Does shear deformability influence the mode II delamination of laminated beams?

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Abstract. Laminated beams affected by interlaminar cracks can be schematised as assemblages of sublaminates, modelled according to some appropriate beam theory. Shear deformability is neglected by the Euler-Bernoulli beam theory, but is taken into account at first order by the Timoshenko beam theory and at higher orders by more refined theories. As well, the connection between the sublaminates can be described by models of growing complexity, ranging from rigid to deformable (elastic or inelastic) interfaces. Consistent with the adopted model, the energy release rate associated to delamination growth can be determined and decomposed into its opening (mode I) and sliding (mode II) contributions. Shear deformability increases the compliance of the structural system and, consequently, may influence the energy release rate. However, the specific influence of shear deformability on the mode II contribution to the energy release rate is not always clear and literature on this point is contradictory. This paper tries to shed light on this controversial issue, by reviewing and critically analysing some of the most relevant studies on this topic. Hence, the circumstances and the ways in which shear deformability may (or may not) influence mode II fracture of delaminated beams should be clarified.

Introduction

Analysis of delaminated plates and beams. The structural response of a laminated plate affected by delaminations can be analysed via structural theories, schematising the delaminated plate as an assemblage of sublaminates [1, 2, 3]. In particular, beam theories can be effectively applied to model plane problems such as those describing the laboratory tests used for assessing delamination toughness of unidirectional fibre-reinforced laminates [4]. The Euler-Bernoulli beam theory, also referred to as classical or simple beam theory (SBT), is the most elementary structural theory that can be used to this aim, but it neglects completely shear deformability which, however, can be relevant for composite materials. Shear deformability can be taken into account at first order by the Timoshenko beam theory (TBT) and at higher orders by more refined theories [5, 6]. Likewise, the connection between the sublaminates can be described by models of growing complexity, ranging from rigid to deformable (elastic or inelastic) interfaces [7, 8].

Delamination growth. Fracture mechanics concepts are commonly used in order to describe the growth of a delamination, by considering it as an interlaminar crack [9, 10]. The energy release rate (ERR), defined as the total potential energy available for a unit crack advance, is frequently used as a parameter to define delamination growth criteria. Accordingly, the delamination is presumed to propagate when the ERR, G, reaches a critical value, G_c. In a linearly elastic laminated beam the energy release rate, G, can be computed via many alternative, equivalent methods including the application of its definition, the use of energetic theorems, the computation of invariant integrals, and so on. On the other hand, interlaminar fracture toughness, G_c, measured by experiments, can vary significantly depending on the active fracture mode (I or opening, II or sliding). More generally, delamination growth occurs under mixed mode conditions, i.e. with a combination of both modes. In this case, the energy release rate, G, is the sum of two parts, G_I and G_{II}, related respectively to fracture modes I and II. As a result, in order to predict crack growth it is necessary to partition the energy release rate into its modal contributions. Equivalently, the mode mixity, i.e. the ratio between the mode II and I contributions to G, has to be determined. To this end, various alternative, but not equivalent methods have been proposed. Williams [11] proposed an analytical method for partitioning the ERR based on analysis of the global forces acting on a cracked laminate. Suo and Hutchinson [12] considered the local singular stress field at the crack tip and deduced the mode mixity for a semi-infinite crack between two infinite isotropic elastic layers. Schapery and Davidson [13] proposed a similar method based on classical plate theory. Li et al. [14] modified the local method [12] in order to take into account the effects of shear forces at the crack tip. Andrews and Massabò [15] extended the method of Ref. [14] to orthotropic materials.

All the aforementioned mode partitioning methods consider the sublaminates to be rigidly connected to each other. This modelling choice simplifies the analysis, but may lead to incorrect results as it may, for instance, underestimate the compliance of real laminates. While following different approaches, many models have been proposed in which the separating layers of a delaminated beam are connected by deformable interfaces, generalising an approach originally proposed by Kanninen [16] for the DCB specimen. Bruno and Greco [17] considered a linearly elastic interface exerting both normal and tangential stresses and computed the mode mixity by examining the contributions to the energy release rate stemming from these stress components in the limit case of a rigid interface. Qiao and Wang [18] compared rigid, semi-rigid and deformable interface models and determined the mode mixity via a suitable adaptation of the local method of Ref. [12]. Bennati et al. [19] presented a model of the ADCB test where the modal contributions to the ERR are determined based on the interfacial normal and tangential stresses at the crack tip.

