Use of a Crack Box Technique for Crack Bifurcation in Ductile Material

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ABSTRACT

In this paper, an automatic Crack Box Technique (CBT) [1] is used to perform fine fracture mechanic calculations with iterative remeshings in elastic-plastic materials found in literature. The crack failure and bifurcation criteria used in this work are J-M^p based criteria [3]. Two types of experiments are analysed i.e. experiments carried out by Tohgo and Ishii [2] and those done by Li, Zhang and Recho [3].

The experiments carried out by Tohgo and Ishii are based on single-edge-cracked specimens subjected to bending moment and shearing force, on aluminium alloy 6061-T651 (hardening exponent $n \approx 7$). 2 three-point-bending and 3 four-point-bending specimens have been tested, from pure mode I to pure mode II . Experimental results give two critical fracture toughnesses (Critical J Integral) : J_{IC}=14N/mm and J_{IIC}=46N/mm. Furthermore they show that cracks follow two fracture types i.e. **T**-type (tensile type fracture) with Mode I predominant and **S**-type (Shear type fracture) with mode II predominant loading.

In reference [3], the static tests were conducted on 10mm thickness and 90 mm width CTS (Compact Tension Shear) specimens. A fatigue pre-crack was introduced up to $a/w \approx 0.5$. The specimens were tested under 0 to 90° loading with respect to the crack axis. The specimen is made of 7005 aluminum alloy.

Plasticity calculations using CBT are based on small strain assumption and were carried out using ABAQUS [4]. The two associated J-integrals J^*_I and J^*_{II} are computed accurately to determine the plastic mixity M^p parameter. It's transition value between **T** and **S** type fracture is obtained from the experimental results which give the critical mixity parameter M^p_c. Using ABAQUS [4], the calculated M^p is compared to M^p_c in order to obtain the bifurcation angle for each specimen.

The fracture type and the crack growth path predicted by using CBT are compared with the experimrntal results. It was shown that the numerical results of S-type crack growth are close to the experimental data. On the other hand, the T-type crack growth angles calculated numerically follow a different trend, with respect to the experimental results. This work shows that the proposed technique is an efficient tool to simulate the crack bifurcation under mixed mode loading in elastic-plastic materials.

1 INTRODUCTION

In industrial complex structures, the determination of the crack path is necessary to assess the fatigue life and the failure. Bifurcation criteria are also needed to predict the crack path. Their choice depends on the mechanical material characteristics and the loading levels.

The local stress field near a crack tip is determined by the use of asymptotic analysis. This allows to predict the critical loading level to crack propagation and to determine the crack bifurcation angle. However, this local asymptotic stress field presents a very high gradient, that's why a specific and regular finite element mesh is required in this zone.

Also during the crack propagation this mesh has to move with the crack tip and evolve as a function of the crack path to optimize the element number. Apart of crack failure and bifurcation criteria, two major problems remain, i.e. what are the mesh characteristics of the crack tip region and how to connect it to the overall structure?

The purpose of this work is to apply the automatic Crack Box Technique (CBT) [1] to the simulation of the crack growth under mixed mode loading in 2-D structures made from elastic-plastic materials. CBT is realized with the ABAQUS code [4]. Using this Technique, series of numerical calculations by Finite Element Method of the mixed mode crack growth are carried out. In order to compare the results of the proposed technique and those of existing methods, two examples of cracks subjected to different mixed mode loads are analysed. The crack growth paths are determined by using elastic and elastic-plastic bifurcation criteria. It is shown that the proposed technique is an efficient tool to simulate the crack bifurcation under mixed mode loading in elastic-plastic materials.

