Ductile Tearing Resistance of Metal Sheets

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Keywords: Ductile tearing, R-curves, *J*-integral, cracked panel, sheet metal.

Abstract. The concept of R-curves has been adopted to characterise stable crack extension and predict residual strength of thin-walled structures particularly in the aircraft industry. The present contribution uses results of FE simulations of crack extension in panels by the cohesive model to validate analytical procedures for determining *J*-integral values at large crack extension from measurable quantities, namely the force vs. displacement records. The numerically determined *J*-integral is taken as the benchmark for the outcome of the analytical formulas.

Introduction

The familiar concept of J_R-curves originally established for thick-walled components under plane-strain conditions has also been adopted to characterise stable crack extension and predict residual strength of thin-walled components under plane-stress conditions, particularly in the aircraft industry. It requires other specimen types, primarily large cracked tensile panels allowing for pronounced ductile crack extension prior to failure. The respective test procedures and standards are not as well founded as those for plane-strain specimens. ASTM E 1820 [1] does not include M(T) specimens at all, and ASTM E 561 [2] yields R-curves in terms of the stress intensity factor as a function of the "effective" crack length. Though the knowledge about the evaluation of J_R-curves from force-displacement records is quite old, namely more than 30 years, the formulas extracted from the literature are still controversial, which impedes any sound discussion on the validity of R-curves, as experimental investigations on this matter suffer from inconsistent data. The present contribution uses results of FE simulations of crack extension in panels by the cohesive model to validate analytical procedures for determining *J*-integral values at large crack extensions from measurable quantities. The numerically calculated *J*-integral is taken as the benchmark for the outcome of the analytical formulas. As no discussion on the *significance* of R-curves for characterising ductile crack extension is intended, no respective *validity* conditions have to be considered. However assuring that at least a *correct* *J*-value has been determined is a necessary prerequisite for discussing the problem of *validity* of J_R-curves!

Basic Equations

The *J*-Integral as Energy Release Rate. The basic idea of determining J from an experimental force-displacement record as depicted in Fig. 1 for a stationary crack is more than 30 years old. It utilises the nature of J being an energy release rate in the deformation theory of plasticity. At some displacement, v, a small increase of the crack surface, ΔA, under "fixed grips" (constant displacement) results in a release of mechanical work, ΔU, and the negative ratio $\Delta U/\Delta A$ for $\Delta A \to 0$ is the J-integral. For a panel shaped specimen of thickness B with a through-crack and a straight crack front we have $\Delta A = B\Delta a = -B\Delta b$ for each crack tip, where $b = W - a$ is the ligament length. Note that for an M(T) specimen the width is $2W$ and the crack length is $2a$. Thus we obtain

$$J = \begin{cases} -\left(\dfrac{\partial U}{B\partial a}\right)_v = \left(\dfrac{\partial U}{B\partial b}\right)_v & \text{for C(T)} \\ -\left(\dfrac{\partial U}{2B\partial a}\right)_v = \left(\dfrac{\partial U}{2B\partial b}\right)_v & \text{for M(T)} \end{cases} . \tag{1}$$

An alternate and equivalent definition of J [4] is given by

$$J = \begin{cases} \dfrac{1}{B}\int_0^F \left(\dfrac{\partial v}{\partial a}\right)_F dF = -\dfrac{1}{B}\int_0^F \left(\dfrac{\partial v}{\partial b}\right)_F dF & \text{for C(T)} \\ \dfrac{1}{2B}\int_0^F \left(\dfrac{\partial v}{\partial a}\right)_F dF = -\dfrac{1}{2B}\int_0^F \left(\dfrac{\partial v}{\partial b}\right)_F dF & \text{for M(T)} \end{cases} . \tag{2}$$