Taking into account shear deformability increases the compliance of the structural system and, consequently, may influence the energy release rate. However, the specific influence of shear deformability on the mode II contribution to the ERR is not always clear and literature on this topic is contradictory. In order to clarify this controversial point, it is convenient to distinguish pure mode II from mixed mode conditions. Actually, for mixed mode conditions, the identification of the mode II contribution depends on which method is adopted for separating fracture modes. Most methods, however, agree in the case of pure mode II conditions, i.e. when no mode I contribution is present.

Pure mode II fracture

Symmetrically delaminated laminates. Pure mode II conditions are attained when a symmetrically delaminated laminate is subjected to an anti-symmetrical load with respect to the laminate's mid-plane. As a representative example of this case, we consider the end notched flexure (ENF) test, used to assess mode II interlaminar fracture toughness (Fig. 1), but similar considerations can be extended to other systems undergoing mode II delamination. The specimen has length $L = 2l$, thickness $H = 2h$, and width B (not shown in the figure); let a be the delamination length. For the sake of simplicity, we consider here a homogeneous orthotropic laminate, whose elasticity moduli are in the material reference are E_x, E_z, and G_{zx}. A load P acts at the mid-span section.

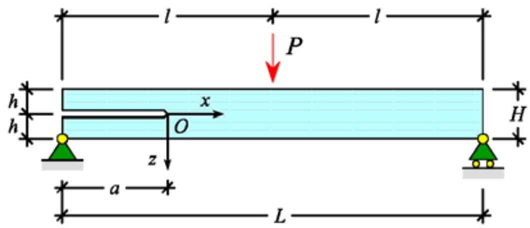

Fig. 1 – Scheme of the end notched flexure (ENF) test.

The ENF test for composite materials was used first by Russell and Street [20], who applied simple beam theory to determine the compliance,

$$C_{\mathrm{SBT}}^{\mathrm{ENF}} = \frac{\delta}{P} = \frac{2l^3 + 3a^3}{8E_{\mathrm{x}}Bh^3}, \tag{1}$$

where δ is the displacement of the application point of the load, and the energy release rate,

$$G_{\mathrm{SBT}}^{\mathrm{ENF}} = \frac{P^2}{2B}\frac{dC_{\mathrm{SBT}}^{\mathrm{ENF}}}{da} = \frac{9P^2a^2}{16E_{\mathrm{x}}B^2h^3}. \tag{2}$$

In order to take shear deformability into account, Carlsson, Gillespie and Pipes [21] modelled the ENF test via the Timoshenko beam theory [22] and obtained the following expressions:

$$C_{\mathrm{CGP}}^{\mathrm{ENF}} = \frac{2l^3 + 3a^3}{8E_{\mathrm{x}}Bh^3}\left(1 + 2\frac{E_{\mathrm{x}}}{G_{\mathrm{zx}}}\frac{1.2l + 0.9a}{2l^3 + 3a^3}h^2\right), \tag{3}$$

for the compliance, and

$$G_{\mathrm{CGP}}^{\mathrm{ENF}} = \frac{9P^2a^2}{16E_{\mathrm{x}}B^2h^3}\left[1 + \frac{2}{10}\frac{E_{\mathrm{x}}}{G_{\mathrm{zx}}}\left(\frac{h}{a}\right)^2\right], \tag{4}$$

for the energy release rate. However, Whitney [23, 24] promptly observed that «continuity of displacement at the crack tip is not attained with the approach that yields» Eqs. 3 and 4. Recently, also Fan et al. [25] realised that «a false assumption was made in the derivation» of Eqs. 3 and 4, leading to «inconsistency (…) for the expressions that are used to calculate the energy release rate». Actually, in Ref. [21] the cross-sectional rotation of the specimen at the crack tip was taken equal to the slope of the deflected beam, but this assumption manifestly contradicts the hypotheses on which Timoshenko's first order shear deformation beam theory is based. As a matter of fact, by applying Timoshenko's theory without the above unnecessary approximations, we find

$$C_{\mathrm{TBT}}^{\mathrm{ENF}} = \frac{2l^3 + 3a^3}{8E_{\mathrm{x}}Bh^3} + \frac{3l}{10G_{\mathrm{zx}}Bh}, \tag{5}$$