2 METHODOLOGY

In order to use the crack box technique one has to automatically create a transition zone between the “crack box” and the whole structural unchanged mesh (see figure 1) :

- Zone (A): Crack box (figure 2) : it contains a specific and regular mesh. It’s affected by the asymptotic solution at the crack tip. For elastic calculations, few elements are needed. The crack tip is modeled with degenerated quadratic elements with one side collapsed and midside nodes are moved to the quarter point nearest the crack tip to create a strain singularity in $r^{-0,5}$ (r is the distance from the crack tip). For plastic calculations, more elements are needed to precisely determine the J-integral. To introduce a r^{-1} singularity for perfectly plastic material strains, degenerated quadratic elements are also used but crack tip nodes are allowed to move independently and midside nodes remain at the midside point. For Ramberg-Osgood materials, the latter mesh allows to globally approximate a $r^{-n/n+1}$ strain field (n is the hardening coefficient). Quarter midside nodes can be used for low n values.

- Zone (B): Transition region. It contains an optimized linear (for elastic calculations) or quadratic (to increase precision for elastic and plastic calculations) triangular mesh obtained with the Delaunay triangulation procedure. This allows to connect our specific crack box with the whole ABAQUS [4] model, which can be a 2D plane strain or stress and a 3D shell model.

- Zone (C): Whole Model. It represents a usual finite element mesh. It’s to be noted that this mesh is unchanged during the crack propagation.

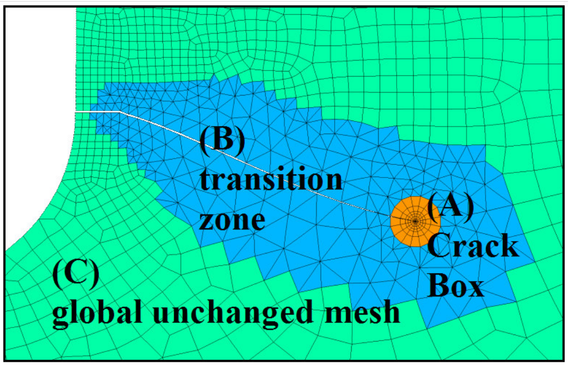

Figure 1: crack box in a structure (regions A, B and C)

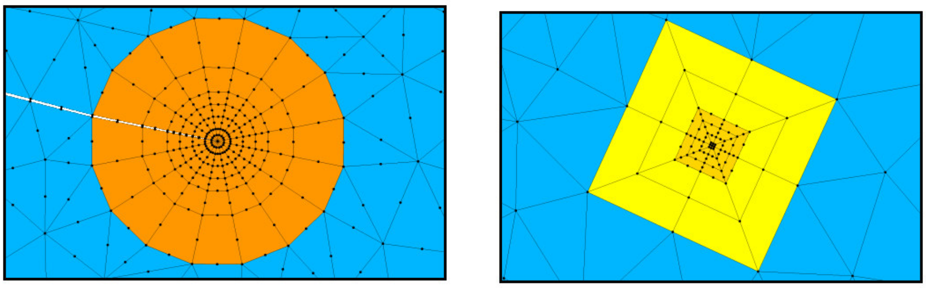

Figure 2: fine crack box (left) and coarse crack box (right)

The automatic crack box technique used in this paper is our own development using the ABAQUS code and consists in the following:

1. Meshing of the three regions for the initial crack
2. Performing FEM calculation associated to crack bifurcation criterion in order to determine the crack bifurcation angle.

3. Taking a crack growth increment in this direction
4. Updating of local crack tip region mesh and connecting it by the use of region (B) to whole structure.

Nota: The region (B) works such a moving contour around the crack tip. It looks like a static condensation of the structural behaviour to the crack tip region.

3 CRACK PROPOGATION AND BIFURCATION CRITERIA

In order to determine the crack growth path under mixed mode loading, one can use different criteria to calculate the bifurcation angle. For example, the maximum circumferential stress $\sigma_{\theta\theta max}$ criterion (Erdogan and Sih [5]), the maximum energy release rate criterion-MERR (Palasniswamy and Knauss [6]), the stationary strain energy density criterion (Sih [7]), the J_{II}=0 (Pawliska et al. [8]) and K_{II}=0 (Cotterell and Rice [9]) criteria (J_{II} is the value of the *J*-Integral corresponding to pure mode II and K_{II} is the value of the stress intensity factor corresponding to pure mode II), the crack tip opening displacement (or angle) criterion (Sutton et al. [10]), and so on. Recently Li et al. [11] have developed the J-M^p based criteria to assess the propagation of a crack in elastic-plastic material under mixed mode loading.

