Crack kinking out of an interface, influence of the T-stress

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Abstract. Cotterell and Rice theory [1] on the kinking of a crack submitted to a biaxial loading in a homogeneous material is revisited. Using both an energetic and a stress fracture criteria [2] allows defining a positive threshold of the T-stress T_c below which no branching can occur [3] provided the inhomogeneities size is small compared to the Irwin length. The absence of such a threshold would definitely condemn experimental procedures like the double-cantilever beam (DCB) or the compact tension (CT) tests, which result in a positive T-stress at the crack tip.

The stress intensity factors K_I and T are computed using a contour integral. Calculations provide a very good agreement with the analytical results of the infinite Centrally Notched (CN) plate in tension for instance. An asymptotic analysis makes it possible to define the branching angle as a discontinuous function of T with a jump from 0° to some significant positive value as T reaches the threshold T_c. Furthermore, a similar analysis for non vanishing K_{II} shows that a positive T-stress increases the kinking angle due to K_{II} alone.

Introduction

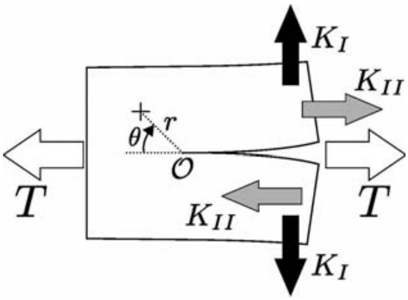

Fig. 1 The three modes and the associated stress intensity factors

Within the plane strain elasticity framework, in homogeneous materials, the displacement field in the vicinity of a crack tip can be described by the so-called Williams' series made of power terms (Eq. 1). The most significant terms (the second and third ones in Eq. 1) involve a real singularity exponent $\lambda = 0.5$ associated with two modes (a symmetric opening and an antisymmetric shear modes denoted respectively by the roman numbers I and II, see Fig. 1). They are well-known universal terms independent of the applied loads and the global geometry of the structure. On the contrary, the scaling coefficients K_i ($i = I, II$) named Stress Intensity Factors (SIF) depend on both the global geometry and the loading. The first non singular term called "T-stress" corresponds to a tension acting in a direction parallel to the crack (see Fig. 1). The related power is 1, the associated mode is denoted $\underline{t}$ and T holds for the corresponding intensity factor, i.e. the remote tension, thanks to an appropriate normalization of the mode $\underline{t}$ (see Eq. 4).

$$\underline{U}^0(r,\theta) = \underline{U}^0(O) + K_I \sqrt{r}\,\underline{u}_I(\theta) + K_{II} \sqrt{r}\,\underline{u}_{II}(\theta) + T\, r\underline{t}(\theta) + \ldots \tag{1}$$

where r and θ are the polar coordinates, with the crack tip O chosen as origin. The index 0 means that Eq. 1 describes the displacement field in the vicinity of the pre-crack tip prior to the initiation of any crack increment (i.e. on an unperturbed domain). The constant term $\underline{U}^0(O)$ is meaningless. It is the rigid translation of the origin (to ensure for consistency that the displacements at O do not a priori vanish). The dots refer to modes with higher order exponents ($3/2$, 2, $5/2$...), all of them being neglected.

Introducing two dimensionless mix mode parameters $m = K_{II}/K_I$ and $M(r) = T\sqrt{r}/K_I$, Eq. 1 can be rewritten as follows:

$$\underline{U}^0(r,\theta) = \underline{U}^0(O) + K_I \sqrt{r}\left(\underline{u}_I(\theta) + m\,\underline{u}_{II}(\theta) + M(r)\,\underline{t}(\theta) + \ldots\right). \tag{2}$$

The parameter m is the classical mix mode parameter (see for instance [4]). The other one, M, depends on r and is similar to the so-called "stress biaxiality ratio" B used by Leevers and Radon [5], the main difference lying in a factor $\sqrt{\pi}$. The r dependency is not completely surprising since it has already been met in interfacial cracks (Rice [6]) or at a V-notch under complex loading (Leguillon and Siruguet [7], Yosibash et al. [8]).
Note that $K_I = 0$ is a particular case which can be straightforwardly treated exchanging the roles of K_I and K_{II}.

