Universality of Fluctuation Statistics under Plastic Deformation of Metals

I. Panteleev[1, a], O. Naimark[1, b], C. Froustey[2]

[1]Institute of continuous media mechanics UB RAS, Perm, Russia

[2]LAMEFIP, ENSAM, Bordeaux, France

[a]pia@icmm.ru, [b]naimark@icmm.ru

Keywords: structural-scaling transitions, instability of plastic flow, non-Gaussian fluctuations, self-organized criticality.

Abstract. Conception of structural-scaling transitions in solids with mesodefects as characteristic type of self-organized criticality is applied for the study of universality of yield stress fluctuation statistics under jerky flow of crystalline solids. It is shown the link of PDF universality for yield stress fluctuations with universality class of collective modes of mesodefects related to the self-similar solution providing the mechanism of momentum transfer under plastic flow.

Introduction

The systems with the so-called "slow dynamics" exhibited high interest during last decades. The examples of such systems are liquids in the condition of turbulence, biological systems, solid under plastic deformation. Particular feature of such systems is the universality of fluctuation statistics (statistical self-similarity) that was established first time in [1] for the experiments in the inertial range of turbulence. The recent experimental and theoretical research showed that crystal (polycrystal) plasticity revealed large spatial-temporal fluctuation, which could be linked with the spatial-temporal scale invariants. Plastic flow proceeds through the intermittent deformation bursts with power law size distributions. Deformation pattern is characterized by long-range correlations, self-similarity and self-affine surface morphology (deformation induced roughness).

Nonlinear dynamics of yield stress conventionally known as the jerky flow is observed for many metals and alloys [2,3]. Classical picture of this phenomenon demonstrates that the instability of homogeneous plastic flow occurs due to the anomalous strain softening. The instability appears in the form of localized plasticity with typical scale of bands $\sim 10-100\mu m$ and the nucleation of bands is associated with the drop of stress (stress fluctuations) on the stress-strain diagram. Different spatial-temporal scenario of the development of shear bands is observed as the consequence of pronounced correlations between the bands. Study of the development of localized shear bands in crystals showed different dynamics of shear bands and, as the consequence, different stochastic dynamics of yield stress fluctuations. The finite value of correlation dimension was found for the stochastic amplitude pulsation of yield stress for middle strain rates and the power-law of the stress fluctuation for the high strain rates. Autocorrelation function revealed also the power-law spectrum for the last case. Mentioned features of stochastic behavior of stress fluctuations is characteristic for the self-organized criticality (SOC) [4], when the events on all temporal and spatial scales demonstrate the power-spectrum and the system behavior doesn't characterized by the finite correlation dimension. It is possible to conclude that the jerky flow dynamics revealed the transition from the dynamic chaos to the SOC-statistics under the increase of strain rate. This transition is usually observed, when the relaxation time of plastic flow can approach to the characteristic loading time.

From the point of view of continuum theory the plastic deformation of crystalline solids is considered as the quasi-laminar flow. To follow the traditional paradigm in the continuum plasticity theory the crystalline solid is the quasi-homogeneous continuum, where the plastic flow can proceed spatially homogeneous in the absence of the plastic instability. It means that the fluctuations don't reveal themselves on the scales more than the representative volume scale that, as it is conventionally assumed, is smaller than the specimen size. To follow this assumption the temporal-spatial non-homogeneity occurs if the deformation processes is macroscopically unstable, for instance, due to deformation softening. Macroscopically observed instabilities appear as the spatial-time structures with the dynamics of running solitary waves or stochastic deformation modes [5]. The paradigm of stable plastic flow was in fact in the contradiction with the results in the physics of plastic flow for crystalline materials, where the mechanism of momentum transfer under the plastic flow was linked with the discrete part of deformation due to the dislocation motion. Recently the view on the plastic flow as stable deformation has been challenged both from an experimental and from a theoretical point of view. Qualitative new picture of plastic flow arose, when instead uncorrelated motion of dislocations the numerous burst deformation areas appear in the conditions of pronounced long-range spatial-temporal correlations. This behavior can be observed in critical phenomena, in self-organized criticality, in the problems related to turbulent flow. By analogy with turbulence these finite-size critical systems can be determined as "inertial systems" and the range of scales between a and L as the "inertial range". The remarkable statistical property of mentioned systems is that they can not be divided into mesoscopic regions that are statistically independent. As the consequence these systems don't satisfy the basic criterion of *central limit theorem* and the spatially averaged quantities (with the Gaussian fluctuation statistics about the mean value) could not be introduced.

