using the analytical estimation papers [6, 7] propose the explanation of mechanisms of deformation structure formation at the strike slip and the sequence pattern of their formation with consideration for the possibility of the medium transition to the inelastic state under gravity. In spite of the numerous experimental data, their generalizations and formed process schemes there remain questions about the structure and configuration of deformation zones as well as about the stress-stain state in the rock mass, which cannot be clarified within experimental and analytical estimation. Therefore, along with the experimental and analytical investigations there is need for mathematical modeling of the geomedium behavior under given conditions.

Numerical modeling of such a process intends the solution of a 3D problem. In paper [8] the shear band structure formation is numerically investigated in the 2D statement with the imitation of 3D

conditions by solving analytically an elastic task [6, 7]. In spite of the correlation of the obtained results with the existing ideas about the process as well as with the 3D simulation results [9], mechanisms of deformation development and spatial structure of fracture zones cannot be completely studied within this approach on account of its spatial character.
Deformation in geological media can proceed in various modes depending on the structure, porosity, and stress state of a medium. Shear deformation often involves volumetric changes and depends strongly on pressure. In this case, large cracks and fractures can appear for which the preceding processes are normally loosening, microcrack accumulation with an increase in volume, and strain localization.

1. Problem statement

Let us consider a problem on the deformation of a medium layer with a longitudinal notch at the bottom, Fig. 1.The layer is under the action of gravity and lies on the rigid base. According to the assigned depth and medium properties (density of overlying layers) in the elastic state stresses induced by gravity in the absence of additional forces are:

$$\sigma_z(z) = -g\int_0^z \rho(z)dz\,, \qquad \sigma_x(z) = \sigma_z(z)\xi\,, \quad \sigma_y(z) = \sigma_z(z)\xi\,, \tag{1}$$

where $\xi = \dfrac{\nu}{1-\nu}$, and ν – is the Poisson ratio. At the additional lateral load the stress state is assigned by the introduction of k_1 and k_2 coefficients in the expressions $\sigma_x(z) = k_1\sigma_z(z)\xi$, $\sigma_y(z) = k_2\sigma_z(z)\xi$. Values of k_1, k_2 >1 correspond to additional compression. Stress values are taken as the initial stress state (prestress).

The layer is loaded by assigning the shift of the left and right parts of the base in opposite directions along the notch at a constant speed. On the end (front and back) faces we set closed conditions imitating the infinite extension of the layer. On the side faces initial stresses were retained.

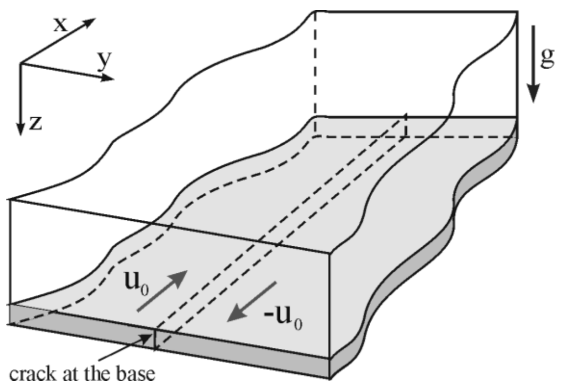


Fig. 1. Schematic pattern of deformation for the medium layer at the rupture in the base

The deformation process of the geomedium layer is modeled using the approach based on the solution of dynamic equations for the elastic-plastic medium by the explicit numerical scheme [10]. In the developed programs for 3D modeling of deformation processes we realized the parallel computing algorithm for multi-core and multi-processor computers, which allows calculating with a sufficient degree of resolution. Deformation above the elastic limit is described by a modified

Drucker–Prager–Nikolaevskii model with a nonassociated flow rule and parameters depending on pressure and accumulated inelastic strain [11-13, 9].

The calculations were carried out using 240×640×45 and larger meshes. In the general case, a tearing mode crack propagating in the base is described by a split node method [10]. In the given paper it was considered by the MPI communication library. Assuming that the crack is formed in the specimen symmetry plane, the computational area is decomposed so that this plane coincides with the interface between two subareas. According to the MPI technology, two nearby processors exchange values in boundary cells, which allows obtaining the continuous solution for the whole computational area. In fact, computational nodes lying at the interface belong to each of the subareas, i.e., they are naturally separated. To describe the motion of tearing mode crack edges velocities in the crack plane are calculated as free one, i.e., computational subareas exchange no data on the stress-state state at the interface, velocities directed perpendicular are set to zero to avoid the interaction of computational subareas. This results in free sliding of crack edges relative to each other.