The global scheme of the bifurcation criteria used for elastic and elastic-plastic purposes is presented in figure 3.

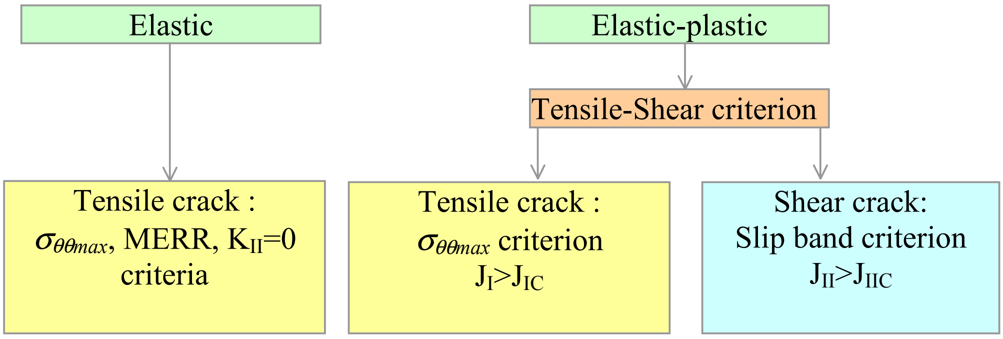

Figure 3: global scheme of bifurcation criteria

3.1 Elastic bifurcation criteria

In the case of a crack in elastic material, the $\sigma_{\theta\theta max}$ criterion is more often used. According to this criterion, the crack propagates always in the direction of the maximum circumferential stress. The bifurcation angle θ_0 can be determined after calculating the values of the stress intensity factors K_I and K_{II} :

$$tg\left(\frac{\theta_0}{2}\right)=\frac{1}{4}\left(\frac{K_I}{K_{II}}\right)\pm\frac{1}{4}\sqrt{\left(\frac{K_I}{K_{II}}\right)^2+8} \quad (1)$$

The numerical simulation of a crack growth is made in this work by using this criterion. Furthermore, the maximum energy release rate criterion (MERR) and the K_{II}=0 can also be considered.

3.2 Elastic-plastic bifurcation criteria

When a crack exists in an elastic-plastic material, the angle of crack growth depends on the competition between cleavage tensile fracture (T-type fracture) and ductile shearing fracture (S-type fracture), essentially depends on the plasticity progression. Recently, Li et al. have developed the J-M^p based criteria [3] in order to determine this crack growth angle. The main idea of the J-M^p based criterion is as follows:

In the case of a crack in an elastic-plastic material under mixed mode loading, Shih (1981) [11] showed that the stresses, strains and displacements fields near the crack tip are dominated by the HRR singularity, and can be characterized by two parameters, the *J*-integral and the mixity parameter M^p. M^p varies from zero to one. When M^p = 0, it is the case of pure mode II and when M^p = 1, it is the case of pure mode I. M^p is defined as follows:

$$M^p=\lim_{r\to 0}\frac{2}{\pi}\tan^{-1}\left|\frac{\sigma_{\theta\theta}(\theta=0)}{\sigma_{r\theta}(\theta=0)}\right| \quad (2)$$

Experimental studies shown that, for an elastic-plastic material, it exists a transition from T-type fracture to S-Type fracture. This transition is controlled by the critical value of the mixity parameter $M^p{}_c$ which can be determined by means of experiments according to the critical fracture toughnesses J_{IC} and J_{IIC} (J_{IC} is obtained from a pure mode I tensile test and J_{IIC} from a pure mode II shear test). If the mixity parameter M^p for a giving loading case is greater than $M^p{}_c$, the crack will propagate by T-type fracture. It means that it will propagate in the direction of the maximum circumferential stress $\sigma_{\theta\theta max}$. The $\sigma_{\theta\theta max}$ criterion can be used to determine the crack growth angle. On the other hand, If M^p is smaller than $M^p{}_c$, the crack will propagate by S-type fracture along one of slip bands. The crack growth angle can be determined according to the slip band criterion [3].