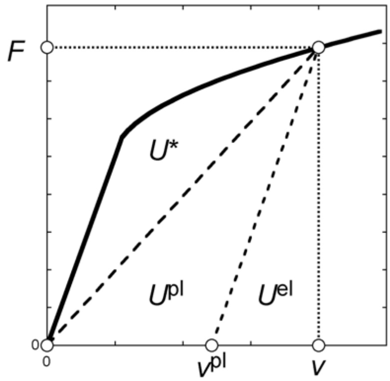

Fig. 1: Force-displacement curve of a fracture specimen with a stationary crack of length a_0

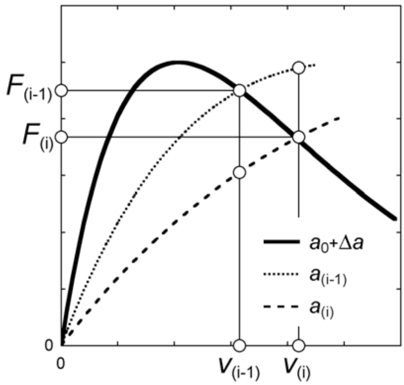

Fig. 2: Force-displacement curves of a fracture specimen of initial crack length a_0 undergoing crack extension and two fictitious specimens with stationary cracks of lengths $a_{(i-1)}$ and $a_{(i)}$

The load-point displacement, v, is split into an elastic (linear, reversible) and a plastic (nonlinear, permanent) part (which of course is contradictory to the assumption of deformation theory), $v = v^{el} + v^{pl}$, and so is the mechanical work or deformation energy and, hence, the J-integral,

$$U = \int_0^v F dv = \int_0^{v^{el}} F dv^{el} + \int_0^{v^{pl}} F dv^{pl} = \tfrac{1}{2} F v^{el} + \int_0^{v^{pl}} F dv^{pl} = U^{el} + U^{pl} . \tag{3}$$

$$J = J^{el} + J^{pl} . \tag{4}$$

Applying this separation in elastic and plastic fractions, eqs. (1) and (2) hold likewise for J^{pl}, U^{pl}, and v^{pl}. The elastic part is calculated from the mode I stress intensity factor, assuming plane stress conditions in metal sheets

$$J^{el} = K^2/E \qquad \text{with} \qquad K = \sigma_\infty \sqrt{\pi a}\, Y(a/W). \tag{5}$$

The K_{eff} Concept. ASTM E 561 [2] regulates the determination of K_{R}-curves, i.e. $K_{\text{eff}} = K\left(a_{\text{eff}}\right)$, as crack growth resistance, "so long as specimens are of sufficient size to remain predominantly elastic throughout the duration of the test". Two options of determining a_{eff} are proposed. Under small scale yielding conditions, the stress field is dominated by an "effective" stress intensity factor, which - according to Irwin - results from a plastic zone correction of the physical crack length by the radius of the plastic zone,

$$a_{\text{eff}} = a + r_{\text{pl}} = a_0 + \Delta a + r_{\text{pl}} = a_0 + \Delta a_{\text{eff}} \qquad \text{with} \qquad r_{\text{pl}} = \frac{1}{2\pi}\left(\frac{K}{R_{\text{F}}}\right)^2, \tag{6}$$

assuming plane stress, again. K can be modified once by a_{eff} according to eq. (6) to yield K_{eff} or iteratively, as a_{eff} depends on K. Alternatively, a_{eff} can be calculated from the compliance of the specimen, regarding the elastic-plastic deformation of the specimen with crack length a as an elastic deformation of a (fictitious) specimen with crack length a_{eff} ,

$$v = v^{\text{el}} + v^{\text{pl}} = C(a)F + v^{\text{pl}} = C\left(a_{\text{eff}}\right)F \ . \tag{7}$$

The compliance is determined experimentally or from analytical formulas. For comparison with other R-curve formulas, K_{eff} will be converted to J according to eq. (5).

