Plastic Limit Loads for Idealized Branch Junctions under Combined Pressure and Bending

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Abstract. Closed-form yield loci are proposed for branch junctions under combined pressure and in-plane bending, via small strain, three-dimensional FE limit load analyses using elastic-perfectly plastic materials. Comparison with extensive FE results shows that predicted limit loads using the proposed solutions are overall conservative and close to FE results. The proposed solutions are believed to be valid for the branch-to-run pipe radius and thickness from 0.0 to 1.0, and the mean radius-to-thickness ratio of the run pipe from 5.0 to 20.0.

Introduction

For design and assessment of piping branch junctions, information on plastic limit loads of piping branch junctions under combined pressure and bending is needed [1-4]. Moreover, for practical engineering application, closed-form approximations of plastic limit loads would be very useful, which is quite a difficult task due to a large number of geometric variables involved.

Plastic limit loads were experimentally determined for piping branch junctions under combined pressure and in-plane bending [5], and under combined pressure and out-of-plane bending [6,7]. Although experimental results are extremely valuable, they have shortcomings. Firstly systematic analyses require a large number of tests, which in turn is very expensive and time-consuming. Moreover, as tests are performed for real materials, it is not clear to define plastic limit loads. Finally, as real branch junctions include some types of reinforcements at branch intersections, experimentally-measured plastic limit loads are in fact dependent on such reinforcements. For clearer interpretation, semi-analytical and/or finite element analyses would be useful. For instance, Nadarajah et al. [8] presented plastic limit loads under combined pressure and in-plane bending only for selected geometric cases. Although Ayob et al. [9] provided yield loci for branch junctions under combined loading via extensive FE elastic analyses, their results were for first-yield loads but not for plastic limit loads. Xuan and Li [10] provided closed-form plastic limit load solutions, derived from force equilibrium between the limit load and the internal force acting on the intersecting line between the main and branch pipes. However, the valid range of their solutions is quite limited in terms of geometric variables, as will be discussed later. As summarized above, existing works are still limited to propose more general plastic limit load solutions for branch junctions under combined loadings in closed forms.

To derive plastic limit load solutions for combined loadings in closed forms, one needs relevant solutions for single loading first. Compared to those for combined loadings, plastic limit load solutions for single loading are relatively well established. For instance, plastic limit pressure solutions for branch junctions were developed both analytically [11,12,15] and numerically [13,14,17,18]. Plastic limit moment solutions for branch junctions under in-plane bending were also developed both analytically [16] and numerically [17,18]. These solutions can be used as baseline solutions to construct plastic limit loads for combined pressure and in-plane bending.

This work presents plastic limit load solutions for piping branch junctions under combined pressure and in-plane bending, based on 3-D FE analyses using elastic-perfectly plastic materials.

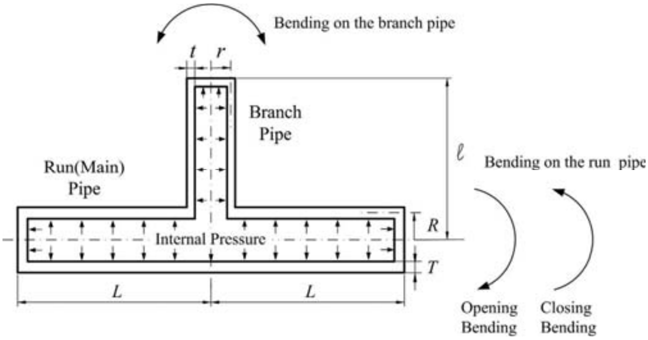

Fig. 1. Schematics of branch junctions with relevant geometric variables.

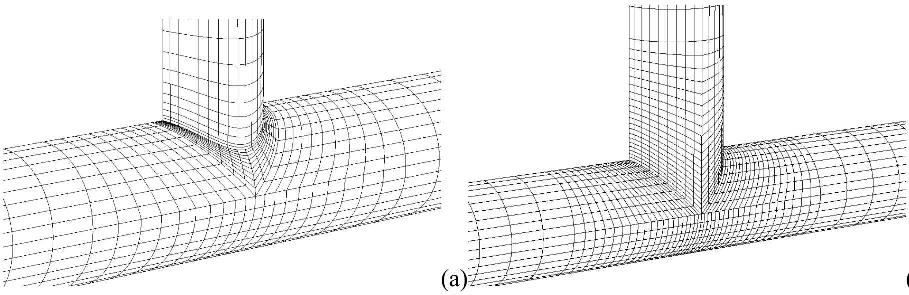

Fig. 2. Finite element meshes.