for the compliance, and

$$G_{\mathrm{TBT}}^{\mathrm{ENF}} = \frac{9P^2a^2}{16E_{\mathrm{x}}B^2h^3} = G_{\mathrm{SBT}}^{\mathrm{ENF}}, \tag{6}$$

for the energy release rate. From Eqs. 5 and 6, we see that shear deformability, at the first order corresponding to the TBT model, influences the compliance but not the mode II energy release rate of the ENF specimen. In confirmation of this, we would like to cite Ozdil, Carlsson and Davies [26], who analysed the ENF specimen using shear deformation laminated plate theory and found that «there is no (…) contribution from shear deformation to the energy release rate for unidirectional (…) laminates with mid-plane cracks», while a «very small contribution» emerges for angle-ply laminates. Also Chatterjee [27] used shear deformation laminated plate theory for studying the ENF specimen and found that «the energy release rate (…) will not be affected if shear deformation effects (in the context of beam theory) are neglected».

The above result is not surprising if we consider separately the deformed shapes due to bending and shear, respectively, of an ENF specimen modelled as an assemblage of rigidly connected sublaminates (Fig. 2). The deformation related to shear appears to be independent of the delamination length, *a*, so its contribution will vanish when we take the derivative of *C* with respect to *a* in order to determine *G*.

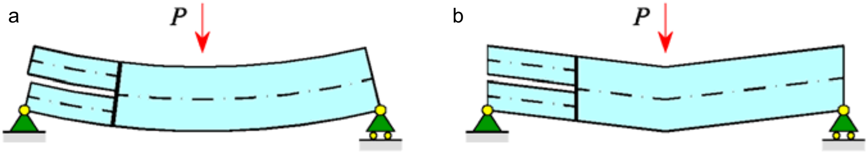

Fig. 2 – Model of the ENF specimen with rigidly connected sublaminates:
(a) bending and (b) shear deformations.

In contrast with the above finding, Eqs. 3 and 4 are still reported in many valuable textbooks [4, 9, 10, 23], used extensively for interpreting experimental results [28, 29, 30, 31] and for comparison purposes [32, 33, 34, 35] as if they were exact expressions.

In this respect, it should be mentioned that if the ENF specimen is modelled as an elasticity problem [27, 36, 37], numerical results show a dependence of the ERR on the shear modulus of the material, G_{zx}. In consideration of his numerical results, Chatterjee [27] suggested the following corrected expression for the energy release rate:

$$G_{\text{Chatterjee}}^{\text{ENF}} = \frac{9P^2a^2}{16E_x B^2 h^3}\left(1 + 0.13\sqrt{\frac{E_x}{G_{zx}}}\frac{h}{a}\right)^2, \tag{7}$$

where the numerical factor 0.13 was obtained by curve fitting. Use of Eq. 7 is suggested in [38]. Wang and Williams [37] used finite element analysis and found a dependence of both the compliance and ERR on the shear modulus of the material. On the basis of their results, they proposed a correction for the delamination length, resulting in the following expressions:

$$G_{\text{Williams}}^{\text{ENF}} = \frac{9P^2(a+\chi h)^2}{16E_x B^2 h^3}, \quad \text{where} \quad \chi = \sqrt{\frac{1}{63}\frac{E_x}{G_{zx}}\left[3 - 2\left(\frac{\Gamma}{1+\Gamma}\right)^2\right]} \quad \text{and} \quad \Gamma = 1.18\frac{\sqrt{E_x E_z}}{G_{zx}}, \tag{8}$$

which have been adopted also in the standard test method for the mixed mode bending test [39].

To sum up, Timoshenko's theory seems unable to reproduce the dependence of the mode II energy release rate on the shear modulus of the material. In order to catch this behaviour, more complex models are needed. These models can be developed in the context of elasticity theory, however resorting to numerical solution methods. Alternatively, higher order beam theories or models introducing deformable interfaces between the sublaminates can be used.