J_{R}-curves for cracked metal sheets

C(T) Specimens. ASTM E 1820 [1] is the standard test method for measurement of fracture toughness on bend-type specimens. Though it is particularly designated to thick (plane strain) specimens, the formulas do not contain any restriction with respect to the thickness of the test piece. The elastic part of J is calculated according to eq. (5). Starting from the J-value at crack initiation, further values $J_{(i)}^{\text{pl}} = J^{\text{pl}}\left(a_{(i)}\right) = J_{(i-1)}^{\text{pl}} + \Delta J_{(i)}^{\text{pl}}$ are calculated stepwise for crack extension increments $\Delta a_{(i)} = a_{(i)} - a_{(i-1)}$. The basic assumption behind this technique is that a specimen, which has undergone crack extension, has the same value of $J_{(i)}$ as a postulated non-linear elastic specimen at the same load, $F_{(i)}$, displacement, $v_{(i)}$, and final crack length, $a_{(i)}$, which was not subject to crack extension, see Fig. 2. The problem reduces to constructing a force-displacement curve for such a specimen. As the respective formulas are well established and written down in the standard, they are not repeated here. The procedure requires the calculation of the plastic work, which is obtained from the force vs. load-line displacement records. The plastic part of the latter is determined from the measured total displacement by subtracting v^{el} calculated from the elastic compliance.
ASTM E 561 [2] provides the respective equations for determining $K_{\text{eff}} = K\left(a_{\text{eff}}\right)$ according to eqs. (6) and (7).
The GKSS test procedure EFAM GTP 02 [3], which is based on the ESIS Procedures P1 and P2, proposes an empirical "crack length correction",

$$J(a) = J(a_0)\left[1 - \frac{\left(0.75\eta - 1\right)\Delta a}{b_0}\right], \tag{8}$$

where $\eta = 2 + 0.522\, b_0/W$, without providing any reproducible background of this equation.

M(T) Specimens. There is no standard like ASTM E 1820 [1] for tensile-type fracture specimens. Several formulas have been derived in the literature but never been standardised. The elastic part results from K, where commonly the "secans formula" for the geometry function is applied

$$Y\left(\frac{a}{W}\right)=\sqrt{\sec\left(\frac{\pi a}{2W}\right)}=\frac{1}{\sqrt{\cos\left(\pi a/2W\right)}}\,. \tag{9}$$

ASTM E 561 [2] provides an alternate expression, which appeared to be improper for $\left(a/W\right)>0.6$, however.
Based on eq. (2), Rice, Paris and Merkle (RPM) [4] have derived the formula

$$J^{\mathrm{pl}}=\frac{1}{bB}\left[\int_0^{v_{\mathrm{L}}^{\mathrm{pl}}} F\,dv_{\mathrm{L}}^{\mathrm{pl}}-\tfrac{1}{2}Fv_{\mathrm{L}}^{\mathrm{pl}}\right]=\frac{1}{bB}\left(U^{\mathrm{pl}}-\tfrac{1}{2}Fv_{\mathrm{L}}^{\mathrm{pl}}\right)=\frac{U^{*\mathrm{pl}}}{bB}=\frac{U^{*}}{bB}\,, \tag{10}$$

for an M(T) specimen with constant crack length. This equation can be extended to crack growth by the assumption explained above, see Fig. 2. It results in

$$J_{(i)}^{\mathrm{pl}}=J_{(i-1)}^{\mathrm{pl}}\frac{b_{(i)}}{b_{(i-1)}}+\frac{F_{(i-1)}v_{(i)}^{\mathrm{pl}}-F_{(i)}v_{(i-1)}^{\mathrm{pl}}}{2Bb_{(i-1)}}\,. \tag{11}$$