Finite Element Limit Analyses

Consider a branch junction, depicted in Fig. 1. For the main (run) pipe, the mean radius and thickness are denoted by R and T, respectively, and for the branch pipe, r and t, respectively. It is assumed that the branch junction has no weld or reinforcement around the intersection. Three-dimensional (3-D), FE limit analyses of the branch junction (Fig. 1), were performed using ABAQUS [19]. Materials were assumed to be elastic-perfectly plastic, and non-hardening J_2 flow theory was used using a small geometry change continuum FE model. To reduce the computing time, a half-model was used by fully considering symmetry conditions, and reduced integration elements (C3D20R within ABAQUS) were used. Typical FE meshes are shown in Fig. 2. For all cases, three elements are used through the thickness, and the resulting number of elements and nodes in typical FE meshes ranges from 3,949 elements/20,598 nodes to 4,914 elements/25,649 nodes. It was found that the use of six elements through the thickness gave essentially the same limit loads [18], and thus the present FE mesh is believed to be sufficiently fine for the present study.

Regarding loading conditions, combined loading was applied in a non-proportional manner. In the first step, internal pressure was applied as a distributed load to the inner surface of the FE model, together with axial tensions equivalent to the internal pressure applied at the end of the branch and run pipes to simulate closed ends. In the second step, in-plane bending was applied to the nodes at the end of the branch pipe constrained through the MPC (multi-point constraint) option within ABAQUS. Sufficiently large deformation (rotation) was applied, and corresponding limit moment was determined directly from the nodal force. For selected cases, the analysis using proportional loading was also performed, where internal pressure and bending were applied in a proportional manner. As expected, both results were almost identical, as the present analysis was performed using the small geometry change option. Two types of combined loading conditions were considered in the present work. The first one is combined pressure and in-plane bending on the branch pipe. The other is combined pressure and bending on the run pipe. The FE limit load can be easily determined from the nodal forces.

Limit Load Solutions for Single Loading

Internal Pressure [18]

$$\frac{P_L R}{\sigma_o T} = \min \begin{pmatrix} \dfrac{Q}{\left(0.25 - 0.5h_1 + h_1^2 + 0.79h_2^2\right)^{0.5}} \\ \dfrac{2}{\sqrt{3}}\dfrac{C}{A} \\ \dfrac{2}{\sqrt{3}} \end{pmatrix} \tag{1}$$

with

$$Q = -(0.4603A + 0.1525)(C-1)^2 + 1.015B^{-0.297}(A-0.52)^4 + 1.127B^{-0.0505}$$

$$f_1 \approx 1 + \frac{1}{3}A^4 \quad ; \quad f_2 \approx \frac{\pi}{2}\left(1 - \frac{3}{16}A^2\right) \quad ; \quad k = \frac{1}{1+C^3}$$

$$h_1 = 1 + \left(0.145kA\sqrt{B}f_2 + 0.3185A^2 f_1\right)\left(1 - \frac{C}{AB}\right)^2 \quad ; \quad h_2 = 0.175kA\sqrt{B}\left(1 - \frac{C}{AB}\right)^2 f_2 \tag{2}$$

$$A = \frac{r}{R} \quad ; \quad B = \frac{2R}{T} \quad ; \quad C = \frac{t}{T}$$

In-Plane Bending on a Branch Pipe [18]

$$\frac{M_L}{4\sigma_o r^2 t} = \min \begin{pmatrix} 1 \\ \dfrac{\pi/2 \cdot Q}{C\left[\left(f_1 A + 0.455 f_2 k\sqrt{B}\right)^2 + 0.2385 B f_2^2 k^2\right]^{0.5}} \end{pmatrix} \tag{A.6}$$

$$Q = (-1.102 - 0.653A)(C-0.7)^2 - 2.583A^3 + 5.462A^2 - 3.544A + 2.009 - 0.0025B$$

Other factors (f_1, f_2, A, B, C and k) are given in Eq. (2).