2. Constitutive relations

In spite of the brittle character of fracture under macroscopic compression, plastic flow models are employed to describe rock deformation processes above the yield strength. Such models provide a rather adequate description of the geomedium behavior at the formation of microcracks and even of strike-slip fault structures. Thus, plastic deformation is understood to mean a deformation process above the yield strength independently of its nature. Use relations of a modified Drucker–Prager–Nikolaevskii model [11-13, 9]. Assume an additive decomposition of strain rate into elastic and plastic parts:

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^{\mathrm{e}}_{ij} + \dot{\varepsilon}^{\mathrm{p}}_{ij} \tag{2}$$

Elastic stresses are derived according to the hypoelastic law:

$$\frac{\mathrm{D}s_{ij}}{\mathrm{D}t} = 2\mu\left(\dot{\varepsilon}_{ij} - \frac{1}{3}\dot{\varepsilon}_{kk}\delta_{ij}\right), \qquad \frac{\mathrm{D}s_{ij}}{\mathrm{D}t} = \dot{s}_{ij} - s_{ik}\dot{\omega}_{jk} - s_{jk}\dot{\omega}_{ik}, \tag{3}$$

$$\dot{\sigma} = -K\frac{\dot{V}}{V}, \tag{4}$$

where K and μ are the bulk and shear moduli, respectively. To write the constitutive relations the stress tensor is decomposed into spherical and diviatoric parts: $\sigma_{ij} = -\sigma\delta_{ij} + s_{ij}$, where $\sigma = -\sigma_{kk}/3$ — is the pressure; s_{ij} are the deviatoric stress tensor components and, δ_{ij} is the Kronecker delta symbol. Here we use the corotational Jaumann derivative that considers the possibility of rotation of medium elements as a whole. The strain rate tensor components $\dot{\varepsilon}_{ij}$ and rotational velocity tensor components $\dot{\omega}_{ij}$ are derived from the relations:

$$\dot{\varepsilon}_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad \dot{\omega}_{ij} = \frac{1}{2}(u_{i,j} - u_{j,i}). \tag{5}$$

The elastic state of the medium in the stress space is limited by the yield surface, outside which a process of inelastic, plastic deformation or fracture occurs. Plastic strain is calculated using the yield surface and plastic potential:

$$f(\sigma_{ij}, \varepsilon_{ij}^{\mathrm{p}}) = 0, \quad g(\sigma_{ij}, \varepsilon_{ij}^{\mathrm{p}}) = 0, \tag{6}$$

$$\mathrm{d}\varepsilon_{ij}^{\mathrm{p}} = \mathrm{d}\lambda \frac{\partial g}{\partial \sigma_{ij}}, \tag{7}$$

where f is the yield surface function,, g is the plastic potential and, $\varepsilon_{ij}^{\mathrm{p}}$ are the plastic (inelastic) strain components. The yield surface is described by the Mises–Schleicher equation as:

$$\tau - \alpha\sigma = Y, \tag{8}$$

where α is the internal friction coefficient, Y is the shear strength of the material or cohesion coefficient and, $\tau = (s_{ij}s_{ij}/2)^{1/2}$.
In upper layers where pressure is low tensile pressures can arise as well as tearing mode cracks can form. For the sake of simplicity, without consideration for the stress components let us limit the yield surface in a tensile region by pressure

$$\sigma = \sigma^*. \tag{9}$$

The plastic potential for the calculation of plastic strain increments will be written as:

$$g(\sigma, \tau) = \tau - \Lambda\sigma, \tag{10}$$

where Λ is the dilatancy coefficient. Note that the use of the non-associated flow law provides the independency of the dilatancy coefficient. In a general case, this parameter is the function of the stress-strain state of the medium [12, 13].
The relation will determine hardening and softening of the medium:

$$Y(\gamma^{\mathrm{p}}) = Y_0[1 + h(A(\gamma^{\mathrm{p}}) - D(\gamma^{\mathrm{p}}))], \tag{11}$$

