# A REVIEW OF THE MAIN TWO-PARAMETER FRACTURE MECHANICS APPROACHES FOR THE ANALYSIS OF CONSTRAINT

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# ABSTRACT

Constraint constitutes one of the major issues in fracture mechanics research. The possibility of performing more adjusted, less conservative assessments requires, in many cases, the consideration of the constraint conditions at the crack tip in order to make better predictions of the critical loads or the critical crack dimensions. Several methodologies have been proposed, from those based on micromechanical models or energy methods to those based on engineering approaches. Among the latter, two methodologies stand out: the biparametric correction to  $K_{IC}$  and the biparametric CTOD constraint correction.

From a practical point of view, the so-called engineering approaches are of maximum interest. Here, a complete overview of these methodologies is presented, with special emphasis on the FITNET FFS Procedure proposal for the assessment of loss of constraint and also on the IST methodology. The contributions, limitations, advantages and disadvantages of both approaches are discussed, as well as the possible interactions and synergies between them.

Keywords: Constraint, Two-parameter fracture mechanics, FITNET, IST, CTOD.

# INTRODUCTION

An implicit source of conservatism in many structural integrity assessments is that the value considered for the material fracture toughness is obtained from deeply cracked specimens subjected to predominantly bending loads. However the material fracture resistance is higher when tests are performed using shallow cracked specimens subjected to tensile loads, as under such conditions there are lower hydrostatic stresses and lower maximum principal stresses near the crack tip. This is described as a lower level of constraint and leads to an increase in the material fracture resistance (in both brittle and ductile conditions). There are several methodologies for the constraint analysis of cracked structures. The most widely accepted are the local approaches, the energy methods and two-parameter fracture mechanics (TPFM). Figure 1 [1] presents a schematic of the different combinations of theories and approaches dealing with fracture analysis. Higher levels provide more accurate results but present more complexity. The lower level is not capable of including constraint in the analysis, the intermediate level (TPFM) explicitly includes a second parameter for the consideration of constraint and the upper level includes constraint implicitly.

Focussing on the upper and middle levels, local approaches (e.g., Beremin model for cleavage [2] and Beremin ductile model [3]) include models for the actual failure mechanisms and are based on a precise knowledge of the stress and strain fields, so that constraint effects are considered implicitly, meanwhile energy methods (e.g., [4,5]) assume that fracture

occurs when the energy available for crack growth is sufficient to overcome the resistance of the material.

On the other hand, TPFM proposes that an additional fracture parameter is sufficient for the consideration of constraint effects. Once such a parameter is defined, the assessment of the corresponding component or structure is relatively simple. This provides as accurate results as other methodologies with, perhaps, a greater physical basis. The analysis presented here denotes as "engineering approaches" all those constraint assessment methodologies based on TPFM, with special emphasis on the constraint correction provided by the FITNET FFS Procedure [6] and the IST methodology [7], which are representative of the majority of engineering procedures or standards for constraint assessment.

Taking the J integral as the crack driving force, two parameter fracture mechanics assumes that the crack growth locally along the crack front, s, is defined by the following expression:

$$J(s; P, a) = J_{R}(\Delta a(s), \kappa(s), Temperature)$$

(1)

where P is the applied load, a is the crack size,  $\kappa$  is the constraint parameter and  $J_{\text{R}}$  is a material function that depends on the crack growth,  $\Delta a$ . Different constraint parameters have been proposed (Figure 1). The most widely used are the T-stress (second term in the William's series for the elastic stress field at the crack tip), the Q-parameter [8], the  $A_2$  parameter [9] and the h-parameter (hydrostatic stress divided by the von Mises effective stress), although the newly developed IST methodology also introduces the  $\beta$  parameter, as explained below in Section 3. Whichever parameter is used, the basis of the two-parameter procedure is the same: it assumes that the fracture toughness for initiation, the crack growth resistance and the fracture mode at a given temperature are governed solely by two parameters.



Fig.1. Schematic of the different theories and approaches for fracture analysis [1].

Here, it is important to notice that, the different kinds of approach are not always entirely separable and, for example, the Weibull shape parameter (local approach), m, is required when accurate results for cleavage fracture from engineering approaches are pursued.

# FITNET FFS CONSTRAINT ASSESSMENT PROCEDURE

### **Review of FITNET FFS constraint assessment procedure**

The methodology proposed by the FITNET FFS Procedure [6] for the assessment of low constraint conditions is based on the constraint section of the R6 procedure but includes new advice in order to make the assessment easier. Therefore, its philosophy is totally analogous

to the one found in R6 and in similar documents such as SINTAP (also, this methodology is being considered for future revised versions of BS7910). However, the FITNET FFS procedure provides more guidance on the definition of the parameters required for the calculations and advice on when constraint assessments are justified. For example: benefit is greatest in components subjected to predominantly tensile loading rather than bending; constraint effects are more significant for components containing shallow cracks and; there is little benefit in an assessment of ductile materials based on initiation toughness (as the fracture toughness at initiation tends to be insensitive to constraint), at low values of primary load ( $L_r \le 0.2$ ;  $L_r$  is defined in eqn (3)) and in collapse-dominated cases.