Proposed Limit Loads (Yield Loci) for Combined Pressure and In-Plane Bending to the Branch Pipe

As a wide range of r/R, R/T and t/T are considered in the present work, plastic collapse could occur not only in the intersection of branch junctions but also in the branch pipe. For instance, when the values of t/T and r/R are small, plastic collapse can occur in the branch pipe, not in the intersection. In such a case, a yield locus is given by

$$\left(\frac{M}{4\sigma_o r^2 t}\right)^2 + \left(\frac{\sqrt{3}r}{2\sigma_o t}P\right)^2 = 1 \tag{4}$$

Note that this yield locus corresponds to a circular interaction rule of a (branch) pipe under combined pressure and bending. Present FE results suggest that plastic collapse in fact occurs in the branch pipe for 5≤R/T≤10 with 0<t/T≤0.2 or 0<r/R≤0.2. For other cases, plastic collapse occurs in the intersection of branch junctions. In this case, yield loci are approximated by

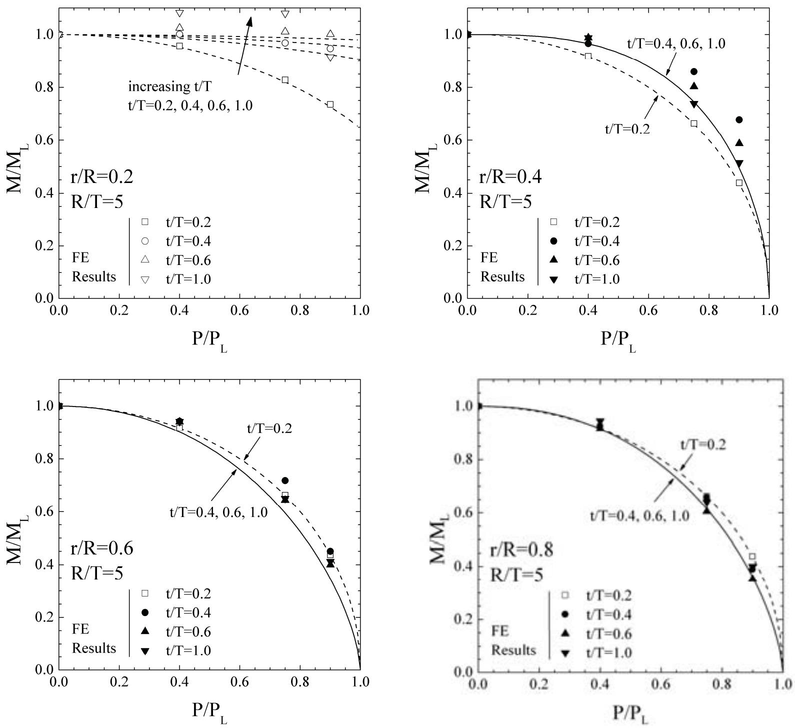

Fig. 3. Comparison of FE results with proposed yield loci for branch junctions under combined pressure and in-plane bending on the branch pipe (R/T=5).

$$\left(\frac{P}{P_L}\right)^\alpha + \left(\frac{M}{M_L}\right)^\beta = 1$$

$$\alpha = \max\left[-4.60\left(\frac{r}{R}\right) - 0.230\left(\frac{R}{T}\right) + 6.01,\ \ 0.813\left(\frac{r}{R}\right) - 0.0255\left(\frac{R}{T}\right) + 1.73\right] \tag{5}$$

$$\beta = 1.57 - 0.0177\left(\frac{R}{T}\right) + 4.030\left(\frac{r}{R} - 0.7\right)^2$$

In Eq. (5), P_L and M_L are limit pressure and limit moment for branch junctions, given by Eq. (1) and Eq. (3), respectively. Note that variables, α and β, are determined simply by fitting FE results, and thus should be valid for the ranges of geometries considered in the present work, 0.0≤(r/R, t/T)≤1.0 and 5.0≤R/T≤20.0. Values of α and β are always greater than unity. For R/T=20, values of α increase linearly from α≈1.2 for r/R=0.0 to α≈2.0 for r/R=1.0. For R/T=5 and 10, two linear curves characterise α. For r/R<~0.5, values of α decrease linearly with r/R, but for r/R>~0.5, values of α are close to those for R/T=20. Values of β are similar for different values of R/T. They decrease with r/R for r/R<~0.7 but increase for r/R>~0.7. For r/R=1.0, values of α and β are close to two, indicating that almost a circular interaction rule is applied.