where h is the hardening coefficient, $\mathrm{d}\gamma^p = 2\left((\mathrm{d}e_{ij})^p (\mathrm{d}e_{ij})^p / 2\right)^{1/2}$ is the plastic shear strain intensity and $e_{ij}^{\mathrm{p}} = \varepsilon_{ij}^{\mathrm{p}} - \frac{1}{3}\varepsilon_{kk}^{\mathrm{p}}\delta_{ij}$. The linear dependence $A(\gamma^{\mathrm{p}}) = 2\gamma^{\mathrm{p}}/\gamma^*$ is used for hardening and a quadratic dependence $D(\gamma^{\mathrm{p}}) = (\gamma^{\mathrm{p}}/\gamma^*)^2$, for softening (damage accumulation), where is the critical strain beyond which material degradation prevails. On achieving certain stress values at the softening stage the shear strength is assumed to be constant and to correspond to residual strength Y* of the material under given conditions.
The mentioned constitutive relations close the system of equations including equations of motion and continuity.

3. Calculation results
The calculations are carried out for the area measured X=18 km, Y=80 km, Z=5 km (= 4.5 km sedimentary rock + 0.5 km elastic base). Rupture is in the center of the base x=9 km.
Rock properties are taken from Table.

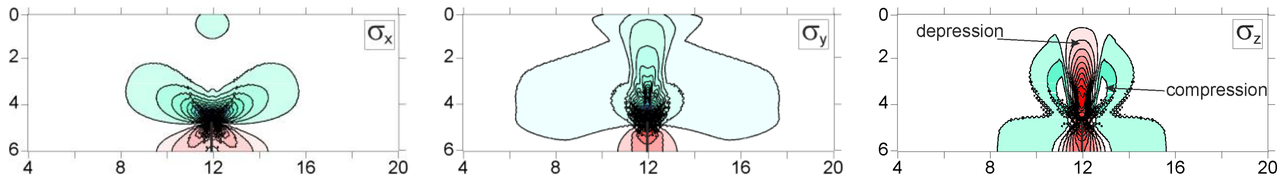
Table 1. Parameters of the model

ρ, g/cm^3	K, GPa	μ, GPa	Y_0, MPa	α	Λ	γ*	*h*	*σ**, MPa
2.2	12.8	5.34	20	0.65	0.1	0.04	0.005	-0.0

In the elastic stage and on the onset of the elastic-plastic deformation the stress-stain state of the medium layer has a character typical for a tearing mode crack when only the tangential component of the stress tensor has the singularity. At analytical account, the stress limitation based on the accepted plasticity law allows obtaining just preliminary estimates of the plasticity zone dimensions without consideration for the influence of nonuniform deformation development and its localization [6, 7]. However, during plastic deformation the shear stress intensity is limited by the shear strength of the medium as well as the character of the stress state changes, there appears a set of localized shear bands typical for geological media [7-9]. The given peculiarity of deformation development above the yield stress is governed by the sensibility to the stress state and by the possibility of shear band formation not only at the softening stage but also at certain values of α and Λ coefficients in Eq. 8, Eq. 10 [13-15].

During deformation, the formation of localized shear zones results in the formation of the depression and excess compression regions. Figure 2 shows the isolines for the increment in the normal stress components relatively to the prestress on the onset of the formation of localization zones. In other words, the difference between the current stress state during deformation and gravity stresses, with consideration for lateral load is demonstrated. At the initial stage as far as plastic deformation develops and localization zones propagate from the notch tip toward the free surface, a zone with positive (tensile) increments of the vertical stress component is seen to form, Fig. 2(c) in one of the section. On sides of the depression region there are excess compression regions.

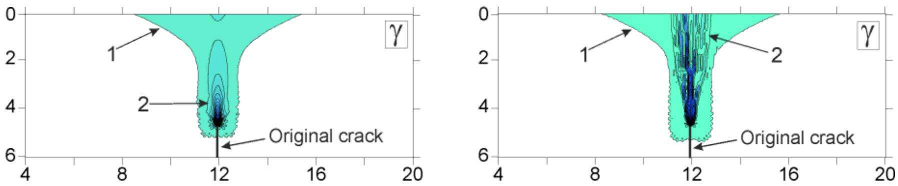
After localization zones are formed, regions with positive increments of stresses are also well determined in other components. Evidently, the presence of tension regions near the surface can lead to failure. Note that the mentioned stress distributions correspond only to one of the vertical sections in a current moment. Due to the nonuniform character of deformation development, what will be below mentioned, the stress distribution will differ in other sections.