Present paper is devoted to the study of fluctuation statistics universality that is characteristic for the mechanism of momentum transfer under plastic flow of crystalline materials. The parameters related to the typical mesoscopic defects were introduced as the localized distortion modes (microshears). Statistical theory of these defects allowed one to establish new type of critical phenomena – structural-scaling transition and to propose the interpretation of self-organized criticality scenario in solid with mesodefects. Two order parameters were found that are responsible for structure evolution – the defect density tensor and structural-scaling parameter, which reflects the scaling transition due to the generation of characteristic collective modes in mesodefect ensemble [5].

Experimental study of jerky flow under plastic deformation of aluminium alloy, steel and Armco-iron supported the universality of PDF for yield stress fluctuations that is similar to PDF universality for the inertial range of liquid turbulence.

Statistically based mesoscopic potential was proposed in the generalization of the Ginzburg-Landau approach for critical phenomena. It was shown that the subjection of out-of-equilibrium system to the kinetics of mentioned collective modes provides the slow dynamics and universality of fluctuation statistics.

Structural-scaling transitions as mechanisms of self-organized criticality in solid with mesodefects

Microscopic order parameters and corresponding Lagrangian for the description of localized instabilities in solid were introduced as the variables characterizing the localization of the symmetry group of distortion tensor, which describes the nucleation and dynamics of localized shears under plastic flow [11]. The formulation of statistical problem for the microshear ensemble showed the existence of two order parameters of solid with micro-shear ensemble: microshear density tensor (microshear induced strain) p_{ik} and additional order parameter δ that is named as the structural-scaling parameter. This parameter represents the ratio of two characteristic scales in solid with mesodefects $\delta \sim (R/r_0)^3$, where r_0 is characteristic size of the defect nuclei, R is the distance between defects. The solution of statistical problem for the case of simple shear ($p = p_{xz}$) induced by the shear stress $\sigma = \sigma_{xz}$ established three characteristic nonlinear regimes for the tensor order parameter p_{ik} in different ranges of structural-scaling parameter $\delta\,(\delta > \delta_* \approx 1.3,\ \ \delta_c < \delta < \delta_*,\ \ \delta < \delta_c \approx 1)$, where δ_c and δ_* are the bifurcation points (Fig.1). Bifurcation points δ_*, δ_c play the role similar to critical temperatures in the Landau theory of phase transformations. The structural scaling parameter δ has the meaning of the second order parameter and the value of δ determines the thermalization conditions of mesoscopic out-of-equilibrium system.

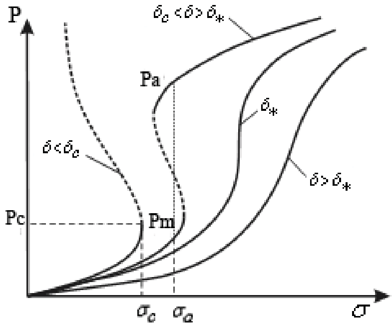

Fig. 1. Characteristic solid responses on the growth of microshears for different values of structural scaling parameter δ

The type of transitions over the critical points is given by the bifurcation type – the group properties of kinetic equations for different ranges of the structural-scaling parameter $\delta\,(\delta > \delta_*, \delta_c < \delta < \delta_*, \delta < \delta_c)$. Qualitative changes of system behavior are illustrated by the family of self-similar solutions – collective modes of microshears for different ranges of δ (Fig.2). The periodic solution transforms for $\delta \to \delta_*$ into the auto-solitary waves $p(\zeta) = p(x - Vt)$ in the orientation metastability area $\delta_C < \delta < \delta_*$, where the collective orientation modes appear at the front of auto-solitary wave (S_2, Fig.2).

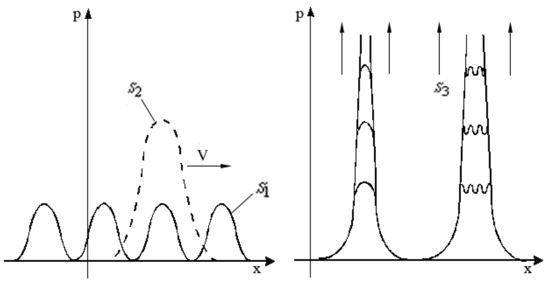

Fig.2. Characteristic types of collective modes in the microshear ensemble

Self-organized criticality in the form of structural-scaling transitions and established universality classes in terms of characteristic self-similar solutions allowed the interpretation of scenario of plastic flow instabilty.

Universality of yield stress fluctuation statistics in jerky flow of metals

It was established that the similarity of mechanisms for liquid turbulence and plastic instability in metals is related to characteristic universality of critical phenomena that was classified as the structural-scaling transitions [4]. Mentioned mechanisms are linked with long-range correlations in the ensemble of localized shears that leads to the strain localization in metals.

