Modeling of Brittle Fracture of Porous Materials (Media) under Multi-axial compression

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Abstract. Multi-axial compression of porous materials is accompanied by formation of specific zones of ordered fracture. In particular, intensive compression with a high portion of all-round pressure of porous materials causes a new form of brittle fracture consisting in formation of a defect elongated transverse to the axis of maximal compression. This defect was named as a crack of compression. The defect is growing if in its end zones local fracture of the material is realized and is accompanied by an effective decreasing of the fracture products volume (their compacting) or by motion and removing of these products. A 2D model of the observed fracture (formation of cracks of compression) was suggested. The model enables to explain some observed effects. Characteristic regimes of the cracks of compression growth were considered within the framework of the model. The obtained results were compared with the experimental observations.

Introduction

Fracture processes at extremal loads, in particular, inherent to tectonic processes and processes of hydrocarbon recovery at the deep deposits, are often accompanied by fracture localization and ordering. These features of fracture are characteristics, for instance, for geophysical media under the action of compression and shear [1-7]. In this connection one can mention the papers devoted to structures of fracture at shear [4] and to formation of hierarchical systems of faults [3,6]. Hence, it is interesting to develop and analyze the models of specific variants of local fracture mechanisms which enable to realize such processes in the structured media. We will consider mode I fracture. Note, that along with the classical mode I fracture – cracks of normal tension [8] – some other fracture types are related to this mode. As an example one can point out a variant of faults extension under compression (cracks of compression) in thin coatings and plates in case when the surfaces of the forming defects – cracks can overlap each other [9]. The model [6,7,10,11], which describes formation of main faults in heterogeneous media and rocks along the main compressive stresses in the stress field close to uniaxial compression, is based on the effects of fracture under the action of local tension caused by structural stress concentrators (pores, inclusions, etc.).

At some testing conditions of high pores materials the processes of local compaction of the material become to be the basic ones and fracture is continued to attaining the form given in Fig. 1 [12-13]. In the tests [13] the samples of high porous sandstone (porosity n>0.2) of sizes 150X150X230mm with a cylindrical hole of diameter 20mm were loaded by nonequicomponent 3-axial compression. The fracture process is accompanied by formation of an elongated fracture zone (cavity) of almost constant thickness located in diametrical plane of the initial hole. The zone thickness is not correlated with the hole size.

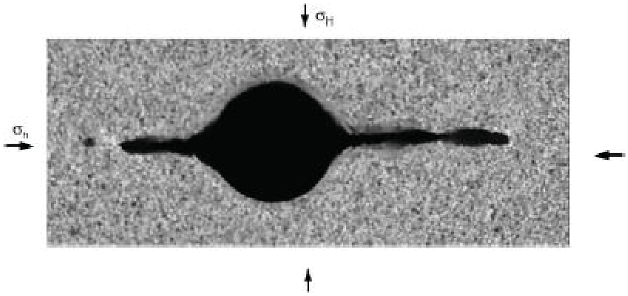

Fig.1. Fracture at intensive compression – fracture zone section ([13]).
Fracture products were removed

The fracture products are shifted to the hole such that the hole is partially filled. Fracture starts at the hole boundary in the zone of maximal compression. Then the fracture zone is growing from the hole boundary according to the concentration of maximal compression. The described fracture scenario corresponds to observations and experimental data published recently in the series of papers [12-16]. It should be emphasized the difference between the new fracture mode from the usual cracks of normal tension which are often formed at unloading of compressed bodies. In this case the cracks of normal tension have similar orientation since the axis of maximal tension at elastic unloading coincides with the axis of maximal residual deformations of inelastic compression. The unloading cracks of normal tension have a typical morphology of rupture, small opening (<0.01mm) and are free of filling, while the fracture zones of the new fracture mode are characterized by opening in their end zones of order of several grain size (>1m) and by partial filling of the fracture cavity with the grinded material. As it will be written later on besides a difference in crack and fracture zone growth regimes under uniform loading is observed.