(*a*) (*б*) (*в*)

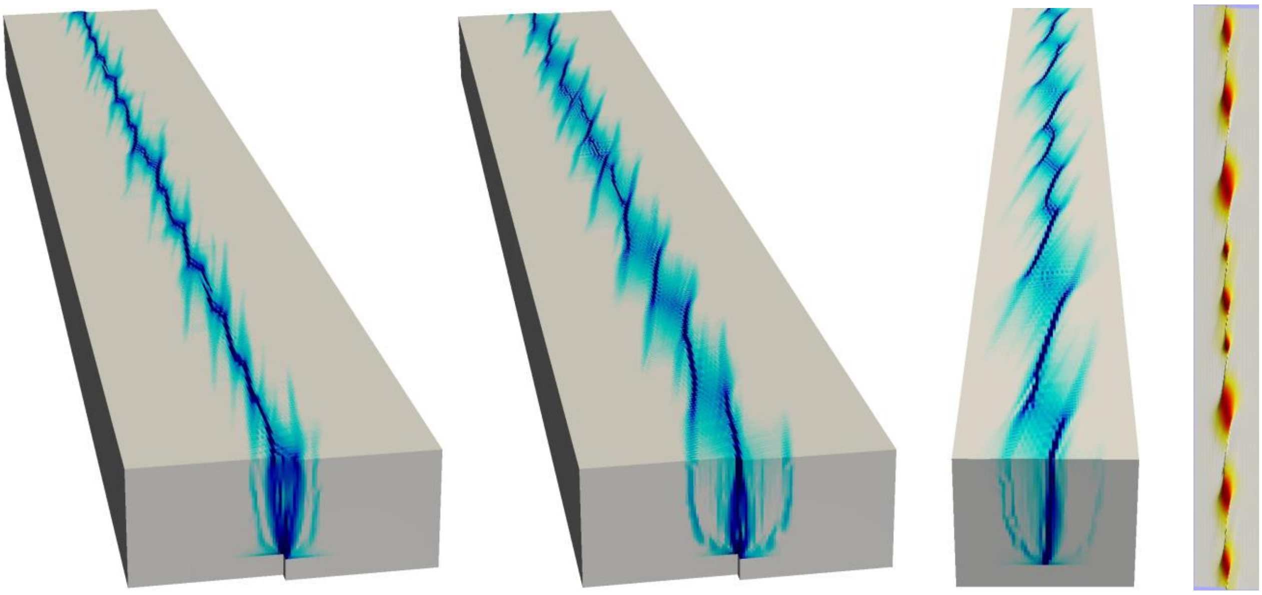
Fig. 2. Distribution of stress component increments in the transverse section of the layer at the elastic-plastic stage of deformation on the onset of the formation of localized shear bands

Figure 3 illustrates the distribution character of inelastic strain intensity in the transverse section. External contour *1* in the figure marks the plastic deformation zone. Deformation localization is observed to start in region *2*. The 3D distribution patterns of plastic strain intensity at the stage of localized shear bands already formed with consideration for opening in the fracture region are given in Fig. 3. As seen from Fig. 2 and Fig. 3, the localization zone widens quickly in the vicinity of the base while this width changes little in the middle and upper parts of the layer. Thus, though the region with irreversible deformation widens intensively as the surface is approached, shear bands are formed only in its central part. Due to the specific pattern of deformation development, such type fractures are termed flower structures of the horizontal shift [1-5].



(*a*) (*б*)

Fig. 3. Distribution of plastic strain intensity in the transverse section of the layer in the beginning (a) and during the formation of localized shear bands (b)



(a) (*b*) (*c*) (*d*)

Fig. 4. Distribution of plastic strain intensity in the medium bulk *a*) k_1=0.8, k_2=1.0; *b*) k_1=1.2, k_2=0.8; *c*) k_1=1.0, k_2=1.0; *d*) surface relief.