3.3 transition criterion

The near-tip field is completely determined by knowing the *J*-integral and the parameter M^p. This parameter combined with the RKR criterion by Ritchie et al. [12], gives the TS transition criterion as function of M^p.

In order to establish the T-S transition criterion for a plane strain crack under mixed mode loading, we require a physically reasonable criterion describing fracture mechanisms in materials. The criterion founded by Ritchie et al. [12], the so-called RKR criterion, is suitable for this purpose. According to this criterion, a tensile crack propagates if the maximum circumferential stress σ_{max} is larger than its critical value σ_c at a characteristic distance r_c immediately ahead of the crack tip. This criterion can be furthermore extended to shear type fracture. In this case, a crack will grow if the maximum shear stress τ_{max} reaches the critical value τ_c at the distance r_c:

$$\sigma_{max}(r = r_c) \geq \sigma_c \quad \text{and} \quad \tau_{max}(r = r_c) \geq \tau_c \tag{3}$$

From these two conditions, the TS transition criterion can be obtained: The tensile fracture takes place if the ratio $\sigma_{max} / \tau_{max}$ at the critical distance r_c from the crack tip is larger than the ratio σ_c / τ_c. Otherwise the shear type fracture occurs: i.e.

$$\frac{\sigma_{max}}{\tau_{max}} > \frac{\sigma_c}{\tau_c} \Rightarrow \text{Tensile crack} \quad \text{and} \quad \frac{\sigma_{max}}{\tau_{max}} < \frac{\sigma_c}{\tau_c} \Rightarrow \text{Shear crack} \tag{4}$$

Even though these RKR based criteria are physically reasonable, their direct application are not easy because the parameters σ_c, τ_c and r_c are difficult to identify. Therefore, we prefer to transform them into the *J-M*^{p} based criterion, which can be used more easily. For this, we need two material characteristic parameters, which can be obtained experimentally. Suppose that from a pure mode I tensile test and from a pure mode II shear test, the critical fracture toughnesses J_{IC} and J_{IIC} can be obtained. Shih [11] showed that, for the material following Ramberg-Osgood hardening rule, the asymptotic stress field at the crack tip is given by:

$$\sigma_{ij} = \sigma_0 \left(\frac{EJ}{\alpha \sigma_0^2 I_n r} \right)^{\frac{1}{n+1}} \tilde{\sigma}_{ij}\left(\theta, M^p\right) \tag{5}$$

where α is a material constant, *n* is the strain hardening coefficient, σ_0 is the yielding stress, I_n is a constant integral. The dimensionless functions $\tilde{\sigma}_{ij}$ depend only on θ and the near-field mixity parameter M^p.

For a given mixed mode, the ratios σ_c/τ_c and σ_{max}/τ_{max} at the distance r_c can be calculated according to the generalised HRR solution. A parameter, named λ, is defined in order to write equation (4) in terms of J_{IC} and J_{IIC}. Equation (4) becomes:

$$\begin{aligned} &\lambda > \lambda_c \quad \Rightarrow \quad \text{tensile crack} \\ &\lambda < \lambda_c \quad \Rightarrow \quad \text{shear crack} \quad \text{with} \quad \lambda_c = J_{Ic} / J_{IIc} \end{aligned} \tag{6}$$

4 APPLICATION ON SINGLE-EDGED-CRACKED BEAMS

In reference [2], experiments are based on single-edge-cracked specimens subjected to bending moment and shearing force, on aluminium alloy 6061-T651 (hardening exponent $n \approx 7$). 2 three-point-bending and 3 four-point-bending specimens have been tested, from pure mode I (beam A) to pure mode II (beam E) (see Table 1). Experimental results give two critical fracture toughnesses : J_{IC}=14N/mm and J_{IIC}=46N/mm. Furthermore they show that cracks in beam A to C seem to initiate for mode I predominant loading (T: tensile type fracture) whereas cracks in beam D and E initiate for mode II predominant loading (S: Shear type fracture).

