The equivalent stress on the critical plane determined by the maximum covariance of normal and shear stresses

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Abstract. The paper presents a new form of the stress criterion of multiaxial random fatigue. The criterion has been defined as a sum of normal and shear stresses with weight coefficients on the critical plane defined as the plane where the maximum covariance between shear and normal stresses occurs. Two forms of the criterion have been proposed.

Introduction

Three groups of multiaxial fatigue criteria can be distinguished depending on the parameter influencing failure. They are stress, strain or energy criteria [1]. It is possible to select some criteria coming from all these groups which include the critical plane position. There are three methods of determination of the critical plane position [2]: the variance method, the method of fatigue damage accumulation, and the method of weight functions. It is obvious that correlations between loadings strongly influence the fatigue life [3, 4]. Correlations between normal and shear stresses on the critical plane seem to be interesting. In [5] it has been shown that the zero correlation between loadings coming (90^0 out-of phase loading) from bending with torsion generates a non-zero correlation between normal and shear stresses on the critical plane.

Criteria of multiaxial fatigue on the critical plane

The stress criteria of multiaxial random fatigue on the critical plane can be written as a sum of normal and shear stresses on that plane with the weight coefficients B and K [1]

$$\sigma_{eq}(t) = B\tau_{\eta s}(t) + K\,\sigma_{\eta}(t). \tag{1}$$

The critical plane can be defined by shear or normal stresses.
When the critical plane is defined by shear stresses, Eq. (1) for equivalent stresses takes the following form

$$\sigma_{eq}(t) = B_1\tau_{\eta s}(t) + (2-B_1)\,\sigma_{\eta}(t), \tag{2}$$

where

$$B_1 = \frac{\sigma_{af}}{\tau_{af}} \tag{3}$$

for parallel S-N characteristics under bending and torsion. This case is typical for elastic-plastic materials.
If the critical plane is defined by normal stresses, Eq. (1) takes the form

$$\sigma_{eq}(t) = B_2\,\tau_{\eta s}(t) + \sigma_{\eta}(t). \tag{4}$$

In this case, the weight coefficient B_2 is obtained from the best correlation between calculated and experimental results for non-proportional loadings. This criterion is usually applied for brittle materials.

Determination of the plane of the maximum covariance between normal and shear stresses

Let us define the critical plane by the maximum covariance. First of all, we should determine covariance in all possible planes α

$$\mu_{ij} = \frac{1}{T_o} \int_0^{T_o} i(t)\, j(t)\, dt \,, \tag{5}$$

where:

$\mu_{i,j}$ - covariance between the signals i(t), j(t),

$i(t) = \sigma_\eta (t)$,

$j(t) = \tau_{\eta s} (t)$,

T_o - time of observation.

The normal and shear stress at the angle α can be determined from

$$\sigma_\eta (t) = \cos 2\alpha \sigma_{xx} (t) + \sin 2\alpha \tau_{xy} (t) \,, \tag{6}$$

$$\tau_{\eta s} (t) = -\frac{1}{2} \sin 2\alpha \sigma_{xx} (t) + \cos 2\alpha \tau_{xy} (t) \,. \tag{7}$$

The stresses $\sigma_{xx}(t)$ and $\tau_{xy}(t)$ are the stresses coming from bending and torsion, respectively. Interpretation of the critical plane position including the considered shear and normal stresses are shown in Fig. 1.

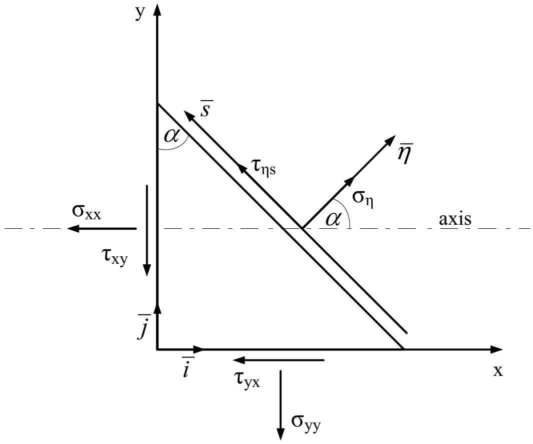

Fig.1. Interpretation of the critical plane position

Covariances at different angles were calculated according to Eq. (5) for various ratios of shear and normal stresses

$$\lambda = \frac{\tau_{axy}}{\sigma_{axx}}. \tag{8}$$

The plane where the covariance reaches its maximum value for the given combination of normal and shear stresses σ_η (t) and $\tau_{\eta s}$ (t) is defined as the critical plane.