Narrow cracklike zones of compacted material, which are interpreted as the cracks of compression (anti-mode 1 crack), are observed in tectonic structures of compression in the vicinity of the fault edge (main tectonic shear [15]).

The conditions of the limit equilibrium for cracks of compression are similar to the conditions of the limit equilibrium for the tensile cracks having an initial opening since the described new defects are geometrically similar to the cracks (their longitudinal size is much larger than their characteristic thickness and the processes of fracture of the initial structure are concentrated in the end zones) [17]. Hence, one can use the fracture mechanics technique to analyze fracture processes for the cracks of compression.

A model of the crack of compression

Let us consider a plane model of a crack of compression which represents an elongated rectilinear cavity filled by fracture products or compacted mass and loaded by compressive stresses in the direction transverse to the cavity direction.

Two characteristics variants of the crack of compression which differ by the degree of fracture products mobility are considered.

Movement of fracture products is difficult. Assume that the fracture products filling the cavity can not move along the direction of the cavity elongation and the value of decreasing the medium volume during the end zone advance on the distance of order of the end zone size is fixed. Denote by h_0 the initial thickness of the medium layer through which the end zone of the defect will advance. Then the variation of the thickness of this layer, Δh, becomes fixed in each section of the elongated defect such that

$$h_0 - \Delta h = \text{const} \tag{1}$$

In these conditions one can model the medium with the defect combining the solutions of two plane problems of fracture mechanics: the problem on the elastic plane wedging by an effective wedge of constant width, Δh, and the problem on energy dissipation at inelastic deformation of a thin layer in the end zone of the defect (the Dugdale type model [8]). By incorporating the known solution of the wedging problem for the wedge of length much larger as compared to its width we can write the following relation if friction is absent on the wedge surfaces [17]

$$K_I = -\frac{\Delta h E}{\left(1-\nu^2\right)\sqrt{2\pi\ell^*}} \tag{2}$$

where K_1 is the stress intensity factor, ℓ^* is the size of the zone free of contact with the wedge near the tip of the defect (crack), E is the Young modulus, ν is the Poisson ration. The sign «-» is related to the decreasing of the medium layer thickness after advance through it the end zone (the wedge of negative thickness). As the second problem we will use the Dugdale one such that

$$K_I = -\sqrt{E\sigma_T\delta} \tag{3}$$

where $\delta \sim \Delta h$ is the limit opening in the defect (crack) tip.

Let us find an interrelation between the size ℓ^* and the value Δh accounting for that similarly to the Dugdale model the process of inelastic deformation (in our case – compacting) of a porous material (accompanied by decreasing of the aforementioned layer thickness on the value Δh) occurs just at that size. For the sake of simplicity assume that the material of this layer is deformed as rigid plastic one then the transverse stresses, σ, are constant at the length ℓ^* such that $\sigma \approx \sigma_T$ where σ_T is a characteristic stress. As in the Dugdale model [8] we obtain

$$\ell^* \sim \frac{\Delta h E}{\pi\sigma_T} \tag{4}$$

Note, that relations (3), (4) also follow from the dimensionality analysis with accuracy to a coefficient.

Further we obtain from Eqs (2), (3) and (4)

$$K_I \sim -\sqrt{\Delta h E\sigma_T}\left(1+\frac{1}{\sqrt{2}(1-\nu^2)}\right) \tag{5}$$

The obtained relation models the effective fracture resistance of the medium relative to local compacting in case of absence of the fracture products motion. This relation differs from the appropriate relation for the Dugdale model only by a coefficient. The value Δh in our model is similar to the limit opening of the crack in its tip in the Dugdale model. According to Eq. (5) minimal fracture resistance relative to cracks of compression (and, hence, minimal energy dissipation at their growth) is observed for the minimal value Δh. In turn, the less is the thickness of the compacting layer, the less is the value Δh. One can wait that the cracks of compression are formed under quasistatic regimes of porous medium loading. The appropriate thickness of the compacting layer for these cracks is close on the order of value to the characteristic size of the medium structural element. According to [18] the thin compacting zones in sandstone have 3-10 quartz grains through the thickness or the thickness of the compacting layer equals h~(0.5-1.5)mm.

