# Proposal of Estimation Method for Notched Specimen Fatigue Limit of Commercially Pure Titanium 

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#### Abstract

In order to identify the fatigue limit for a sharp notched specimen of commercially pure titanium, rotating bending fatigue tests on smooth and notched specimens were curried out. In previous studies, it was found that the fatigue limit for a sharp notched specimen of commercially pure titanium could not be determined and it is not sure whether it fail over $N=10^{7}$ cycles or not. Based on the experimental results, in the smooth specimen and notched specimen ( $\rho=1.0 \mathrm{~mm}$ ), they have fatigue limits and knee points, while the sharp notched specimen ( $\rho=0.1 \mathrm{~mm}$ ) does not. As an estimation method, the fatigue crack initiation of each specimen was then investigated. The investigation results are as follows. (1) The proportional relationship exists between the maximum stress $K_{t} \sigma_{a}$ at the notch root and logarithm of the number of cycles to fatigue crack initiation $N_{i}$. (2) Based on the slope of the relationship between the maximum stress $K_{t} \sigma_{a}$ and the number of cycles to fatigue crack initiation $N_{i}$ at the notch root, there is no possibility that the fatigue crack in the same mechanism initiates over $N=10^{7}$ cycles. Therefore, the stress amplitude that is not initiated the fatigue crack at $N=10^{7}$ cycles is the fatigue limit.


## Introduction

Commercially pure titanium is a highly specific material, and has a good corrosion resistance. Therefore, its demand is increasing as a structural component such as in chemical plants. In order to use a material as a structural component, it is necessary to investigate the fatigue strength characteristics in stress concentration areas.

Until now, the fatigue strength characteristics of commercially pure titanium notched specimens in the form of a plate were investigated by Takao et al. [1] [2]. Takao et al. reported that notch root radii over $\rho=0.02 \mathrm{~mm}$ of commercially pure titanium notched specimens do not initiate a non-propagating crack. Figure 1 shows the $S-N$ diagram for commercially pure titanium. It is understood that the fatigue limits of smooth specimens and dull notched specimen ( $\rho=1.0 \mathrm{~mm}$ ) can be determined because their knee points exist around $N=1.0 \times 10^{6}$ cycles. However, as shown in Fig.1, the fatigue limit of sharp notched specimens ( $\rho=0.1 \mathrm{~mm}$ ) can not be determined because its knee point shifts to the high cycle side. Therefore, we can not define the knee point. Moreover, a non-propagating crack is not initiated. Therefore, from only the $S$ - $N$ diagram, we can not determine whether the stress amplitude of no failure at $N=10^{7}$ cycles is the fatigue limit or the fatigue strength at $10^{7}$ cycles.

In this study, the aim is to identify whether the stress amplitude of no failure at $N=10^{7}$ cycles is the fatigue limit or fatigue strength at $10^{7}$ cycles for a sharp notch specimen of commercially pure titanium. As the estimation method, rotating bending fatigue tests on specimens of commercially pure titanium were carried out in order to investigate the number of cycles to fatigue crack initiation $N_{i}$ and its tendency. The reasons are summarized as follows.
(1) The sharper the notch root radius becomes, the more the number of cycles to failure $N_{f}$ increases. However, we predicted that the fatigue crack in the stress concentration area initiates at very low cycles, and the number of cycles to fatigue crack initiation $N_{i}$ has a certain tendency versus the stress amplitude.
(2) We predict that when the stress amplitude becomes low, fatigue crack does not initiate upon extension of the tendency of fatigue crack initiation $N_{i}$. So, there is no possibility that the fatigue crack in the same mechanism does not initiate over $N=10^{7}$ cycles. Therefore, the stress amplitude that is not initiated the fatigue crack at $N=10^{7}$ cycles is the fatigue limit.
The number of cycles to fatigue crack initiation $N_{i}$ of commercially pure titanium smooth specimens with an oxygen solution layer [3] is investigated by Tokaji et al. [4] [Fig.2]. The results showed that the number of cycles to failure $N_{f}$ has a large scatter for commercially pure titanium with the oxygen solution layer on the specimen surface, and the reason is that the number of cycles to fatigue crack initiation $N_{i}$ has a large scatter. The number of cycles to fatigue crack initiation $N_{i}$ is $7 \sim 60 \%$ versus the number of cycles to failure $N_{f}$ when the stress amplitude is slightly higher than the fatigue limit. We predict that the reason is due to the thickness of the oxygen solution layer on specimen surface widely varies. Based on this report, the number of cycles to fatigue crack initiation $N_{i}$ for commercially pure titanium with the oxygen solution layer on the specimen surface does not have a certain tendency versus the stress amplitude. However, if the matrix of the specimen surface is homogenized by removing the oxygen solution layer, we predict that there is the possibility that the number of cycles to fatigue crack initiation $N_{i}$ has a certain tendency versus the stress amplitude. We then investigated the number of cycles to fatigue crack initiation $N_{i}$ for sharp notched specimens of commercially pure titanium after removing the oxygen solution layer. We discuss whether the stress amplitude of no failure at $N=10^{7}$ cycles is the fatigue limit or fatigue strength at $10^{7}$ cycles for the sharp notched specimens of commercially pure titanium.