Table 1 : mixed mode in elastic calculation

Beam	A	B	C	D	E
Elastic K_I/K_{II}	∞	3.3	2.	1.	0.

Figure 4 shows the numerical results of the crack path for different beams by using CBT according to the elastic bifurcation criteria (Eq. 1).

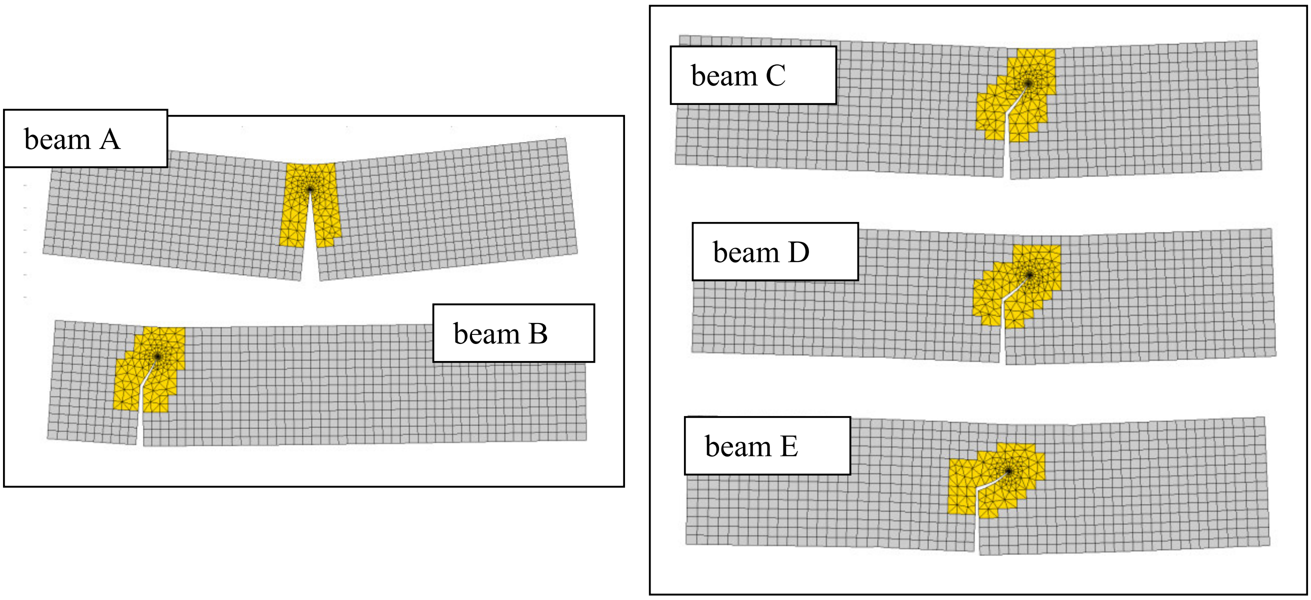

Figure 4 : Crack path for different beams : CBT-Elastic calculation

Plastic calculations using CBT are based on small strain assumption and were carried out using ABAQUS [4]. The two associated J-integrals J^*_I and J^*_{II} are then computed accurately to determine the plastic mixity parameter M^p. It's transition value between **T** and **S** type fracture is obtained from the experimental results which give the critical mixity parameter M^p_c = 0.75. Using ABAQUS, the calculated M^p is compared to M^p_c in order to obtain the bifurcation angle θ_0 for each beam.

The tensile or shear type fracture prediction for all five beams are presented in Table 2. M^p are given for two first steps of propagation to check that the crack keeps following either maximum circumferential stress criterion or slip band criterion. These predictions appear to be in good agreement with experimental observations.

