A Consistent Anisotropic Brittle Damage Model Based on Growing Elliptical Microcracks

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1 Introduction

A micromechanical based continuum damage model based on the reduction of stiffness due to kinking elliptical microcracks is proposed to show the anisotropic irreversible process of damage accumulation due to microcrack kinking and growth in brittle and quasi-brittle materials subjected to non-proportional loadings under fatigue conditions. The model is formulated in a fully analytical way based on the concept of linear elastic fracture mechanics and the degradation of the elastic properties is associated with the irreversible process of crack kinking and growth. In order to make the formulation of the model mathematically traceable, the concept of an equivalent elliptical crack is proposed. The geometry and the orientation of the equivalent crack are resulting from the postulates of equivalent dissipation and equivalent damage induced anisotropy. The evolution of the cracks is governed by the criterion of maximum driving force coupled with a fatigue crack evolution law.

2 Formulation of the damage model

Applying the approach of micromechanics, the rate of the change of the compliance tensor for a volume element V of elastic material, attributable to the extension rate $\dot{s}$, through which a point on the perimeter of a single crack kinks to a new position is given by [1]

$$\dot{\mathbb{S}}_{ijmn}^{\text{Kink}} = \frac{\partial^2 \dot{\psi}^{**}}{\partial \sigma_{ij}\, \partial_{mn}} = \frac{1}{V} \frac{\partial^2 \left(G(s)\,\dot{s} \right)}{\partial \sigma_{ij}\, \partial \sigma_{mn}}, \tag{2.1}$$

where ψ^{**} is the energy dissipation due to crack kinking, and $G(s)$ is the driving force action at the propagated crack front, given by [2]

$$G(s) = \frac{1-\nu^2}{E} \left(K_I^2(s) + K_{II}^2(s) + \frac{1}{1-\nu} K_{III}^2(s) \right) = M_{\alpha\beta} K_\alpha(s) K_\beta(s), \tag{2.2}$$

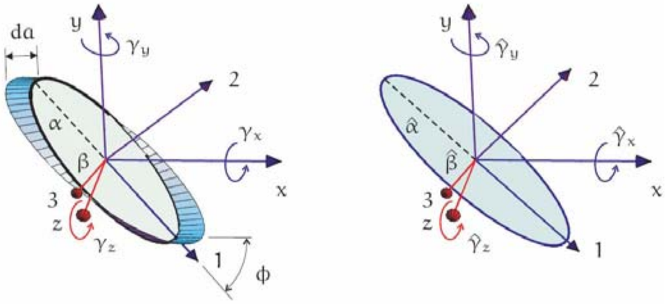

Figure 1: kinked crack and its equivalent elliptical crack

Integrating this along the crack perimeter, the rate of the change of compliance due to the growth of an internal crack is resulting. Considering the expansion of the stress intensity factors in terms of the extension length [3] ($K_\alpha(s) = K_\alpha^* + K_\alpha^{(1/2)}\sqrt{s} + O(s)$) and the crack tip parameters prior to kinking xd $\mathbb{S}$

$$\dot{\mathbb{S}}_{ijmn}^{\text{Kink}} = \frac{1}{V}\oint_c M_{\alpha\beta}\left(\frac{\partial K_\alpha(s)}{\partial\sigma_{ij}}\frac{\partial K_\alpha(s)}{\partial\sigma_{mn}}\dot{s}\right)\mathrm{d}l, \frac{\partial K_\alpha(s)}{\partial\sigma_{ij}} = F_{\alpha\lambda}(\phi)\frac{\partial K_\lambda}{\partial\sigma_{ij}} + G_{\alpha\lambda}(\phi)\frac{\partial T_\lambda}{\partial\sigma_{ij}}\sqrt{s} \tag{2.3}$$

where K_α and T_α are the stress intensity factors and the T-stresses of the crack prior to propagation, and $F_{\alpha\beta}(\phi)$ and $G_{\alpha\beta}(\phi)$ are universal functions in terms of the kinking angle ϕ. The local propagation rate $\dot{s}$, measured in the direction normal to the crack front at a considered point, can be calculated using a fatigue crack evolution law coupled with the selected fracture criterion [1].

After the kinking of the initial cracks, however, mathematical formulation of the next kinking steps is no longer possible. To overcome this difficulty, the concept of an equivalent elliptical crack may be considered. In this regard, a kinked crack is replaced with an equivalent elliptical crack, and the geometry and the orientation of the equivalent crack are resulting from the postulates of equivalent dissipation and equivalent damage induced anisotropy [4]. The proposed continuum damage model based on the reduction of stiffness due to kinking elliptical microcracks can be easily implemented in a finite element code.

Figure 2 shows the evolution of damage in a specimen under sequential cyclic loadings, which is in good agreement with the experimental results given in [5]. It should be noted that the proposed damage model shows a small mesh independency.

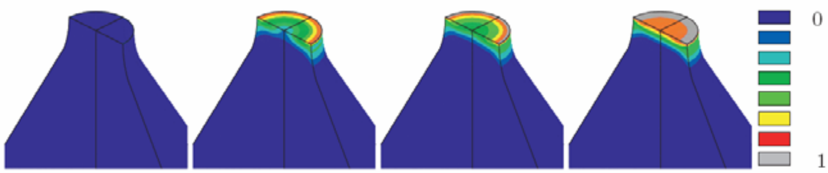

Figure 2: evolution of damage

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References

[1] Schütte, H. & K. Molla-Abbasi (2007). On the evolution of elastic symmetries of growing mixed-mode cracks. *8th International Conference on Multiaxial Fatigue and Fracture.*

[2] Le, K. C., H. Schütte & H. Stumpf (1999). Determination of the driving force acting on a kinked crack. Archive of Applied Mechanics **69** (1), 337-344.

[3] Leblond, J. B. (1993). Crack kinking and curving in three-dimensional elastic solids - application to the study of crack path stability in hydraulic fracturing. *Mixed-mode fatigue and fracture-ESIS 14 (Edited by H.P. Rossmanith and K.J. Miller), Mechanical Engineering Publications, London*, 219-243.

[4] Molla-Abbasi, K. (2008). *A Consistent Anisotropic Brittle Damage Model based on the Concept of Growing Elliptical Cracks*. Ph. D. thesis, Ruhr Universität Bochum, Bochum, Germany.

[5] Jeelani, S. & M. Musial (1986). A study of cumulative fatigue damage in ASIS 4130 steel. *Journal of Materials Science* **21**, 2109-2113.