The length of the crack of compression is much larger (in our example it is of order of tens centimeters). For the typical parameters of high porous sandstone (porosity n ~ 0.2, $\sigma_T \sim 30$MPa, E ~10^4 MPa) we obtain from Eq. (5) within the assumption that porosity is decreased down to n_1 ~0.05 at h = 1mm during the compacting process

$$\Delta h \sim h(n - n_1) \sim 0.15\text{mm} \quad ; \quad K_I^{ef} \sim 12\text{MPa}\sqrt{\text{m}}$$

Remind for comparison that fracture toughness equals $K_{IC} \sim 1\text{MPa}\sqrt{\text{m}}$ for the mode I cracks in rocks.

Free motion of fracture products along the crack of compression. Let us consider the case when the fracture products can move along the crack of compression. Then equalization of medium resistance to drawing together of the surfaces of the crack of compression occurs. Assume, e.g., that in an asymptotic variant the fracture products (similarly to fluid) provide uniform backpressure to the external loads. The resulting difference of the stresses at the surfaces of the crack of compression equals

$$\Delta\sigma = \sigma_\infty - p \tag{6}$$

where σ_∞ is the external compressive stress, p is the value of backpressure caused by fracture products.

An average crack opening in elastic plane under the action of uniform loading on its surfaces has the following form for the given variant of loading [17]

$$\bar{u} = \frac{2}{\ell}\int_0^\ell u(x)dx = \frac{\pi\Delta\sigma}{E}\left(1-\mu^2\right)\ell \tag{7}$$

where ℓ is the half-length of the crack of compression.

Let us estimate the following two variants of deformation of the effective medium consisting of fracture products in the crack of compression cavity.

a) The effective medium is equivalent to an elastic one with the effective Young modulus E^*. We will account for a possibility for changing the material volume at its fracture in the end zone as well as existence of an initial free volume in the region of crack of compression initiation (v_0). Introduce the coefficient of volume changing at medium fracture in the end zone (loosening coefficient)

$$k \sim \frac{v_f}{v_{in}} \tag{8}$$

where v_f is volume in the final (fractured) state, v_{in} is the volume in the initial state.

This coefficient satisfies the inequalities k < 1 (for the case of compacting), k > 1 (for the case of loosening). A balance of the current volumes (areas for the plane problem) for a crack of compression in an elastic massif can be written as follows

$$h_0\ell + v_0 = h_0\ell k\left(1-\frac{p}{E^*}\right) + \frac{\pi}{E}\Delta\sigma\left(1-\mu^2\right)\ell^2 \tag{9}$$

where the first term in the write side is related to the volume changing in the layer of fractured material while the second term is related to the volume changing of the crack cavity at their surfaces deflection under the action of the stresses ($\Delta\sigma$). We obtain from Eqs (6) and (9)

$$\Delta\sigma=\left(1+\frac{v_0}{h_0\ell}-k+\frac{k\sigma_\infty}{E^*}\right)\left(\frac{k}{E^*}+\frac{\pi(1-\mu^2)\ell}{Eh_0}\right)^{-1} \tag{10}$$

Accounting for the observed width constancy of the crack of compression with moving filling [13] one can assume that the end zone of the crack is autonomous. Hence, the material is characterized by a critical level of the stress intensity factor for the given type of loading. The value of the critical stress intensity factor is not changed at an advance of the crack of compression (the Barenblatt type criterion).