Calculations were made for combined cyclic bending with torsion with phase displacements 0^0 and 90^0 and different ratios $\lambda \in (0.01 \div 1000)$ of shear and normal stresses coming from torsion and bending – Eq. (8). Fig. 2 shows positions of the planes in which the maximum covariance between normal and shear stresses was obtained for proportional bending with torsion. Fig. 3 presents the same values obtained for non-proportional bending with torsion (phase displacement - 90^0).

Under pure bending, the maximum covariance corresponds to the critical plane position at the angles 30^0 and $+30^0$. Under torsion, the critical plane is located at the angles -22.5^0, -67^0, 22.5^0 and 67.5^0.

For a small ratio of shear stresses to normal stresses (significant participation of bending) there are two planes of the maximum covariance. As the ratio increases (greater participation of torsion), four planes occur.

How does the phase displacement angle influence position of the defined plane? Such analysis is shown in Fig. 4 for λ = 1. For proportional loadings and phase displacement 30° there are two positions of the critical plane. Under phase displacement 90° we obtain two positions of the critical plane. For other phase displacements, we have three positions of the critical plane. From analysis it appears that under any loadings it is necessary to search all the possible critical planes where there is a local extremum of covariances between components of shear and normal stresses.

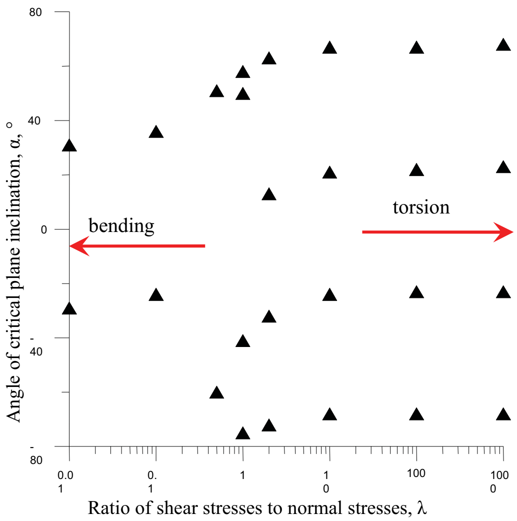

Fig. 2. Angles of the critical plane inclination between stresses coming from bending and torsion for phase displacement 0^0

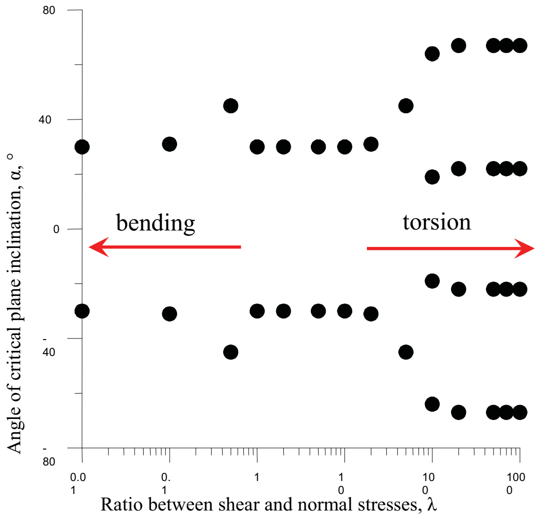

Fig. 3. Angle of the critical plane inclination for phase displacement 90^0 between stresses coming from bending and torsion

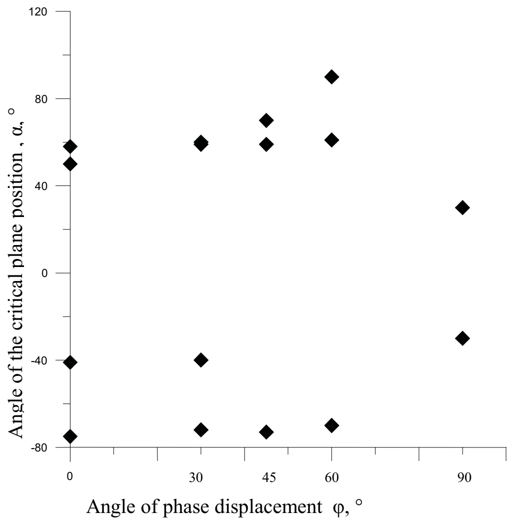

Fig.4. Angle of the critical plane inclination depending on the phase displacement angle for λ=1