Table 2 : M^P results and tensile/shear prediction

	Beam A	Beam B	Beam C	Beam D	Beam E
M^P 1st increment	0.98	0.84	0.77	0.62	0.04
M^P 2nd increment	0.99	0.98	0.99	0.03	0.03
bifurcation angle θ_0 (°)	0.	–32.1	-40.5	49.4	-0.1
T-S prediction	**T**	**T**	**T**	**S**	**S**
Experiments	T	T	T	S	S

From Table 2, it can be seen that the values of M^p are larger than the critical value (M^p_c= 0.75) for beams A, B and C. So according to the T-S transition criterion, we predict tensile fracture for these beams. On the other hand, the values of M^p for beams D and E are smaller than 0.75. Therefore, shear fracture shall occur. One can observe that the value of M^p tends rapidly to 0 (pure mode II) or to 1 (pure mode I). It means that the crack continue to grow in the same fracture type. These predictions agree with the experimental observations of Tohgo and Ishii.

Figures 5 and 6 show the numerical results of the crack path for different beams by using CBT according to J-M^p based criteria. One can observe obviously the difference of the crack path between the results of beams D and E obtained by elastic calculation (Fig. 4) and those obtained by plastic calculation (Fig. 6).

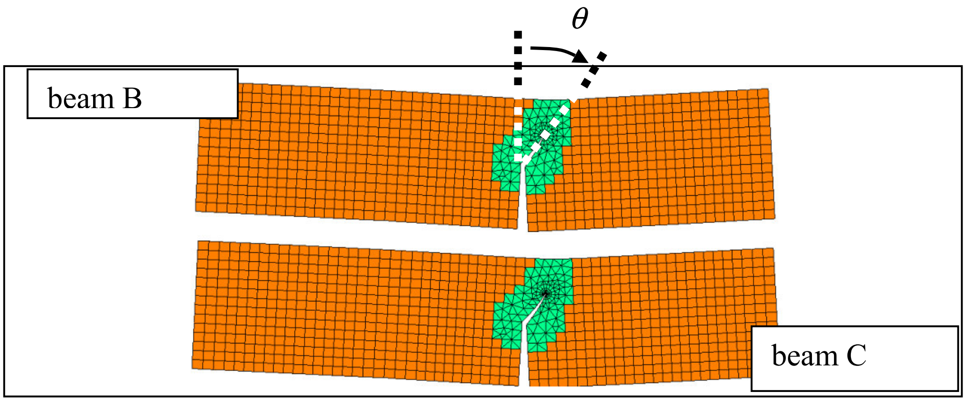

Figure 5 : Crack path for beams B and C (T-type fracture) - CBT-Elastic Plastic calculation with n=7

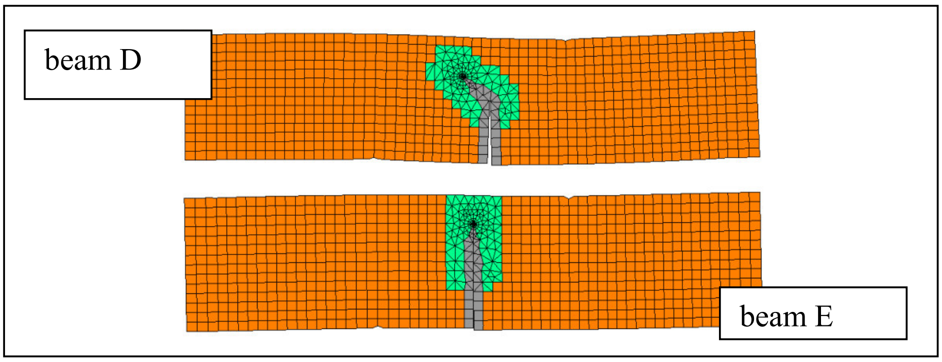

Figure 6 : Crack path for beams D and E (S-type fracture) - CBT-Elastic Plastic calculation with n=7

It is to be noted that the principal limitation of the use of J-M^p based criteria is the respect of the plane strain condition. However, this condition can be satisfied only for the case when the thickness of the specimen is big enough with respect to the crack length and to the size of the plastic zone. In fact, once the crack grows, it is difficult to respect this condition. Especially, the plastic zone near the crack tip increases as the crack length grows. The plane strain condition is true only in the middle of the specimen but at the surface of the specimen the plane stress condition dominates.