Whitney [24] used second order shear deformation beam theory (SOBT) and obtained

$$G_{\text{SOBT}}^{\text{ENF}} \cong \frac{9P^2a^2}{16E_x B^2 h^3}\left[1 + 2\frac{h}{\lambda a} + \frac{131}{75}\left(\frac{h}{\lambda a}\right)^2\right], \quad \text{where} \quad \lambda = 4\sqrt{\frac{14}{5}\frac{G_{zx}}{E_x}}. \tag{9}$$

Pavan Kumar and Raghu Prasad [40, 41] compared simple beam theory with first (Timoshenko's), second and third order shear deformation beam theories. They found that the SBT and TBT models of the ENF specimen yield the same values of the energy release rate, while increments in the ERR were obtained via both the second and third order beam-theory models.

Corleto and Hogan [42] developed a model of the ENF test considering the upper half laminate as a Timoshenko beam on a generalised elastic foundation consisting of extensional and rotational distributed springs. They found that the energy release rate is independent of the shear stiffness of the sublaminate, $C_1 = 5/6\ G_{zx}\ h$, but not of the shear modulus of the material, G_{zx}, which enters the expressions of the elastic constants of the foundation. Based on this result, Ding and Kortschot [43] used simple beam theory instead of Timoshenko's theory to deduce a simplified model of the ENF test where a foundation made of tangential springs only is considered. Wang and Qiao [44] modelled the ENF specimen using the Timoshenko beam theory and considering an elastic foundation made of distributed tangential springs. They also found that the shear stiffness of the laminated beams influences the specimen's compliance but not the energy release rate. The latter, however, is dependent on the shear modulus of the material through the elastic constant of the foundation.

As for models with rigidly connected sublaminates, this result is perfectly intuitive if we consider separately the deformed shapes of an ENF specimen due to bending and shear, respectively (Fig. 3).

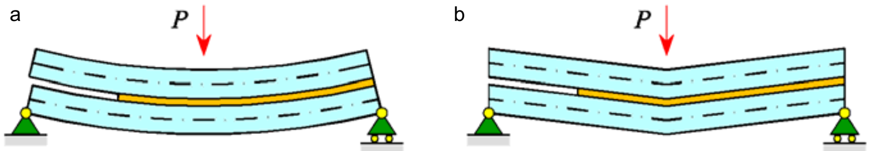

Fig. 3 – Model of the ENF specimen with elastic interfaces connecting sublaminates: (a) bending and (b) shear deformations.

Mixed mode fracture

Symmetrically delaminated laminates. When mixed-mode fracture is considered, the determination of the mode II contribution to the energy release rate is generally dependent on the method used to separate fracture modes. However, at least for symmetrically delaminated laminates, most mode-partitioning methods agree, since the contributions related to modes I and II can be obtained by considering, respectively, the symmetric and anti-symmetric parts of the loads acting on the laminate. This is the case of the mixed mode bending (MMB) test, which is here taken as a representative example (Fig. 4).

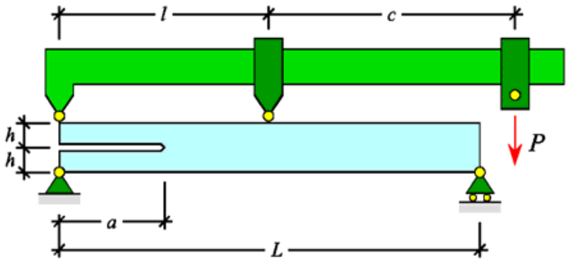

Fig. 4 – Scheme of the mixed mode bending (MMB) test.

The MMB test can be considered as the superposition of a DCB and an ENF tests. Therefore, all the above considerations concerning the ENF specimen can be immediately extended to the mode II contribution to the energy release rate of the MMB specimen.

The current Authors have recently developed an enhanced beam-theory (EBT) model of the MMB test [45], where the specimen is considered as an assemblage of two identical sublaminates, modelled as Timoshenko beams, partly connected by an elastic interface consisting of a continuous distribution of normal and tangential springs. Correction factors for the ERR with respect to the

SBT model, $\mu_{\mathrm{I}} = G^{\mathrm{MMB}}_{\mathrm{I,EBT}} / G^{\mathrm{MMB}}_{\mathrm{I,SBT}}$ and $\mu_{\mathrm{II}} = G^{\mathrm{MMB}}_{\mathrm{II,EBT}} / G^{\mathrm{MMB}}_{\mathrm{II,SBT}}$, have been obtained and are shown in Fig. 5 as functions of the elastic constants of the interface. Only the mode I contribution to the ERR is influenced by the sublaminate's shear stiffness, C_1. This finding confirms a general result valid for symmetrically delaminated laminates subjected to shear forces at the crack tip [14, 46, 47].