Garwood, Robinson and Turner (GRT) [5] proposed a procedure to calculate the total J from the load-displacement curve. It yields

$$J_{(i)}=J_{(i-1)}\frac{b_{(i)}}{b_{(i-1)}}+\frac{F_{(i-1)}v_{\mathrm{L}(i)}-F_{(i)}v_{\mathrm{L}(i-1)}}{2Bb_{(i-1)}}+\frac{K_{(i)}^{2}b_{(i)}-K_{(i-1)}^{2}b_{(i-1)}}{Eb_{(i-1)}}\,, \tag{12}$$

which is slightly different from the total J calculated according to eq (11) and adding the elastic part from eq. (5).
Hellmann and Schwalbe [6] provide a formula with reference to GRT [5] but have apparently missed that the increment of crack surface in an M(T) specimen is $\Delta A=2B\Delta a$. Recently, Neimitz et al. [7] tried to calculate the increment $\Delta J_{(i)}^{\mathrm{pl}}$ via the total differential of eq. (10). They overlooked, that the latter consists of two part, namely for constant displacement and for constant crack length, see Fig. 2,

$$dJ^{\mathrm{pl}}=\left(\partial J^{\mathrm{pl}}\right)_{v^{\mathrm{pl}}}+\left(\partial J^{\mathrm{pl}}\right)_{a}=\left(\frac{\partial J^{\mathrm{pl}}}{\partial a}\right)_{v^{\mathrm{pl}}}da+\left(\frac{\partial J^{\mathrm{pl}}}{\partial v^{\mathrm{pl}}}\right)_{a}dv^{\mathrm{pl}}\,. \tag{13}$$

The GKSS test procedure EFAM GTP 02 [3] recommends the empirical crack length correction of eq. (8) with $\eta=1$.
Most authors apply ASTM E 561 [2] for large thin panels as used in the aircraft industry, e.g. [8, 9].

Validation of the R-curve formulas

Tests and Numerical Models. Investigating the accuracy of the various equations for evaluating J requires reference solutions to compare with. This is only possible by applying numerical models accounting for crack extension, which provide consistent data for the quantities used in the above equations, namely F, v and a, as well as for the J-integral. As has been shown in several papers, e.g. [10 - 12], ductile crack extension in metal sheets can be adequately modeled with cohesive elements. The data presented here have been obtained for an aluminium-magnesium alloy Al 5083 H321, $E=71600$ MPa, $R_{\mathrm{p0.2}}=240$ MPa, which is widely used in shipbuilding and automotive industry. Several fracture specimens with different sizes have been manufactured from rolled plates of 3 mm thickness, which were tested under quasi-static conditions. The parameters of the cohesive model, namely the cohesive strength, $\sigma_c = 560$ MPa, and the separation energy, $\Gamma_c = 10$ kJm^{-2}, have been determined from C(T) specimens of $W = 50$ mm and validated for other specimens geometries and sizes; for details see [10]. Fig 3 shows the experimental and numerical force vs. displacement curves of C(T) and M(T) specimens of width $W = 150$ mm. The coincidence between test and simulation results is reasonable, keeping in mind that a unique set of cohesive parameters has been used to model crack extension in various specimen geometries and sizes. Any discrepancies occurring between test and simulation data do not affect the following conclusions, anyway, as the intention is just to have a set of consistent data to check the accuracy of the above J formulas.