## Material and Test Procedure

The material used in this study is commercially pure titanium. The material is supplied in the form of round bars. Table 1 shows the chemical composition. Figure 3 shows the microstructure of the specimen surface. The average grain size is about $64 \mu \mathrm{~m}$. Figure 4 shows the shapes and dimensions of the specimen. The stress concentration factor $K_{t}$ at the notch root loaded bending moment of the smooth specimen is 1.03 . This is a very small value, therefore, we regarded the specimen as a smooth specimen. The diameter of the minimum cross section is 5 mm , and notch depth is 0.5 mm . The dimension of the notch is a 60 degree V type, with a circular groove. The notch root radii $\rho$ is $\rho=1.0 \mathrm{~mm}$ and $\rho=0.1 \mathrm{~mm}$. The specimens were annealed at 823 K for 60 min in order to relieve the residual stress. The oxygen solution layer was present on the specimen surface because the degree of vacuum for the annealing furnace were about $10^{-3}$ torr. Therefore, chemical polish was carried out in order to remove the oxygen solution layer, and a $30 \mu \mathrm{~m}$ thickness was removed from the specimen surface. The specimens were then polished using alumina powder (powder size: $0.05 \mu \mathrm{~m}$ ), and buffing was additionally curried out by OP-S. Finally, etching was done using a by corrosive liquid (HF $3 \mathrm{wt} \%, \mathrm{HNO}_{3} 10 \mathrm{wt} \%$ and distillated water) in order to easily observe the fatigue crack initiation. The fatigue tests were carried out using a rotating bending fatigue testing machine, whose stress ratio is -1 . And test environment is in atmospheric air at room temperature. The testing frequency was 50 Hz . The nominal stress amplitude of the fatigue test, $\sigma_{\mathrm{a}}$, is defined at the minimum cross section. The fatigue cracks for the smooth specimens and notched specimens $\rho=1.0 \mathrm{~mm}$ were observed by the replica method. The notched specimens $\rho=0.1 \mathrm{~mm}$ were detached from the fatigue testing machine at the predetermined cycles. The fatigue crack was then directly observed by an optical microscope because it is difficult to observe the fatigue crack at the notch root by the replica method.

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The number of cycles to fatigue crack initiation $N_{i}$ is defined by the cycle when the fatigue crack length reached $100 \mu \mathrm{~m}$. The fatigue crack behavior of commercially pure titanium developed due to microcrack connection [5]. In this study, the microcrack connection in the commercially pure titanium occurred when the fatigue crack length is shorter than one grain size (average: $64 \mu \mathrm{~m}$ ). We consider that the length of one grain size (when the micro cracks connection finished) is the minimum unit for the fatigue crack of commercially pure titanium. Therefore, we defined that the fatigue crack initiation length is $100 \mu \mathrm{~m}$, because the length is sufficiently longer than minimum fatigue crack initiation unit of commercially pure titanium.


Fig. 1 Results of fatigue tests for smooth and notched specimens of commercially pure titanium [2].


Fig. 2 Results of fatigue tests for smooth specimens of commercially pure titanium [4].

Table 1 Chemical composition ( $\mathrm{wt} \%$ ).

| Fe | O | H | Ni | C | Cr | Si | N | Ti |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.058 | 0.116 | 0.0021 | 0.009 | 0.008 | 0.008 | 0.007 | 0.005 | Bal. |

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Fig. 3 Microstructure of commercially pure titanium.


Fig. 4 Shapes and dimensions of specimen and detail.