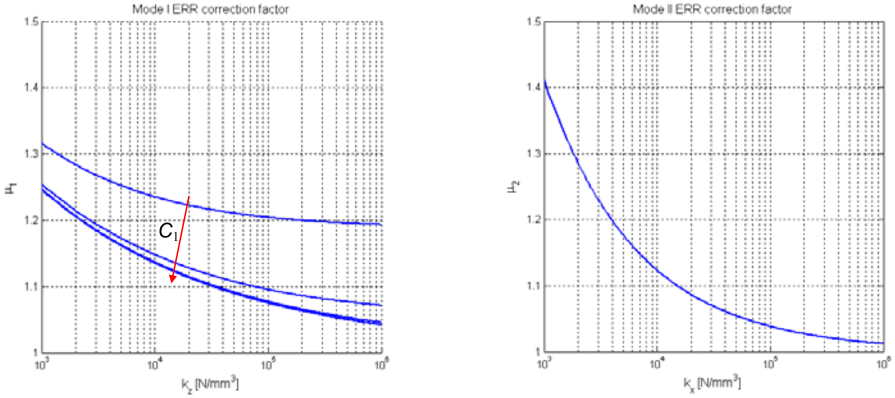

Fig. 5 – EBT model of the MMB test: mode I and II correction factors for the ERR.

Asymmetrically delaminated laminates. For asymmetrically delaminated laminates, the quantification of the mode II contribution to the ERR depends on the adopted mode-partitioning method, besides other modelling hypotheses. For instance, the asymmetric double cantilever beam (ADCB) test (Fig. 6), now unanimously recognised as a mixed mode test, has been the subject of debate in the years following its introduction, since according to the global method [11] it should be considered as a pure mode I test.

The current Authors have recently developed an enhanced beam-theory model of the ADCB test [19] and determined explicit expressions for the compliance, energy release rate, and mode-mixity angle. These results show that in general for asymmetrically delaminated laminates, both G_{I} and G_{II} depend on the shear stiffnesses, C_1 and C_2, of the sublaminates.

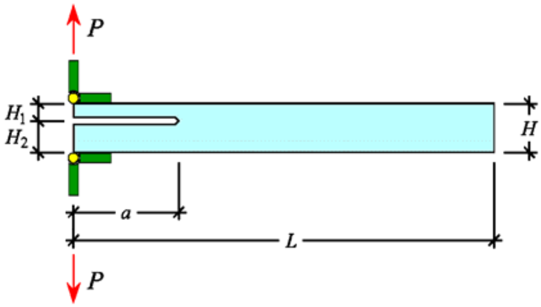

Fig. 6 – Scheme of the asymmetric double cantilever beam (ADCB) test.

Conclusions

Shear deformability, which can be relevant for composite materials, increases the compliance of laminated beams affected by delaminations and, consequently, may influence the energy release rate associated to delamination growth.

Pure mode II conditions have been examined taking the ENF specimen as a representative example. Several models of this test reported in literature have been reviewed. The simple beam-theory model neglects completely any shear deformability. The Timoshenko beam-theory model, considering shear deformability at first order, has proven to be unable to catch the influence of shear deformability on the energy release rate. Actually, in order to highlight this contribution, more complex models are needed, developed in the context of elasticity theory and with recourse to numerical solution methods. Alternatively, higher order beam theories or models introducing deformable interfaces between the sublaminates can be used. In particular, when elastic-brittle interface models are used, an dependence of G_{II} on the shear modulus of the material, G_{zx}, is found through the elastic constants of the interface, but still no direct dependence on the shear stiffness of the laminated beam, $C_1 = 5/6\ G_{zx}\ h$, is observed.

It may be interesting to notice that, as far as mixed mode conditions are concerned it is necessary to distinguish between symmetrically and asymmetrically delaminated laminates. In symmetrically delaminated laminates, such as the MMB specimen, fracture modes I and II are related, respectively, to the symmetric and anti-symmetric parts of the loads. As for pure mode II conditions, based on Timoshenko's theory there is no influence of shear deformability on the ERR, which instead may be caught by more complex models. On the contrary, in asymmetrically delaminated laminates, such as the ADCB specimen, generally both modal contributions to the ERR may be influenced by the shear deformability, even if only first order shear deformation beam theory is considered.

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