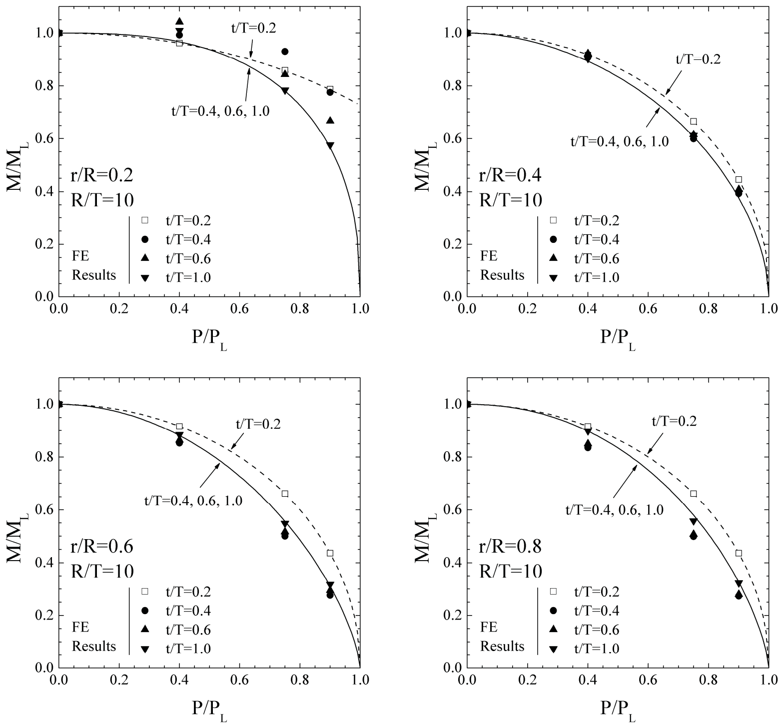

Fig. 4. Comparison of FE results with proposed yield loci for branch junctions under combined pressure and in-plane bending on the branch pipe (R/T=10).

For branch junctions under combined pressure and in-plane bending on the branch pipe, two solutions are proposed, Eqs. (2) and (3). According to the plastic theory [20], actual limit load for branch junctions can be found by taking the smaller one from Eq. (2) and Eq. (3). Resulting yield loci for R/T=5 are compared with present FE results in Fig. 3. Corresponding results for R/T=10 and R/T=20 are shown in Fig. 4 and in Fig. 5, respectively. In Fig. 6, the results for r/R=1.0 are compared with FE results for three values of R/T, R/T=5, 10 and 20. In all figures, the predictions using Eq. (4) are shown in dotted lines, and those using Eq. (5) are in solid lines. Note that pressure and moment are normalized with respect to Eq. (1) and Eq. (3), respectively.

In Figs. 3-5, it can be seen that plastic collapse in the branch pipe occurs when values of r/R, t/T and R/T are small. For instance, for R/T=5, plastic collapse occurs in the branch pipe for r/R=0.2, regardless of t/T. For higher values of r/R, it occurs only for t/T=0.2. For R/T=20, on the other hand, plastic collapse does not occur in the branch pipe. When plastic collapse occurs in the branch pipe, the circular interaction rule, Eq. (4), agree well with FE results. For the case when plastic collapse occurs in the intersection of branch junctions, the proposed yield loci, Eq. (5), are overall conservative, compared to FE results. The degree of conservatism increases with decreasing r/R. For a given r/R, it increases with decreasing t/T. Although for the case of R/T=10 and r/R=0.8, however, Eq. (5) gives slightly higher limit loads than FE results and thus are slightly non-conservative.