The process is most peculiar in that deformation localization and fracture occur as a series of surfaces changing their shape with depth rather than as the only surface. As for the central part of the layer above the notch line in the base, there are observed intermittent fracture zones. In every horizontal section especially before rupture on the surface a sufficiently regular system of *R* bands (Riedel shear bands) can be seen. With depth, the zone where shear bands are formed narrows. As the initial fault (notch) is approached, the plastic zone retracts into the fault line. In horizontal sections an inclination angle of localization bands changes slightly with depth in the horizontal plane. Depending on loading conditions, edges of shear bands are often observed to deviate from a rectilinear profile towards side walls or otherwise in the shear direction. This results in elongated S-like bands. Such deviation from the rectilinear profile of localization bands in horizontal sections of the layer is induced by additional compression (tension) on side faces, whose increase reinforces this effect, see Fig. 3.

Note that localized shear zones are formed and develop not in a stepwise manner as compared to crack propagation. A region of uniform plastic deformation is formed at initial stages. With deformation, a fine network of bands appears in this region. The strain intensity in them differs little while the strain-stress state is no more homogeneous. Gradually localization grows with increasing deformation and the structure of certain periodicity and configuration is formed, which is governed by parameters in constitutive relation, i.e., by the medium properties and deformation conditions.

Further behavior of localization zones has a character close to a filled shear crack. Stresses are found to concentrate near band tips and to decrease within them.

Fracture zones in the central part of Fig. 3 marked in dark are explained by the yield surface limitation in tension, i.e., by the consideration for an opening mode of fracture. Firstly, the formation of regions connecting inclined zones of localization is associated to this mode. These regions are seen to form in the central part of Fig. 3(*b*). Failure regions formed in upper layers in tension and regions of complete degradation often change the regularity of the localization pattern. A pair of localized bands can propagate from the tips, one of which follows the fracture zone and the other is directed at a large angle to the shear axis, Fig. 3(*b*, *c*). Sometimes, as for a given fracture zone, a localized band propagates from only one of its edge while as for the neighboring zone from the other edge.

The character of propagation of shear bands depends significantly on medium properties. For a brittle medium with rapid softening and zero residual strength the inelastic deformation zone covering localized shear bands turns to be narrow enough. The higher (sign considered) is rupture stress σ^*, the narrower is the fracture zone. A zigzag-like surface of fracture is formed, which in fact divides the layer into two parts. On the surface and in horizontal sections the localization band looks like a fault with branched cracks that at further deformation can coalesce and form a zigzag-like fault similar to the localization line in the central part. In this case, the strain intensity in certain regions of localization zones near the surface can exceed its value at depth where deformation develops more uniformly along the fracture line. In the limiting case of brittle fracture, a tearing mode crack propagates from the base to surface with branched cracks from its edges. If the yield surface limitation is given no consideration in tension and residual strength remains, the localization zone is wider. This effect is governed by more inhomogeneous deformation within the localized band. In the central part where fracture occurred a deformation degree is sufficiently higher than on band edges.

In vertical sections parallel to the initial fracture plane, localized bands are well seen to curve in the near of the base. They are slightly sloping in the bottom, and become almost vertical on reaching the surface. The angle of sloping changes maximally approximately up to the middle of the layer thickness.

The non-uniform pattern of inelastic deformation development in the layer and the induced volume change governed by dilatancy is also manifested in the surface deformation, Fig. 3(*d*). Changes in the surface relief turn to be complex enough as along with fold formation induced by shifts along localization lines dilatancy has a considerable contribution to the relief change. Dilatancy development in the localization zone leads to the surface elevation. Besides, stress and strain increments of opposite sign arise around the shear band similarly to a process near inclined or sliding mode cracks. Depression and excess compression zones appear and the surface falls or elevates.

4. Conclusion

The performed investigations allowed us to obtain and analyze the three-dimensional structure of shear bands formed at a longitudinal shift of the rock mass (strike slip). Under certain deformation conditions, localization zones that form the zigzag-like fracture surface and the spatial system of branched shear bands are seen to appear. These zones change their form with depth and coalesce into the fault line at depth. In the localization zone depression and excess compression regions are formed, which causes deformation and change of the surface relief.

The authors are grateful to G.N. Gogonenkov and A.I. Timurziev (CGE JSC) for the discussion and useful remarks.

The paper is performed at the support of RFBR (Grant No. 11-05-00661-a) and SB RAS Integration Project No. 127.

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