Using the linear elastic constitutive law, one obtains the following expansion of the stress field

$$\underline{\underline{\sigma}}^0(r,\theta) = \frac{K_I}{\sqrt{r}}\left(\underline{\underline{s}}_I(\theta) + m\,\underline{\underline{s}}_{II}(\theta) + M(r)\,\underline{\underline{\tau}}(\theta) + \ldots\right), \tag{3}$$

bringing into evidence the singular behaviour in the vicinity of the crack tip. The functions $\underline{\underline{s}}_I(\theta)$, $\underline{\underline{s}}_{II}(\theta)$ and $\underline{\underline{\tau}}(\theta)$ obviously derive from Eq. 1 or 2 and the elastic constitutive law. From now on, we will use the following normalizations of the modes:

$$s_{I\theta\theta}(\theta=0) = 1\,;\, s_{IIr\theta}(\theta=0) = 1\,;\, \tau_{rr}(\theta=0) = 1. \tag{4}$$

Based on an asymptotic analysis of the stress field near the crack tip, the crack deflection is usually attributed to the shear mode II and the crack kinking angle α is a function of the mix mode parameter m. The prediction varies continuously from $0°$ (no deflection, $m = 0$, pure mode I) to a value between $70°$ and $78°$ (for pure mode II) depending on the selected criterion: maximum hoop stress, maximum energy release rate (a discussion can be found in Bui [4] or Leblond [9]) or local symmetry principle (Goldstein and Salganik [10], Amestoy and Leblond [11], Leblond and Mouro [12]).

In general the T-stress is invoked only for the further growth and the curvature of the new crack branch but is supposed to play no role at the very beginning of the process [11]. Moreover, experiments on crack kinking are generally carried out trying to avoid the spurious effect of the T-stress (Maccagno and Knott [13], Jernkvist [14]). Nevertheless, Cotterell and Rice [1] have carried out a stability analysis of the path of a crack under mode I loading ($K_{II} = 0$) concluding that the straight crack path is stable only if $T < 0$. In a recent paper Ayatollahi and Aliha [15] propose a criterion for crack branching which takes the T-stress into account. Nevertheless, it is based on a point-stress approach which involves a critical distance r_c, assumed to be a material parameter and which must be "reasonably" (*sic*) chosen, a major drawback. It leads to a slightly different conclusion: the crack kinking angle α is a continuous function of T for $T > 0$ and vanishes for $T < 0$. These results will be discussed further.

The aim of this paper is to extend Leguillon's criterion [2] in order to include the T-stress in the analysis. This criterion is a combination of an energy and a stress conditions which avoids any arbitrary choice of a critical length and has proved to work well to predict crack initiation at V-notches under symmetric [2] and complex loadings [7],[8] in homogeneous materials. Williams' expansion is used together with a matched asymptotic expansions procedure (see [8]) to derive the stress field and the energy release rate. The conclusion differs from the two aforementioned papers, the straight crack growth under mode I loading remains stable up to a strictly positive threshold $T_c > 0$, as experimentally shown in [3].

It is worth noting that many authors [16],[17],[18] emphasize on the difficulty to extract T from a finite element solution. They propose different procedures leading to quite large inaccuracies. Herein, the path-independent integral H [19],[20] is used, keeping in mind that T is nothing but a stress intensity factor associated to mode $\underline{t}$.

Finally, the authors would like to remind that all along the present paper, only the first step of crack initiation is considered, no matter the further crack growth and curvature.

The fracture criterion

Using a matched asymptotic analysis [8], it has been shown recently (Leguillon [2]) that the initiation of a crack at a v-notch can be accurately predicted using the simultaneous satisfaction of both a stress and an energy conditions. Furthermore, the proposed criterion coincides with Griffith's one for a pure crack.
As a consequence of exponents greater than $1/2$ in the Williams' expansion in the general case, this initiation process is shown to be unstable; the crack jumps a short length. The presence of the T-stress term leads to a similar reasoning which is carried out herein.

Energy condition. The first condition results from an energy balance between two states of the structure prior and after the onset of a short crack increment. It states that the incremental energy release rate $G = -\delta W_p / \delta S$ has to exceed the toughness G_c of the material, δW_p being the elastic potential energy change and δS the newly created crack surface. In plane elasticity, using Eq. 2 and 3 along with an asymptotic expansion with respect to the crack increment length ℓ ($\delta S = \ell \times d$, where d stands for the specimen thickness), the energy condition leads to

$$G = A_{11}(\alpha)K_I^2 + A_{12}(\alpha)K_I K_{II} + A_{22}(\alpha)K_{II}^2 + B_1(\alpha)K_I T\sqrt{\ell} + B_2(\alpha)K_{II} T\sqrt{\ell} + C(\alpha)T^2\ell + ... \geq G_c, \quad (5)$$

which rewrites using the previously defined dimensionless mix mode parameters m and M

$$G = K_I^2\,[A_{11}(\alpha) + A_{12}(\alpha)\,m + A_{22}(\alpha)\,m^2 + B_1(\alpha)\,M(\ell) + B_2(\alpha)\,mM(\ell) + C(\alpha)\,M^2(\ell) + ...] \geq G_c, \quad (6)$$

where A_{ij}, B_i and C are coefficients (MPa^{-1}) depending on the crack initiation angle α. They are defined using the two-scale matched asymptotics procedure (Leguillon [21]) and can be computed using the path-independent integral H as well (see Leguillon and Sanchez-Palencia [20]).