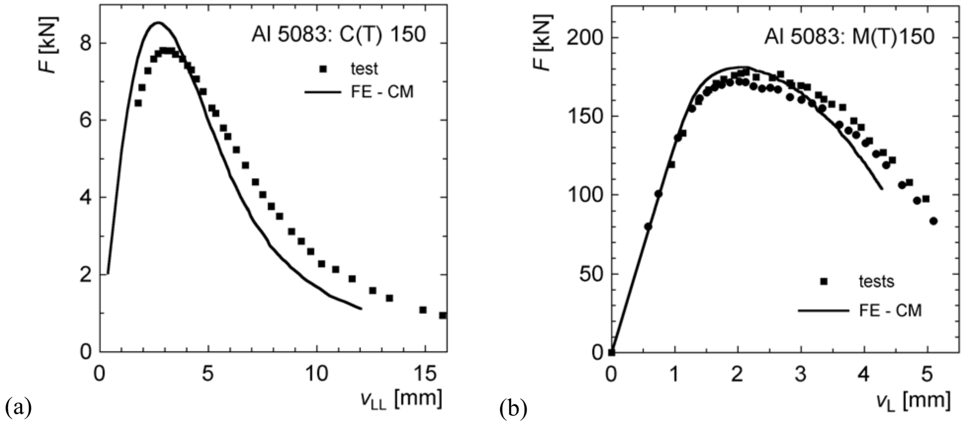

Fig. 3: Force-displacement curves of 3 mm thick, cracked panels of Al 5083, width 150 mm, comparison of tests and simulations with the cohesive model;
(a) C(T) $a_0 = 75$ mm, (b) M(T) $a_0 = 30$ mm

Evaluation of Numerical Data The data of the numerical simulations are taken to evaluate J_R-curves. The J-integral value calculated by ABAQUS is used as reference. Special care has to be taken to ensure obtaining a "far-field" value of J [13], which is comparable to the values calculated from a global force-displacement curve. Fig 4 shows the results.

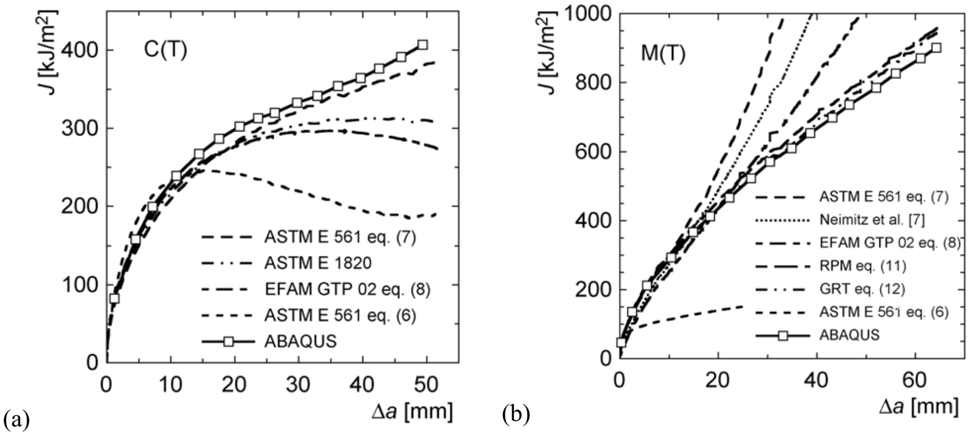

Fig. 4: J_R-curves of cracked panels of Al 5083, thickness 3mm, width 150 mm, comparison of various evaluation procedures; (a) C(T), (b) M(T)

Generally, little differences occur up to crack extensions of $\Delta a \leq 10\,\text{mm}$, which is $0.13(W-a_0)$ for the C(T) and $0.08(W-a_0)$ for the M(T), respectively, but the deviations may become significant for large crack growth. ASTM E 561 based on the plastic zone correction, eq. (6), fails beyond $\Delta a > 10\,\text{mm}$ for the C(T) and did not produce any useful result at all for the M(T) specimen. EFAM GTP 02 [3], eq. (8), starts becoming improper beyond $\Delta a > 0.27(W-a_0)$, and so does ASTM E 1820. An analysis of the data revealed a discrepancy between the FE data and the ASTM E 1820 procedure in splitting elastic and plastic energies according to eq. (3). Both ASTM E 561, eq. (6), and EFAM GTP 02 show a decreasing J_R-curve for large crack extension, which is physically meaningless. Whereas ASTM E 561 based on the compliance, eq. (7), gives the best approximation up to $\Delta a = 50\,\text{mm} = 0.66(W-a_0)$ for the C(T), it overestimates J significantly beyond $\Delta a > 0.12(W-a_0)$ for the M(T). The eq. of Neimitz et al. [7] for the M(T) and the EFAM GTP [3] start overestimating J beyond $\Delta a > 0.17(W-a_0)$ and $\Delta a > 0.25(W-a_0)$, respectively. The RPM eq. (11) and the GRT eq. (12) produce perfect approximations up to $\Delta a = 65\,\text{mm} = 0.54(W-a_0)$.