5 APPLICATION ON CTS (COMPACT TENSION SHEAR) SPECIMENS

Another example is issued from Ma et al.'s experimental work [14], using a specific cracked specimen so called the compact-tension-shear (CTS) specimen developed by Richard and Benitz [15]. The location of loading holes provides a range of loads which results in a full spectrum of mode mixities. The test specimens were made from 7005 aluminium plates, with a hardening coefficient about n=7. The critical mixity parameter M^p_c is about 0.75. A fatigue crack was introduced up to $a_0 / w \approx 0.5$, where a_0 is pre-cracked length, w is specimen width. The specimen thickness is 10 mm. The loading is applied in different directions with respect to the crack axis. The experimental results of the crack growth path under 60° and 30° loading are presented in Figure 7. It has been observed that the initial crack growth angle are respectively about 40° and 50° to the crack axis and then the cracks seem to follow slip bands in the directions of -110° and -90°.

Figure 8 shows the numerical results of crack growth paths obtained by CBT according to different criteria.

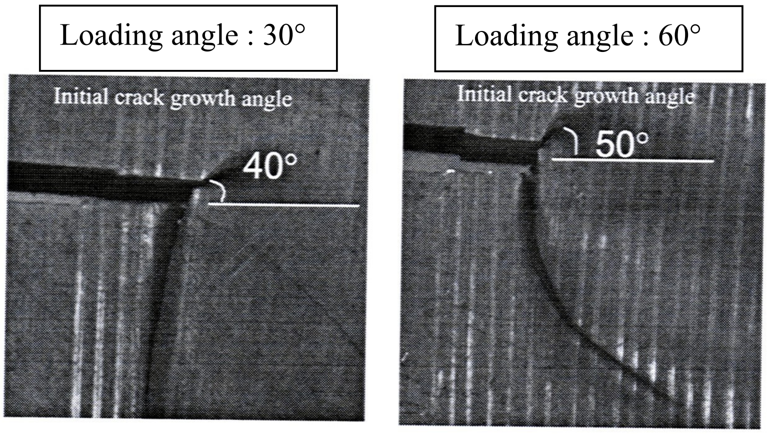

Figure 7 : Experimental results of CTS specimen under 30° and 60°loading

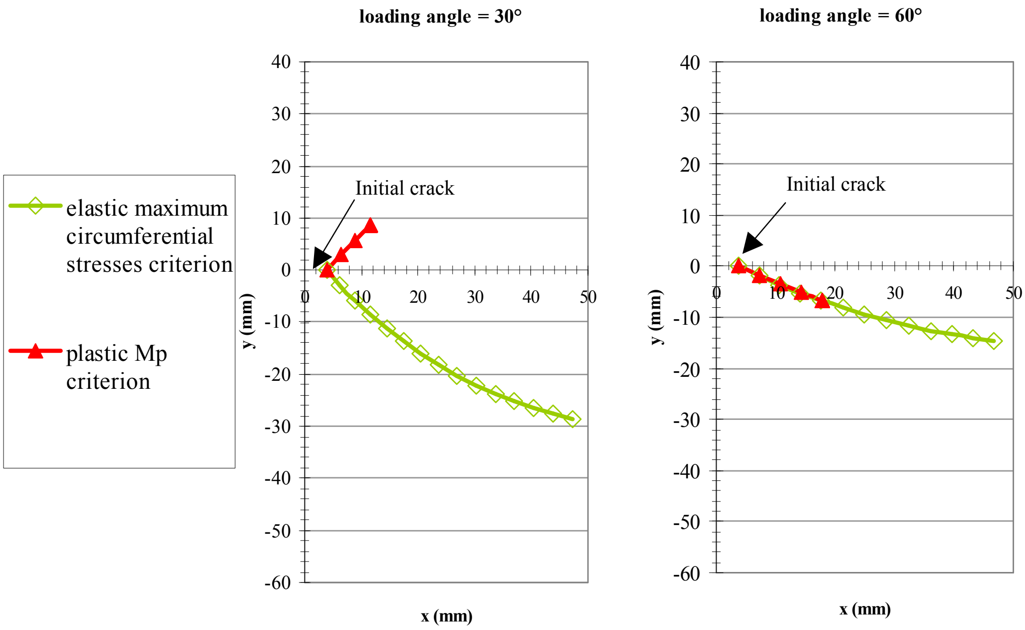

Figure 8 Numerical simulation of crack growth path by CBT

The comparison between the numerical and experimental results is as follows:

(1) When the loading angle is 30°, the value of M^p calculated is about 0.63 which is smaller than M^p_c. According to *J*-M^p criterion, the fracture type is shear type. The numerical results of the initial crack growth angle predicted by *J*-M^p criterion in the first steps follow the experimental results (The initial crack growth angle is about 40°). The experimental observation shows that the crack follows another slip band (about -110°) after the initial growth.