Let us determine the stress intensity factor for the given scheme of the uniform loading of the effective crack by incorporating the known solution of the crack problem [8] for the plane with uniformly loaded crack

$$K_I=\Delta\sigma\sqrt{\pi\ell}=\left(1+\frac{v_0}{h_0\ell}-k+\frac{k\sigma_\infty}{E^*}\right)\left(\frac{k}{E^*}+\frac{\pi(1-\mu^2)\ell}{Eh_0}\right)^{-1}\cdot\sqrt{\pi\ell} \tag{12}$$

The effect of changing the fracture regime can be illustrated by the variant $v_0 \neq 0$, $k = 1$. For this variant we obtain from Eq. (12)

$$K_I=\left(\frac{v_0E^*}{h_0\ell}+\sigma_\infty\right)\cdot\sqrt{\pi\ell}\left(1+\frac{\pi(1-\mu^2)E^*\ell}{Eh_0}\right)^{-1} \tag{13}$$

Function (13) has two extreme points. An example of the function is given in Fig.2 for the series of the parameter values close to the conditions of the experiments described in [14-16] $\frac{v_0E^*}{h_0\sigma_\infty}=0;\,0.002;\,0.0.03\mathrm{m}^{-1}$ and $\frac{\pi(1-\mu^2)E^*}{Eh_0}=20\mathrm{m}^{-1}$ (curves 1, 2, 3 in Fig.2, respectively). If we assume as above that the Barenblatt type fracture criterion is fulfilled then the following four stages of crack of compression growth can occur.

First a fracture nucleus of length ℓ_a occurs in the vicinity of the initial free volume (e.g., of a pore or hall) (see the left point of intersection of a horizontal line with curve 3 in Fig.2). Then the stage of fracture deceleration occurs (the first descend part of the curve in Fig.2). To increase the size of the crack of compression one needs to increase the external load. Attaining the length ℓ_{ao} related to the minimum value of K_I the crack of compression becomes unstable and its length continues to grow up to attaining the stable branch of function (13) (point ℓ_{a1} in Fig.2). The intermediate stable state related to the inequality $\ell_a<\ell_{a0}$ can serve for prediction of the conditions of the holes stability. In this variant of fracture instability at fracture initiation (at the length ℓ_{ao}) occurs at the minimal value K_I caused by the presents of the initial free volume. In the general case this K_I level does not coincide with the value of the critical stress intensity factor for the local deformation and fracture processes in the tip of the extended crack of compression. A transition on the stable branch of function (13) is related to attaining the critical K_I level.

We believe that just this regime of the crack of compression growth was observed in experiments on a porous model material [18] and on sandstone [14-16].

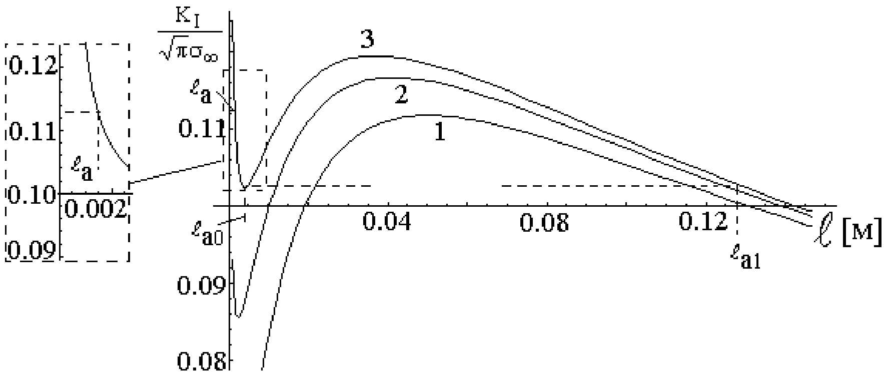

Fig.2. Regimes of crack of compression growth

b) Let us consider the case when the deformation resistance of a moving layer of fractured material represents a nonlinear function of the load. We will use the relation which corresponds to the interaction of blocks in block massif at its elastic deformation [18]

$$\sigma \sim a\varepsilon^2 \tag{14}$$

where a is an empirical constant.