For simplicity, let us rename $X(\alpha, m, M(\ell))$ the term in brackets in Eq. 6. Due to the symmetry properties of the coefficients, in the cases of interest, $X(\alpha, m, M(\ell))$ is an increasing function of ℓ and the condition in Eq. 5 provides a lower bound for the admissible crack increments lengths (except if $\alpha = 0$ or $T = 0$). This is the reason why the crack jumps over a short distance $\ell > 0$ if it kinks due to the presence of the T-stress.

Stress condition. The second condition is based on the maximum tension that a material can bear before failure. It states that failure can occur only if the opening stress along the expected crack path (defined above by the angle α and the length ℓ corresponding to the lower bound provided by Eq. 5) exceeds the material strength σ_c. This criterion, once combined with Eq. 3, writes

$$K_I^2 (s_{I\theta\theta}(\alpha) + m\, s_{II\theta\theta}(\alpha) + M(\ell)\, \tau_{\theta\theta}(\alpha) + ...)^2 \geq \ell\, \sigma_c^2, \tag{7}$$

where the index $\theta\theta$ stands for the hoop component of the tensors $\underline{\underline{s}}_I$, $\underline{\underline{s}}_{II}$ and $\underline{\underline{\tau}}$. The above Eq. 7 provides an upper bound for the admissible crack increment lengths.

The mix criterion. The compatibility between inequalities Eq. 6 and 7 gives an equation for the crack initiation length ℓ_c as a function of α. As a first and essential consequence, this length cannot be considered as a material parameter:

$$\ell_c \frac{\sigma_c^2}{(s_I(\alpha) + m\, s_{II}(\alpha) + M(\ell_c)\tau(\alpha))^2} X(\alpha, m, M(\ell_c)) = G_c \tag{8}$$

Finally, either Eq. 6 or Eq. 7 gives a condition on K_I for crack initiation in the direction α:

$$K_I \geq K_{I\alpha} = \sqrt{\frac{G_c}{X(\alpha, m, M(\ell_c))}} \tag{9}$$

The critical value $K_{I\alpha}$ depends on α and the actual kink angle α_c maximizes the denominator, i.e. minimizes $K_{I\alpha}$ giving K_{If} (i.e. K_I at failure). This parameter must not be confused with the toughness K_{Ic}; it generalizes the K_I at failure introduced in [13], taking the T-stress into account. It relies on K_{Ic} using the Irwin relation between G_c and K_{Ic} (biased by $\sqrt{2\pi}$ due to the normalization in Eq. 4):

$$K_{If} = \sqrt{\frac{G_c}{X(\alpha_c, m, M(\ell_c))}} = K_{Ic}\sqrt{\frac{1-\nu^2}{E}\frac{2\pi}{X(\alpha_c, m, M(\ell_c))}}, \tag{10}$$

Moreover $K_{If} = K_{Ic}$ if $\alpha = 0$.

In case of a pure antisymmetric loading, $K_I = 0$ and m and M are ill-defined. However, the above procedure is simplified and can be reemployed by swapping the roles of K_{II} and K_I. The procedure includes the search for the crack initiation angle. It is worth noting that in this case, fracture occurs while $K_I = 0$. Another particular case is when $T = \sigma_c$: both K_I and K_{II} can be equal to 0 and nevertheless, a crack initiates.

Influence of the T-stress

Preliminary simulations allowed us to come up with the following conclusions as to our procedures. The first one is that no pollution is induced between modes I and t, i.e. loading a structure with pure mode I results in $T \approx 0$ and vice-versa. We also matched one of the results obtained by Leevers and Radon in [5] on an infinite CN (Centrally Notched) plate, that is $T = -p_y$ (see Fig. 2), as well as another one by Larsson and Carlsson [17] on a Compact Tension (CT) sample (see Fig. 3), that is positive T-stress at the V-notch tip. Finally, plotting the initiation angle α_c vs. mix-mode parameter m perfectly fitted the curve given by Maccagno and Knott [13].