In Section 6.4.3, the FITNET FFS proposes a TPFM approach for the constraint assessment that can be applied following two distinct equivalent procedures (applicable to both initiation and tearing analyses). Both approaches use the normalised ratios

$$K_{r} = \frac{K^{p} + K^{s}}{K_{mat}}$$
(2)
$$L_{r} = \frac{P}{K_{mat}}$$
(3)

$$L_{r} = \frac{1}{P_{L}(a,\sigma_{y})}$$
(3)

where K<sup>p</sup> and K<sup>s</sup> are the stress intensity factors for the primary and secondary loads, K<sub>mat</sub> is the fracture toughness, P is the applied load and P<sub>L</sub> is the corresponding collapse load which depends on crack size, a, and yield stress  $\sigma_y$ . Note, the procedure uses a correction factor in eqn (2) to cover interactions between primary and secondary loads; this is omitted here for simplicity as it does not affect the discussion of constraint effects.

Procedure I consists of a modification to the Failure Assessment Diagram (FAD) while retaining the fracture toughness used to define  $K_r$ ; Procedure II modifies the definition of  $K_r$  and retains the FAD used in ordinary assessments. With Procedure I, the modified FAD is:

(4)

$$K_r = f(L_r)(1 + \alpha(-\beta L_r)^k)$$
;  $L_r \le L_{rmax}$ 

where  $f(L_r)$  is the shape of the failure assessment line,  $\alpha$  and k are material constants describing the influence of constraint on fracture toughness and  $\beta$  is a normalised measure of the structural constraint. As shown in Figure 2, an assessment point initially placed above the Failure Assessment Line (therefore, considered as in an unsafe condition) can be located within the area defined by the FAL and the coordinate axes using the constraint approach (thus demonstrating that it is in a safe condition). The constraint measure  $\beta$  can be defined in terms of the T-stress or in terms of the Q-parameter.



Fig. 2. Different FADs for different constraint assessments, through Procedure I.

Procedure II modifies the value of the fracture toughness used in eqn (2) from  $K_{mat}$  to  $K_{mat}^{c}$  and retains the FAD used in ordinary assessments, being totally analogous to Procedure I. In this case, the constraint consideration affects the value of the K<sub>r</sub> parameter

(ordinate of the assessment point), which is vertically displaced downwards (the higher the loss of constraint, the higher the vertical displacement).

Regarding the input parameters required for the assessment, the FITNET FFS procedure incorporates a methodology developed by Sherry et al. [10] within the framework of the VOCALIST project [1] that allows the  $\alpha$  and k parameters to be obtained from the tensile data and the Beremin cleavage model parameter, m, Here, the most difficult point is the estimation of m for the specific case being analysed. Therefore, this is a clear example of how a TPFM approach uses input data obtained from a local approach.

Beyond the explicit constraint assessment methodology proposed by the FITNET FFS Procedure, a key point is its flexibility (both scientific and technological) to connect constraint analysis with two issues of significance in fracture mechanics: the Master Curve and the notch effect. Thus, the FITNET FFS Procedure includes the formulation proposed by Wallin [11] for the estimation of the shift of the Master Curve caused by loss of constraint. The Master Curve expression under low constraint (T < 0) conditions is:

$$K_{mat}^{c} = 20 \text{ MPa} \sqrt{m} + (K_{mat} - 20) \exp(0.019[-T_{stress}/10 \text{ MPa}])$$
(5)

This means that the T-stress produces a shift towards lower temperatures in the Master Curve equal to T-stress/10 (for negative values of T-stress). Likewise, the stress relaxation at notches has not been previously included in any fracture assessment procedure. Section 12.5 of the FITNET Procedure includes methodologies for the estimation of the increase of fracture resistance caused by the notch effect. Such methodologies are basically the Critical Average Stress Model (CASM) [12] and Finite Fracture Mechanics (FFM) [13]. When using the CASM, the apparent fracture toughness ( $K_{IN}$ ) is:

$$K_{\rm IN} = K_{\rm IC} \sqrt{1 + \frac{\rho}{2X_{\rm ef}}} \tag{6}$$

where  $\rho$  is the notch radius and X<sub>ef</sub> is the effective distance, as defined in [12]. Analogous relations can be found when using FFM. The FITNET FFS Procedure, Section 12.5.5, also provides a simple methodology that allows FAD assessments of components with low constraint due not only to shallow defects or tensile loading but also to notch-type defects, that is, a procedure that provides a global treatment for the in-plane loss of constraint, as developed in [14]. This methodology assumes that the loss of constraint due to a notch is independent of the loss of constraint due to T or Q type stresses. Therefore, combining Procedure I and the CASM, the assessment of a component with a shallow notch subjected to tensile loading is performed using the following equation for the modified FAD:

$$K_{r} = f(L_{r}) \cdot (1 + \alpha(-\beta L_{r})^{k}) \sqrt{1 + \frac{\rho}{2X_{ef}}} \quad L_{r} \leq L_{rmax}$$

$$\tag{7}$$

For Procedure II, the formulation would be analogous and straightforward. Figure 2 shows a schematic of the assessment of a component subjected to low constraint conditions, with all sources of in-plane loss of constraint occurring simultaneously. The increase in the safe area in the FAD gained through the different constraint corrections is clearly revealed.

Finally, the procedure provides a methodology for assessing components with cracks that start from a notch tip, as developed by NASA [15], and some advice regarding the interaction between constraint and biaxiality.

### Contributions and limitations of FTITNET FFS constraint procedure

The main contributions of the FITNET FFS are the following:

a) It provides guidance on whether or not to consider constraint effects in the analysis..

- b) The FITNET analysis using the T-stress parameter is rather simple, given that the structural constraint parameter  $\beta$  ( $\beta_T$ ) does not depend on the applied load.
- c) FITNET provides much of the data required in the analysis as input parameters (material constants, T-stress solutions....). One difficulty in use of this approach for cleavage fracture is the determination of the Beremin parameter (m) as an input to obtaining the constraint corrected fracture toughness. However, the values proposed by IST (as seen below) could also be taken as reference values in FITNET.
- d) FITNET FFS can be applied to a large number of geometries.
- e) Its application is not restricted to brittle situations.
- f) It can be applied in terms of both Failure Assessment Diagrams or in terms of Crack Driving Force Diagrams. Also, its application can be extended to situations with ductile tearing [6] (with a more complex analysis process).
- g) FITNET FFS includes the Master Curve as a constraint assessment tool.
- h) It enables the analysis of notches and cracks emanating from notch tips.
- i) Finally, FITNET FFS proposes a methodology for the assessment of those situations where all the sources of in-plane loss of constraint occur simultaneously.

However, there are several conceptual matters that need to be considered when applying the FITNET FFS Procedure constraint approach:

- a) The α and k numerical solutions for cleavage fracture are based on 2D plane strain finite element (FE) analysis with the modified-boundary layer (MBL) model, whereas a crack in a component is not necessarily subjected to plane strain [7].
- b) The  $\alpha$  and k numerical solutions for cleavage fracture are limited to 5 < m < 20.
- c) It does not consider the volume effect on the Weibull stress for cleavage fracture [7].
- d) In the case of performing the analysis using the Q-parameter, the parameter measuring the structural constraint,  $\beta_Q$ , may depend on the applied load, so it is necessary to know its value at fracture (this would require some iteration).

### IST CONSTRAINT ASSESSMENT METHODOLOGY

### Review of IST constraint assessment methodology

The Japanese project IST was developed from 2002 to 2005 with the aim of developing a standard procedure for the fracture assessment of steel components from the fracture toughness results obtained in laboratory specimens and is presented in the recently developed ISO 27306 [16]. It allows the cleavage fracture of ferritic steels to be analysed, so it is not recommended for those situations where there is significant stable crack propagation [7]. It is also a two-parameter approach, as it uses a conventional fracture parameter, the CTOD, and a second parameter named "equivalent CTOD ratio", here called  $\beta_{IST}$ . Thus, the critical CTOD,  $\delta_{cr}$ , obtained from standard fracture toughness specimens, is converted to the critical CTOD for the component being analysed,  $\delta_{WP,cr}$ , at the same level of Weibull stress:

$$\delta_{\rm WP,cr} = \delta_{\rm cr} / \beta_{\rm IST} \tag{8}$$

This is based on the result from the Beremin cleavage local approach, that the Weibull stress for a given failure probability is the same regardless of the geometry of the cracked component (standard specimen or structural component). Again, this two-parameter methodology uses an input parameter (m) obtained from a local approach. It should be noted here that IST provides a constraint correction that is independent of the applied load.

The factor  $\beta_{IST}$  basically depends on the yield to tensile strength ratio of the material (YR), the Weibull stress parameter (m) and the crack geometry. Within Small Scale Yielding (SSY) conditions, it also depends on the deformation level of the structural component. The value of

 $\beta_{IST}$  for each particular case can be obtained following three different assessment levels (the greater the information available, the lower the conservatism of the analysis):

- Level I (Simplified Assessment): applied to those cases in which the information required for the estimation of  $\beta_{IST}$  is not fully available. In such cases,  $\beta_{IST} = 0.5$ .
- Level II (Normal Assessment): applied to those cases where both the mechanical properties and the crack geometry are known, but m is not available. In such situations, the IST proposes two default values for m: 10 when  $\delta_{cr} \leq 0.05$  mm; and 20 when  $\delta_{cr} > 0.05$  mm. Once m is defined,  $\beta_{IST}$  is obtained from nomographs as a function of the component crack type and size, the material yield-to-tensile ratio, and the parameter m.
- Level III (Material Specific Assessment): applicable to those situations where m is statistically determined from a sufficient number of fracture toughness test results [7].