For this variant of nonlinearity we obtain (similarly to the analysis performed for case (a)) the following formula if the effects of the free volume and loosening are neglected

$$\Delta\sigma = \frac{1}{2a}\cdot\frac{h_0E}{\left(\pi(1-\mu^2)\ell\right)^2}\left((h_0E)^2 + a\sigma_\infty\left(\pi(1-\mu^2)\ell\right)^2\right)^{1/2} \tag{15}$$

Respectively

$$K_I = \frac{h_0E\sqrt{\pi\ell}}{2a\left(\pi(1-\mu^2)\ell\right)^2}\left((h_0E)^2 + a\sigma_\infty\left(\pi(1-\mu^2)\ell\right)^2\right)^{1/2} \tag{16}$$

The function $K_I(\ell)$ in Eq. (16) is monotonically decreasing with ℓ increasing. Hence, initiation and further growth of an initial defect is stable (to increase the length of the crack of compression one needs to increase the external load (σ_∞)).

Thus different variants of growth of cracks of compression are possible in dependence on deformation properties of the layer of fracture materials in the crack cavity and its mobility. The variants of growth from global stable to unstable are possible. In an intermediate variant the sizes of the crack of compression are finite.

The results of the performed asymptotic analysis show that in the considered model of the crack of compression the regimes of its growth are rather complex including, in particular, unstable stage, a transition to the regime of stable growth under uniform loading. Note, that the stage of the stable growth under uniform tension is impossible for the cracks of normal tension [8, 17].

To compare the obtained results with experimental data given in [12-14] these data are summed up in Fig. 3. The data are related to the tests performed for three different types of porous sandstone at different levels of the minimal stress.

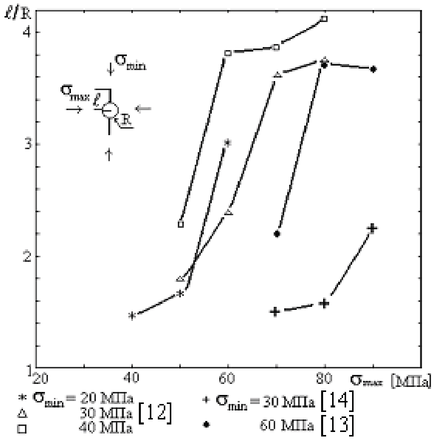

Fig.3. Summed experimental data about the relative sizes of cracks of compression in different types of sandstone

One can see three characteristic stages of crack of compression growth which qualitatively coincide with the stages predicted by the suggested model. Slow (stable) crack advance with the increasing of maximal compression is followed by the stage of fast crack growth. Then this fast growth is stopped and the further crack growth becomes stable.

Note, that a physical restriction exists for applicability of the suggested model. Indeed, a contact of the crack of compression surfaces will occur in the middle part of the crack at some crack length. The condition of crack surfaces contact has the following form for the plane model of the crack of compression in case of free motion of fracture products

$$\frac{\ell_{max}\Delta\sigma(1-\mu^2)}{E} \le h_0 \tag{17}$$

By incorporating Eq. (6), we obtain

$$\ell_{max} \le \left(\frac{kh_0E}{(1-\mu^2)E^*} - \frac{v_0}{h_0}\right)\left(1-\pi+k\left(\frac{\sigma_\infty}{E^*}-1\right)\right)^{-1} \tag{18}$$

One can see that the size of the crack of compression at the occurring of its surfaces contact is proportional to crack opening in the end zones (h_0) if $v_0 \to 0$. Further, for the crack of compression with a contact zone the fracture process is continued in two parts of the crack separated by the contact zone. The stress intensity factor in their active tips is stabilized (in case of large relative sizes of the contact zone the situation is similar to one considered earlier for the case when the restrictions for fracture products motion exist).

Thus in the paper it is suggested a mechanism and model of quasi-brittle fracture of the porous media by local compacting accompanied by cracks of compression formation. The growth of these cracks can occur in stable and unstable regimes with changing the regimes in spite of uniform external loading. In turn, the combinations of stable and unstable regimes enable to realize the fracture scenarios related to formation of more complex structures of fracture such as echelons of cracks of compression.

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