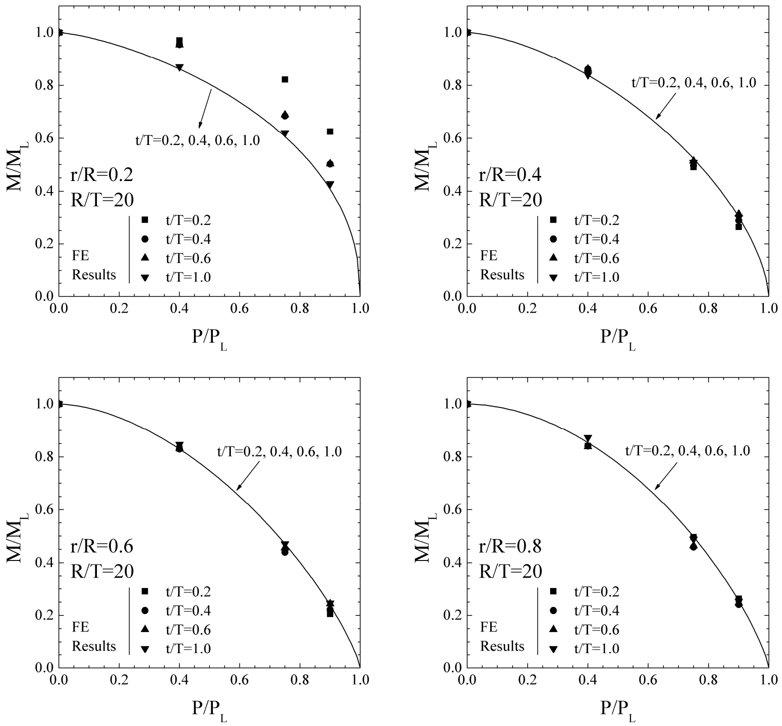

Fig. 5. Comparison of FE results with proposed yield loci for branch junctions under combined pressure and in-plane bending on the branch pipe (R/T=20).

Concluding Remarks

This paper provides closed-form yield loci for branch junctions under combined pressure and in-plane bending, via small strain, three-dimensional FE limit load analyses using elastic-perfectly plastic materials. For in-plane bending loading, both bending on the branch pipe and on the run pipe are considered. Furthermore, for the case of bending on the run pipe, the effect of the bending direction is also considered. Comparison with extensive FE results shows that predicted limit loads using the proposed solutions are overall conservative and close to FE results. As the proposed solutions are based on FE results, they are valid for the range of geometries considered in this paper, that is, $0.2 \leq (r/R,\ t/T) \leq 1.0$ and $5.0 \leq R/T \leq 20.0$. Furthermore, as the proposed solutions are constructed using limit load solutions for single loading that are valid for $0.0 \leq (r/R,\ t/T) \leq 1.0$, they are believed to be valid even for $0.0 \leq (r/R,\ t/T) \leq 1.0$. As such ranges cover practically interesting problems of branch junctions, the proposed solutions would be useful to design and assessment of branch junctions for excessive plastic deformation and creep rupture.

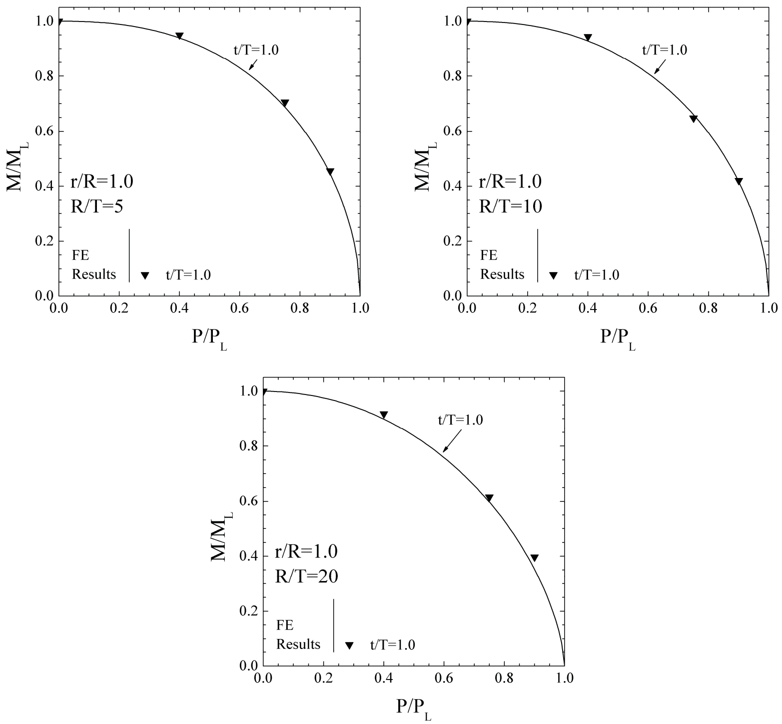

Fig. 6. Comparison of FE results with proposed yield loci for branch junctions under combined pressure and in-plane bending on the branch pipe (r/R=1.0).

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