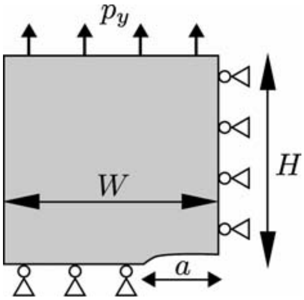

Fig. 2 Semi-infinite CN specimen

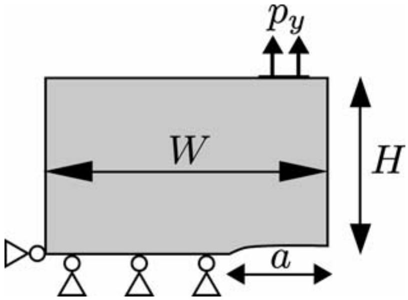

Fig. 3 CT specimen used for the computation of T

We will now focus on the influence of the T-stress assuming that $K_{II} = 0$ and then $m = 0$. Based on an assumption of perfect symmetry, the usual analysis leads to the conclusion that if K_I exceeds K_{Ic} the crack can grow but no branching can occur whatever T. On the other hand, it seems clear that if $T > 0$ is sufficiently high and approaches the tensile strength of the material, the crack will kink even for small values of K_I. It is shown in this section that under these conditions, $\alpha = 0$ can become an unstable crack direction naturally rejected because the hypothesis of perfect symmetry actually fails due to micro inhomogeneities and flaws along the crack path.

Note that without mode II, it is impossible to predict the sign of the kink angle (i.e. if the crack turns right or left); it is either positive or negative as a consequence of the already mentioned random distribution of micro imperfections, which also discards a double symmetric branching.

Computations are carried out on PMMA ($E = 3200$ MPa, $\nu = 0.3$, $G_c = 350$ J.m^{-2}, $\sigma_c = 75$ MPa [22]). Fig. 4 plots $K_{I\alpha}$ defined by Eq. 9 vs. the kink angle α, for a range of T / K_I ratios.

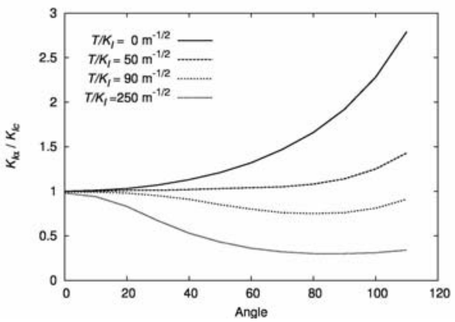

Fig. 4 Factor $K_{I\alpha}$ **vs.** the kink angle α

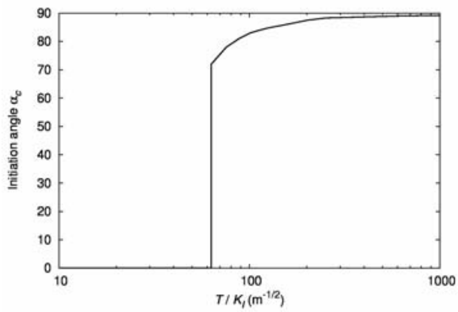

Fig. 5 Initiation angle α_c vs. $T > 0$

The initiation angle α_c is extracted from Fig. 4 through a comparison of $K_{Ic} = K_{If} = K_{I\alpha}$ for $\alpha = 0$ with $K_{I\alpha}$ (Eq. 9) at other angles. For a given T, the minimum value directly gives the actual initiation angle. The threshold T_c corresponds to the smallest value of T for which $\alpha_c \neq 0$.

As already mentioned, for symmetry reasons $\alpha = 0$ is always a solution from a theoretical viewpoint. It corresponds to a minimum of $K_{I\alpha}$ for small values of T and is therefore a stable direction. But, as the tensile stress increases, beyond the threshold T_c the location of the minimum of $K_{I\alpha}$ skips to a non zero value of α giving rise to a kinked solution. The direction $\alpha = 0$ remains a solution corresponding now to an extremum but is unlikely to occur. It is an unstable direction, any micro inhomogeneity or flaw being able to trigger the kink.

Obviously, the greater T, the more important the deflection. Fig. 5 illustrates this phenomenon: for T below its critical value T_c, $\alpha_c = 0$ is a stable solution (i.e. a global minimum); but as soon as the threshold is reached, the initiation angle jumps to a value around $\alpha_c \simeq 72°$ and then keeps increasing up to the asymptote $\alpha_c = 90°$ as T increases. In the limit, the influence of mode I becomes negligible and the criterion reduces to the stress condition, $K_I \simeq 0$, $T = \sigma_c$ and $\alpha_c = 90°$.