(2) When the loading angle is 60°, the expected initial crack growth angle corresponding to the tensile-type is about –32° (M^p = 0.84). The experimental observation shows an initial crack growth angle of about 50°. This is in disagreement with the expected one. Nevertheless, the initial crack growth angle seems to follow a shear band (shear type fracture) but not the cleavage (tensile type) direction. It has been noted obviously that, after the initial growth, the crack grows along the border of the plastic zone associated to a necking effect. The effect of plastic zone on crack growth is significant. In this case, the plane strain condition is totally not satisfied because of great deformation. So *J*-M^p criterion can not be used in this case.

Other numerical simulations under plane stress condition are carried out on the same specimen. Figure 9 shows the plastic zones near the crack tip when the crack begins to grow. It can be clearly seen that the crack growth directions are those of the plastic bands.

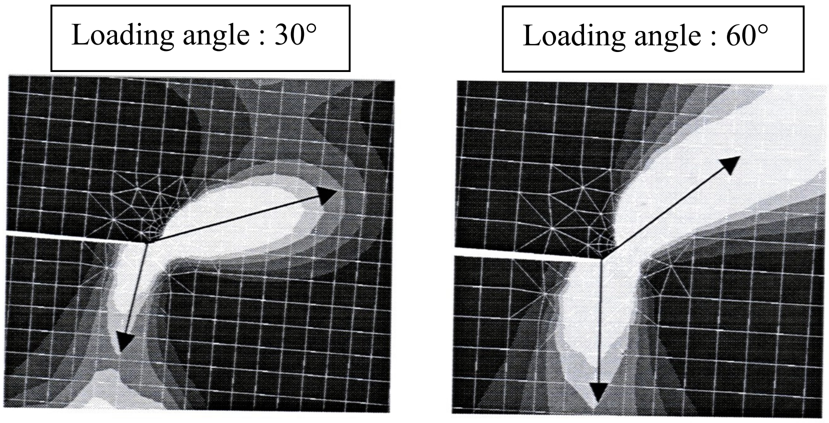

Figure 9 : Numerical results of plastic zones near the crack tip

6 CONCLUSIONS

The automatic Crack Box Technique (CBT) is applied to simulate the crack growth under mixed mode loading in 2-D structures made from elastic-plastic materials. Using this technique, the simulation of crack growth under mixed mode can be made easily. The crack growth paths determined by using different elastic and elastic-plastic bifurcation criteria are compared. It was shown that the results obtained according to elastic-plastic bifurcation criteria approach better to the experimental results. However, J-M_p criterion can not be used in the case when the plane strain condition is not satisfied. This work shows that the proposed technique is an efficient tool to simulate the crack bifurcation under mixed mode loading in elastic-plastic materials. Nevertheless more experiments are still needed to improve the efficiency of this technique in elastic-plastic materials.

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