Criterion of multiaxial fatigue on the critical plane determined by the maximum covariance

The general criterion of multiaxial fatigue (Eq. (1)) can be applied for formulation of the criterion on a new critical plane. This criterion must be valid for simple stress states. For purposes of this analysis pure bending and pure torsion were assumed as basic stress states. Two critical planes were obtained for bending and four planes were obtained for torsion. Two solutions were obtained for symmetry in relation to the plane inclined at 0°. The basic relations between stresses versus the critical plane position give us two forms of the criterion of multiaxial fatigue:

$$\sigma_{eq}(t)=\frac{4\sqrt{3}+3\sqrt{2}B_1}{3\left(\sqrt{3}+1\right)}\sigma_n(t)+\frac{\sqrt{3}\left(3\sqrt{2}B_1-4\right)}{3\left(\sqrt{3}+1\right)}\tau_{ns}(t), \tag{9}$$

$$\sigma_{eq}(t)=\frac{4\sqrt{3}-3\sqrt{2}B_1}{3\left(\sqrt{3}-1\right)}\sigma_n(t)+\frac{\sqrt{3}\left(4-3\sqrt{2}B_1\right)}{3\left(\sqrt{3}-1\right)}\tau_{ns}(t), \tag{10}$$

where the coefficient B_1 in given in Eq. (3) like in Eq. (2).

Thus, it is possible to propose a new method of determination of the critical plane position, different than three known methods (the method of variance, the method of damage accumulation, the method of weight functions). Moreover, two new forms of the criteria of multiaxial random fatigue on the critical plane were obtained. Similar analysis and new forms of fatigue criteria could be proposed for strain and energy criteria based on analysis of both stresses and strains in the given plane. Calculations of the equivalent histories should be performed according to Eqs. (9) and (10) for all the possible positions of the critical planes. The obtained histories of the equivalent stress can be applied for determination of calculation fatigue lives according to standard methods presented in many papers (e.g. [6 - 8]).

It should be checked if the proposed expressions for the equivalent stress (Eqs. (9) and (10)) are valid for different materials. Thus, experimental verification of the presented criterion is necessary.

Conclusions

1. The presented calculations and analysis allow to propose a new stress criterion of multiaxial random fatigue on the critical plane including normal and shear stresses.
2. The critical plane can be defined as the plane of the maximum covariance between normal and shear stresses.
3. Under complex stress states we can observe some positions of the critical plane, depending on the considered complex stress state.
4. On the basis of analysis of simple stress states, two forms of the expression for the equivalent stress were proposed.

References

[1] T. Łagoda, P. Ogonowski, Criteria of multiaxial random fatigue based on stress, strain and energy parameters of damage in the critical plane, Mat.-wiss. u. Werkstofftech, 2005, Vol.36, No 9, pp.429-437.

[2] E. Macha, Simulation investigations of the position of fatigue fracture plane in materials with biaxial loads, Mat. -wiss. U. Werkstofftech. nr 20, 1989, Teil I, Heft 4/89, pp.132-136, Teil II, Heft 5/89, pp.153-163.

[3] T. Łagoda, E. Macha, A. Dragon, J. Petit, Influence of correlations between stresses on calculated fatigue life on machine elements, Int. J. Fatigue, Vol.18, No 8, 1996, pp.547-555.

[4] J. Sempruch, Kryterium wytrzymałości zmęczeniowej stali konstrukcyjnej 25CrMo4 w warunkach złożonego stanu naprężenia, Wydawnictwo Uczelniane Akademii Techniczno-Rolniczej w Bydgoszczy, Rozprawy 51, 1992, s.76 (In Polish).

[5] K. Walat, T. Łagoda, Korelacja między naprężeniami stycznymi i normalnymi w płaszczyźnie krytycznej, XXI Konferencja Naukowa Problemy Rozwoju Maszyn Roboczych, Zakopane 2008. Streszczenia Referatów, Politechnika Opolska, Opole 20088, ss.167-168, Teksty referatów, CD, ss.629-635 (In Polish).