## Experimental result

## S-N diagram

Figure 4 shows the $S-N$ diagram for the smooth and notched specimens of commercially pure titanium. The fatigue limits of the smooth specimen and notched specimen ( $\rho=1.0 \mathrm{~mm}$ ) can be determined, because clear knee points exist in the $S-N$ diagram. On the other hand, for the notched specimens ( $\rho=0.1 \mathrm{~mm}$ ), the lower the stress amplitude becomes, the more the number of cycles to failure $N_{f}$ increases. At the stress amplitude $\sigma_{a}=80 \mathrm{MPa}$, the specimen is fractured at $N=4.0 \times 10^{6}$ cycles. Therefore, the knee point does not exist, i.e, the fatigue limit can not be determined. In addition, non-propagating cracks did not exist on the smooth and notched ( $\rho=1.0 \mathrm{~mm}, \rho=0.1 \mathrm{~mm}$ ) specimens of no failure at $N=10^{7}$ cycles. The presence of existence for non-propagating cracks will be discussed in the next section.

## Presence of non-propagating crack

Takao [1] reported that a non-propagating crack does not exist in commercially pure titanium of no failure at $N=10^{7}$ cycles. However, there is no study that investigated quantitatively the presence of non-propagating crack. So, in this study, confirmation experiment for the existence for non-propagating crack was then curried out as follows. For the smooth specimen of no failure at $N=10^{7}$ cycles, a 10 MPa additional load were again applied. If a non-propagating crack exists, the non-propagating crack becomes the weakest point in the specimen, and the fatigue crack grows from the non-propagating crack. On the other hand, if the non-propagating crack does not exist, the weakest point is the slip system. However, this discussion is valid under the assumption that commercially pure titanium does not have strain age hardening. Figure 5 shows the experimental results. For the smooth specimen of no failure at $N=10^{7}$ cycles, $\sigma_{a}=220 \mathrm{MPa}$ was loaded. However, fatigue crack in the specimen was not initiated at $N=10^{6}$ cycles. So $\sigma_{a}=230 \mathrm{MPa}$ was loaded, and the specimen was fractured at $N_{f}=1.5 \times 10^{5}$ cycles. Figure 6 shows the surface observation result for the specimen using the replica method. Figure 6 shows that a fatigue crack initiated from a crystal in which no non-propagating crack existed. Based on this result, we can determine that no non-propagating crack exists in the specimen of no failure at $N=10^{7}$ cycles.


Fig. 5 Results of fatigue tests of smooth and notched specimens of commercially pure titanium.


Fig. 6 Confirmation experiment for existence of non-propagating crack of commercially pure titanium.

(a) $\sigma_{a}=210 \mathrm{MPa}, N=1.0 \times 10^{7}$ [cycles]

Loading direction


(b) $\sigma_{a}=220 \mathrm{MPa}, N=1.0 \times 10^{6}$
$\rightarrow \sigma_{a}=230 \mathrm{MPa}, N=3 \times 10^{4} \quad$ [cycles]
$\longrightarrow:$ Tip of crack

Fig. 7 Fatigue crack initiation from the weakest crystal of commercially pure titanium.

## The number of cycles to fatigue crack initiation $N_{i}$

The number of cycles to fatigue crack initiation $N_{i}$ for smooth and notched specimens of commercially pure titanium was investigated. Figure 8 shows the experimental results. For the notched specimen $\rho=0.1 \mathrm{~mm}$, at $K_{t} \sigma_{a}=355 \mathrm{MPa}$ (stress amplitude $\sigma_{a}=90 \mathrm{MPa}$ ), the number of cycles to fatigue crack initiation $N_{i}$ was not investigated, because we investigated that the effect on fatigue strength by removing the specimen from fatigue testing machine. There is no effect on the fatigue strength by removing the specimen from the fatigue testing machine, because it is not different from the other results as shown in Fig.8. In this result, the number of cycles to fatigue crack initiation $N_{i}$ does not have any scatter, and by comparison with Tokaji's report [3], the reason why the number of cycles to fatigue crack initiation $N_{i}$ has scatter is due to the oxygen solution layer on the specimen surface. Therefore, for the specimen after removing the oxygen solution layer, the proportional relationship exists between the maximum stress $K_{t} \sigma_{a}$ at the notch root and logarithm of the number of cycles to fatigue crack initiation $N_{i}$. Also, the slope of the relationship between the number of cycles to fatigue crack initiation $N_{i}$ and the maximum stress $K_{t} \sigma_{a}$ at the notch root is compared. Then it is turned out that the sharper the notch root radius become, the steeper the slope of relationship between the maximum stress $K_{t} \sigma_{a}$ at the notch root and the number of cycles to fatigue crack initiation $N_{i}$ becomes.