Significance of the R-curves

Once a *correct* R-curve can be generated from the experimental data, the question of its significance for describing ductile crack extension may be raised, i.e. its "validity". No respective recommendations or guidelines are provided in the standards. This question cannot be answered here, as it would require systematic experimental and numerical investigations. The present data may be used however to give a first impression about the transferability of J or CTOD based R-curves. Fig. 5 displays the respective results for the C(T) and the M(T). The CTOD R-curves are based on the δ_5-parameter by Schwalbe [14].

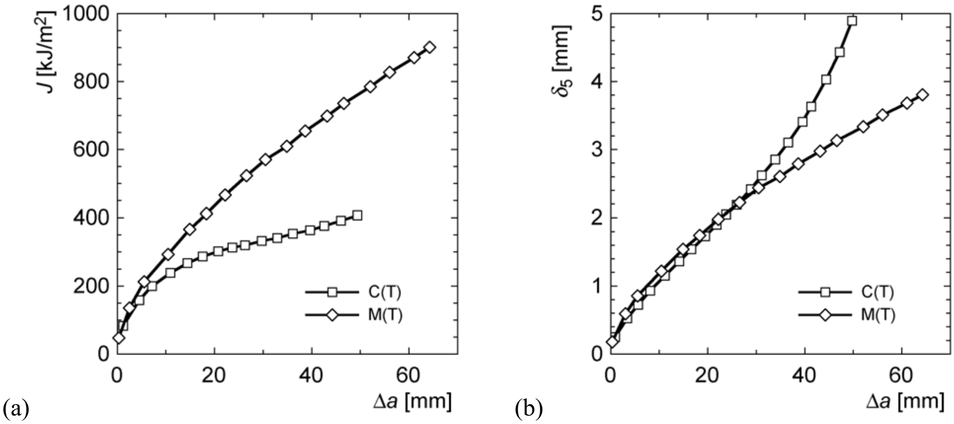

Fig. 5: R-curves of 3 mm thick C(T) and M(T) specimens of Al 5083, width 150 mm, as obtained from simulations with the cohesive model; (a) J-integral, (b) CTOD δ_5

The range of validity, i.e. of geometry independence is obviously much larger for the CTOD R-curves.

Summary

Though the basic concepts for determining J from experimental force-displacement records are quite old, they have not yet found their way into standards or guidelines for other than thick bend-type specimens. The K_R-curve concept of ASTM E 561 is accepted for structural assessment in the aerospace industry, but its background is obsolete. It might work well in some cases but completely fail in others. No sound discussion on the validity of R-curves for thin-walled structures and M(T) specimens is possible, as long as experimental investigations on this matter suffer from inconsistent

data. As J cannot be measured directly but has to be evaluated from measurable data like forces and displacements, there is no possibility for an experimental validation of the respective evaluation procedures.
The present contribution uses results of FE simulations of crack extension in panels by the cohesive model to validate analytical procedures for large crack extension. The conclusions are

- ASTM E 1820 has a limited range of application for C(T)-type specimens of $\Delta a_{max} \approx 0.25\,(W - a_0)$.
- The K_{eff}-concept of ASTM E 561 is questionable. The option of a plastic zone correction is virtually useless. The alternate option of determining a_{eff} from the specimen compliance worked even better than the ASTM E 1820 procedure for the C(T), but with the experimental techniques of present days given, there is no real necessity of defining an "effective" crack length.
- J_R-curves of thin M(T) specimens can be correctly determined up to large crack extensions by the formulas of RPM, eq. (11), or GRT, eq. (12).
- There seems to be no necessity, either, to use *empirical* equations like in EFAM GTP [3]. They do not provide any advantage of simple application and their validity range is limited.

ESIS would be well advised to consider these results and conclusions in the revision process of their test procedure P3-08 for determining the fracture behaviour of materials.

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