This result and the existence of the threshold are contradictory with the results of Cotterell and Rice [1]. These authors also invoke the inhomogeneities and flaws in order to assume that K_{II} cannot be exactly vanishing. Thus the crack slightly kinks (see Maccagno and Knott [13]) and depending on the sign of T kinks again or not. Finally, they draw the conclusion that the straight crack path is stable when $T < 0$ and unstable when $T > 0$.

It is difficult to find experiments making it possible to discriminate between the existence of a positive threshold or not. Nevertheless, a first answer is brought by the CT test by Larsson and Carlsson in [17] as previously mentioned, exhibiting a positive T at the crack tip without any crack kinking. Another one can be brought by the Double Cantilever Beam (DCB) test. Computations have been carried out on slender beams: height $h = 2$ cm and two lengths, respectively $L = 20$ cm and $L = 40$ cm (aspect ratio 1/10 and 1/20, material data are given at the beginning of the section). The coupons used have the same geometry as shown on Fig. 3 up to the aspect ratio. The crack length is one fourth of the full specimen length. The intensity factors K_I and T are extracted using the path-independent integral H [19],[20]. Results bring into evidence a non negligible positive T-stress at the crack tip, respectively $T = 5.3$ MPa and $T = 5.7$ MPa although far below the threshold $T_c = 52.9$ MPa. Following Cotterell and Rice, these results would definitely condemn these kinds of experiments, since a positive T-stress at the crack tip would irrevocably lead to an unstable crack deflection causing transverse failure of the specimens.

Our conclusion also differs from that of Ayatollahi and Aliha [15], who use the expansion in Eq. 3 and the maximum hoop stress criterion to derive an expression of the deflection angle α'_c. Nevertheless, it shows that a kink can occur for any small T, and exhibits a strong dependency with a critical distance r_c, which to the authors' mind is a difficult data to estimate a priori.

The present threshold T_c is very high ($T_c \simeq 0.7\sigma_c$) and by far larger than that experimentally determined by Selvarathinam and Goree [3] on CT coupons which seems to be surprisingly small. One reason is probably that, herein, there is an implicit assumption concerning the flaws and inhomogeneities size. It must be much smaller than the characteristic length ℓ_c (Eq. 8) which turns out to be around $\ell_c \simeq 70\mu m$ for $T = T_c$ in PMMA (in the present case it is around twice the Irwin length). Otherwise, if larger than a few micrometers, it would interact and influence the failure mechanism.

It has been observed that the ratio T_c/σ_c remains constant (around 0.7) for several materials. This is not really surprising since both mode I and T-stress fields are independent of the material elastic parameters (due to the normalization in Eq. 4) [4].

Simulations have also been carried out in order to determine the influence of a negative T-stress: they highlight the fact that straight propagation is stable in any case, this solution being a minimum whatever $T < 0$. This conclusion is obviously in agreement with that of Cotterell and Rice.

We also investigated the general mix mode case ($K_I > 0$, $K_{II} > 0$ and $T > 0$), and not surprisingly, the non vanishing K_{II} triggers a crack deflection without any threshold and this mechanism is enhanced by T (remember that K_{II} can be chosen positive without restriction to the general case, the only difference being the sign of the kink angle).

Conclusion

It has been shown that, even under a symmetric loading, the T-stress at a crack tip can induce a crack kinking as soon as it reaches a given positive threshold T_c which is strongly related to the strength σ_c of the material. No kinking can occur below the aforementioned critical value provided the inhomogeneities size is much smaller than the characteristic length involved in the model. Only large flaws can account for a lower threshold. Selvarathinam and Goree [3] explain that if a crack slightly kinks due to micro-inhomogeneities with T below the threshold T_c, it will instantaneously realign itself along the direction of the primary crack. This mechanism has been confirmed in wood specimens in [23], but is neglected here, the crack being assumed to grow straight in that case.

One particular situation has not been treated herein, the general mix-mode case with a compressive T-stress: $K_I > 0$, $K_{II} > 0$ and $T < 0$. Clearly, if the compressive T-stress is small and K_{II} large, the crack kinks and remains open. But if $T < 0$ decreases, the combination with K_{II} leads to a decreasing opening hoop stress of the kinked crack and finally this class of solution disappears. Depending on the intensity of the load, it can remain a straight propagation in mix mode. However, the toughness G_c corresponding to mode I propagation is no longer involved. One has to consider the toughness as a function of the mix mode parameter m: $G_c = G_c(m)$ [23][24].

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