## Discussion

## Fatigue limit for the $\rho=0.1 \mathrm{~mm}$ notched specimen

Based on the above results, we can discuss the fatigue limit for the $\rho=0.1 \mathrm{~mm}$ notched specimen. Figure 9 shows the relationship between the maximum stress $K_{t} \sigma_{a}$, the number of cycles to fatigue crack initiation $N_{i}$, and the number of cycles to failure $N_{f}$ for the $\rho=0.1 \mathrm{~mm}$ notched specimen. The proportional relationship exists between the maximum stress $K_{t} \sigma_{a}$ at the notch root and logarithm of the number of cycles to fatigue crack initiation $N_{i}$. Figure 9 shows that, at the maximum stress of $K_{t} \sigma_{a}=276 \mathrm{MPa}\left(\sigma_{a}=70 \mathrm{MPa}\right.$ ), a fatigue crack does not initiate upon extension of the proportional relationship of the fatigue crack initiation. So, there is no possibility that a fatigue crack in the same mechanism initiates over $N=10^{7}$ cycles. Therefore, the stress amplitude that is not initiated fatigue crack at $N=10^{7}$ cycles is the fatigue limit.


Fig. 8 Relationship between $K_{t} \sigma_{a}$ and $N_{i}, N_{f}$ for smooth specimen and notched specimens of commercially pure titanium.

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## Fatigue limit for notched specimens sharper than $\rho=0.1 \mathrm{~mm}$

We now discuss the fatigue limit for other notched specimens that are sharper than $\rho=0.1 \mathrm{~mm}$. Like the $\rho=0.1 \mathrm{~mm}$ notched specimen, from only $S-N$ diagram, it is predicted that the fatigue limit for the notched specimens that is sharper than $\rho=0.1 \mathrm{~mm}$ can not be determined. Figure 10 shows that the number of cycles to fatigue crack initiation $N_{i}$ of $0.45 \% \mathrm{C}$ steel notched specimens [6] and the number of cycles to fatigue crack initiation $N_{i}$ of commercially pure titanium notched specimens. For the $0.45 \% \mathrm{C}$ steel, the sharper the notched root radius becomes, the steeper the slope of the relationship between the maximum stress $K_{t} \sigma_{a}$ and the number of cycles to fatigue crack initiation $N_{i}$ becomes. Based on these results, it is predicted that the slope of the relationship between the maximum stress $K_{t} \sigma_{a}$ and the number of cycles to fatigue crack initiation $N_{i}$ for notched specimens that are sharper than $\rho=0.1 \mathrm{~mm}$ is steeper than that of $\rho=0.1 \mathrm{~mm}$.


Fig. 9 Relationship between $K_{t} \sigma_{a}$ and $N_{i}, N_{f}$ for $\rho=0.1 \mathrm{~mm}$ notched specimen of commercially pure titanium.


Fig. 10 Relationship between $K_{t} \sigma_{a}$ and $N_{i}$ for $0.45 \%$ C steel [6] of commercially pure titanium.

Therefore, from the discussion given in the previous section, the fatigue crack in the same mechanism does not initiate over $N=10^{7}$ cycles, and the fatigue limit for notched specimens that are sharper specimens than $\rho=0.1 \mathrm{~mm}$ can be determined.

## Conclusions

When only using the $S$ - $N$ diagram, the fatigue limit for sharp notched specimens of commercially pure titanium can not be determined. So the number of cycles to fatigue crack initiation $N_{i}$ for sharp notched specimens of commercially pure titanium after removing the oxygen solution layer was investigated. We discussed whether the stress amplitude of no failure at $N=10^{7}$ cycles is the fatigue limit or the fatigue strength at $10^{7}$ cycles. The results of this study are summarized as follows.
(1) By removing the oxygen solution layer, the number of cycles to fatigue crack initiation $N_{i}$ does not have scatter.
(2) For the specimen after removing the oxygen solution layer, a proportional relationship exists between the maximum stress $K_{t} \sigma_{a}$ at the notch root and logarithm of the number of cycles to fatigue crack initiation $N_{i}$.
(3) From the slope of the relationship between the maximum stress $K_{t} \sigma_{a}$ and the number of cycles to fatigue crack initiation $N_{i}$ at the notch root, there is no possibility that a fatigue crack in the same mechanism initiates over $N=10^{7}$ cycles. Therefore, the stress amplitude that is not initiated fatigue crack at $N=10^{7}$ cycles is the fatigue limit.

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