The study of statistics of yield stress fluctuation induced by strain localization zones of uni-axially deformed specimens was carried out for 5454 aluminum alloy and Armco-iron. Deformation diagram for aluminium alloy is presented in Fig.5 and one reveals the pronounced fluctuations of yield stress of plastic flow. Qualitative difference in the fluctuation statistics allowed the establishment of four deformation stages for aluminium alloy (Fig.3) according to characteristic statistical moments and the probability distribution function (PDF) for yield stress fluctuations. The distribution of the third moment (skewness coefficient) along the stress-strain diagram is represented in Fig.4 and reflects the different of deformation stages related to the mechanisms of plastic flow. As it follows from Fig.5 the PDF has slight deviation from the Gaussian form at the stage I of diagram. The second stage is transient to the third stage with the universal PDF form. It is important to note that the shape of PDF doesn't change along the third stage that reflects the statistical self-similarity of phenomena of plastic instability and the scale invariance of mechanisms of plastic flow at this stage.

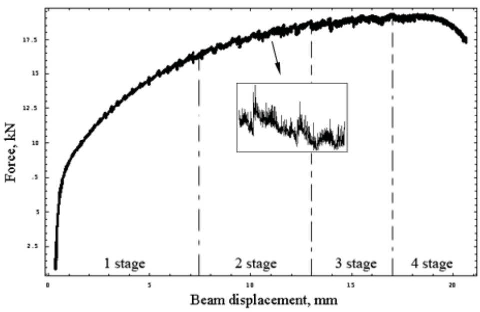

Fig. 3. Stress-strain diagram of 5454 aluminium alloy

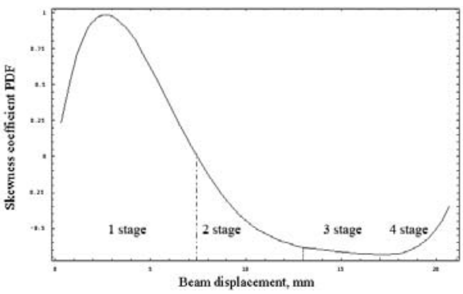

Fig 4. The skewness coefficient for different stages of deformation

These features of stress fluctuation statistics allowed us to assume the self-organized criticality of mechanism of plastic flow that corresponds to the set of collective modes of the microshear ensembles for the range of structural-scaling parameter $\delta_c < \delta < \delta_*$. The self-similar auto-solitary wave solution that was established for the kinetics of microshears ensemble determines the specific universality class of collective modes, which could provide the PDF universality for plastic flow as the finite-size critical phenomenon. The generation of multiple self-similar auto-solitary wave structures along the specimen can provide numerous stress fluctuation in the condition of long-range correlation. The self-criticality features of mechanism of plastic flow can take place namely along the second stage of deformation with pronounced "slow dynamics", when the generation of the set of collective modes provides the effective rescaling in the terms of characteristic length determining the current value of structural-scaling parameter δ. The formation of mentioned collective modes and the "coarsening" of structural scales induced by defects lead to the deviation from the "self-critical scenario", that is realized along the "upper thermodynamic branches" of metastability curves (Fig.1). This transient stage of decreasing of structural relaxation (related to the kinetics of auto-solitary waves) represents the transition to the hardening branch of deformation curve, where the dislocation substructures are loosing the mobility providing the conventional mechanism of plasticity. The fourth stage can be considered as the precursor of nucleation of failure hotspots due to the qualitative change of the universality class from the auto-solitary wave self-similar solution to the blow-up self-similar solution [*].

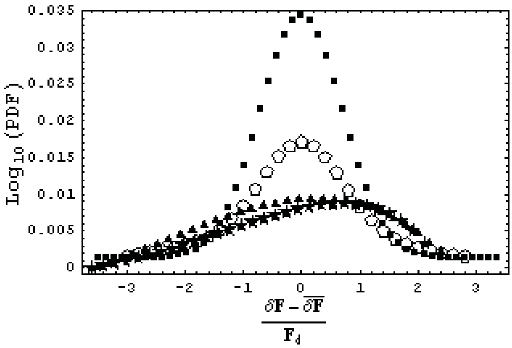

Fig.5. PDF forms for different stages (1 - ■; 2 - ▲; 3 -★; 4 - ◊) of the stress-strain diagram.

The statistics of yield stress fluctuation was studied also for the uni-axial plastic flow of Armco-iron specimen in the condition of compression test at 300 K. Probability distribution function for the yield stress fluctuations for aluminum alloy, Arco-iron and steel specimens are presented in Fig.6, 7, 8.