Whichever level is used, once  $\beta_{IST}$  is defined, the fracture resistance of the component is obtained through eqn (8) and the fracture assessment can be performed through a FAD.

## Contributions and limitations of IST methodology

The main contributions of IST are the following:

- a) It has been adopted as an ISO standard [16].
- b) It is a rather simple constraint assessment procedure. The  $\beta_{IST}$  parameter is easily obtained even in those situations where the information available is very limited.
- c) It presents a hierarchical procedure with different levels depending on the data available.
- d) In those situations where the mechanical properties of the material are known, IST proposes an intermediate level of analysis (Level II), less conservative than the Simplified Assessment (Level I). In this way, an advanced constraint assessment level is achieved with limited calculation efforts.
- e) The  $\beta_{IST}$  parameter is obtained by comparison of two situations (standard fracture toughness specimen vs. structural component) with the same Weibull stress. This is not the case for FITNET because of the volume effect mentioned above.
- f) It allows fracture analyses to be performed by using the FAD.

However, as with FITNET, there are several issues that should be considered when applying the IST methodology:

- a) It is limited to cleavage fracture [7].
- b) Its simplicity is restricted to a limited number (4) of geometries. For any other geometry, the estimation of β<sub>IST</sub> is only simple at Level I (that may be too conservative in many situations). Moreover, even for the four geometries covered in the procedure, the solutions are sometimes limited (i.e., thickness ≥ 25 mm).
- c) IST does not consider the notch effect. It does not include constraint effects on the material Ductile to Brittle Transition Regime (i.e., Master Curve).
- d) IST does not explain why its constraint correction is not sensitive to the applied load.

### COMPARISON, INTERACTIONS AND SYNERGIES BETWEEN FITNET FFS AND IST.

At this point, it is possible to establish the existing relationship between the FITNET FFS and IST constraint assessment procedures. Considering the relation between  $K_I$  and CTOD ( $\delta$ ), if constraint is included in the assessment and the FITNET and IST methodologies are equivalent then:

$$K_{r,constraint} = \frac{K_r}{K_{IC}(1 + \alpha(-\beta L_r)^k)} = \sqrt{\delta_r} \cdot \sqrt{\beta_{IST}} = K_r \cdot \sqrt{\beta_{IST}} \qquad \frac{1}{(1 + \alpha(-\beta L_r)^k)} \Leftrightarrow \sqrt{\beta_{IST}}$$
(9)

That is, the comparison of the constraint corrections provided by FITNET FFS and IST corresponds to comparing the terms in expression (9). It should be noted here that FITNET FFS correction depends on the applied load (or  $L_r$ ), while the IST constraint corrections do not depend on load level. Figure 3 presents a comparison of the different corrections obtained through FITNET and IST in a particular case analysed in [17].



Fig. 3. Comparison between FITNET FFS and IST constraint corrections.  $\alpha$ =2.07; k=1.43.

As mentioned above, the FITNET FFS constraint procedure could consider (limited to ferritic steels) using the default values given by IST for the Weibull shape parameter, m, as input to determination of the constraint corrected cleavage fracture toughness (essentially  $\alpha$ , k). This would turn the FITNET FFS constraint procedure in the cleavage regime into a hierarchical methodology with two different levels: Level I, in which default m values are assumed, and; Level II, requiring the specific evaluation of m.

# CONCLUSIONS

Two constraint approaches have been described and discussed: FITNET FFS (which includes others such R6 and SINTAP) and the IST methodology (also in ISO 27306). The contributions and limitations of the approaches have been explained, showing their ability to be applied to the analysis of complex problems through a relatively simple methodology.

While FITNET FFS provides advice about the value of performing constraint assessments, as well as solutions or references for a greater amount of crack geometries (and for the analysis of other types of defects such as notches and cracks emanating from notches), IST is focussed on a limited number of geometries. FITNET FFS is applicable to both cleavage fracture and ductile tearing, whereas IST is only suitable for cleavage. Moreover, FITNET FFS allows constraint assessments to be made with the Master Curve.

IST presents a hierarchical procedure adopted as an ISO standard which can be applied with very limited information about the material properties (Level I, where the user simply has to confirm that the material is ferritic steel). Also, IST provides helpful default values of the Weibull shape parameter m.

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