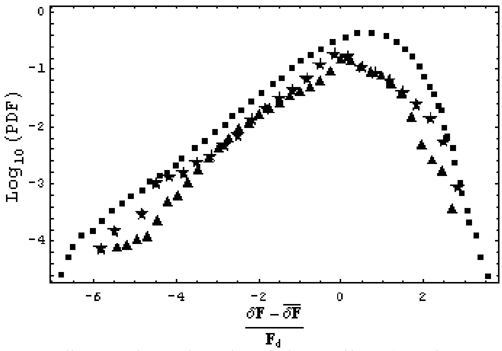

Fig. 6. PDF of yield stress fluctuations for aluminium alloy (tension test for strain rates ▲ – $\acute{\varepsilon} = 0{,}01\ s^{-1}$, ★ - $\acute{\varepsilon} = 0{,}02\ s^{-1}$). ■ - PDF for inertial range of turbulence.

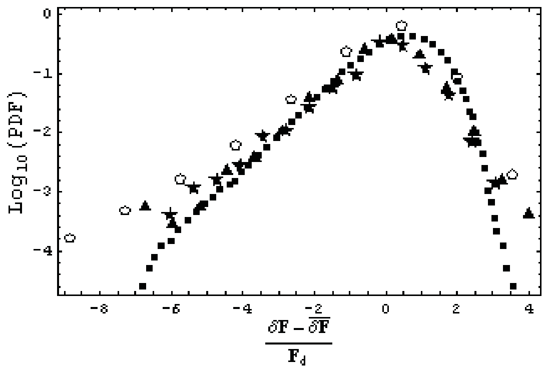

Fig.7. PDF of yield stress fluctuations for steel specimens (tension test for the strain rates ▲, ★ - $\dot{\varepsilon}$ = 0,01 s^{-1}, ◊ - $\dot{\varepsilon}$ = 0,02 s^{-1}).

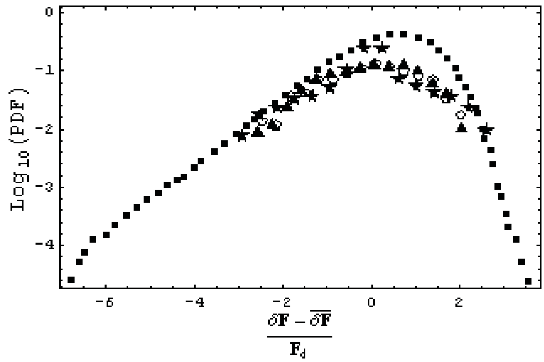

Fig 8. PDF of yield stress fluctuations for Armco-iron (compression test for the strain rates▲,◊ - $\dot{\varepsilon}$ = 0,01 s^{-1}, ★- $\dot{\varepsilon}$ = 0,02 s^{-1}).

Experimental curves reveal pronounced statistical self-similarity as the universality of PDF that allowed us to assume, that phenomenon of plastic instability is the finite-size critical phenomena related to the characteristic "universality class".

Discussion

Jerky flow instability under plastic flow of metals and alloys is the unique phenomenon for the study of long-range correlation properties in the out-of-equilibrium systems with slow dynamics. According to discrete nature of the carriers (dislocations, dislocation substructures) providing the momentum transfer under plastic flow, both fluctuations induced dynamics and scaling invariance can be studied analyzing the temporal statistics of yield stress fluctuations and spatial scaling invariants of the roughness due to the shear banding. It is shown that statistics universality in the term of PDF of yield stress fluctuations can be linked with self-similar nature of collective modes of mesodefects – autosoliary waves, generated in the ensemble of microshears. Spatial-temporal invariance is the consequence of this self-similarity of collective modes related to specific non-linearity type of critical phenomena for mesoscopic system with defects – structural-scaling transition.

The research was supported in part by grants of the Russian Foundation of Basic Research (07-01-96004-р_урал_а, 08-01-00699-а, 07-01-91100-АФГИР_а).

References

[1] S.Bramwell, P.Holdsworth and J.F.Pinton, Nature (London) 396, 552 (1998).

[2] G.Ananthakrishna, S.J.Naronha, C. Fressengeas, and L.P.Kubin, Phys.Rev., E 60, 5455 (1999).

[3] M.Zaiser, Advances in Physics, 55,n.1-2, 185 (2006).

[4] P.Bak, C.Tang and K.Wissenfeld, Phys.Rev.Lett., 59, 381 (1987).S.T. Bramwell, K. Christensen., J.-F. Fortin et al., Phys.Rev.Lett. 84, 3774 (2000).

[5] O.B. Naimark., in Advances in Multifield Theories of Continua with Substructure, ed. by G. Capriz and P. Mariano, Birkhäuser, Boston (2004), p.75-114