THE THEORY OF CRITICAL DISTANCES: APPLICATIONS IN FATIGUE

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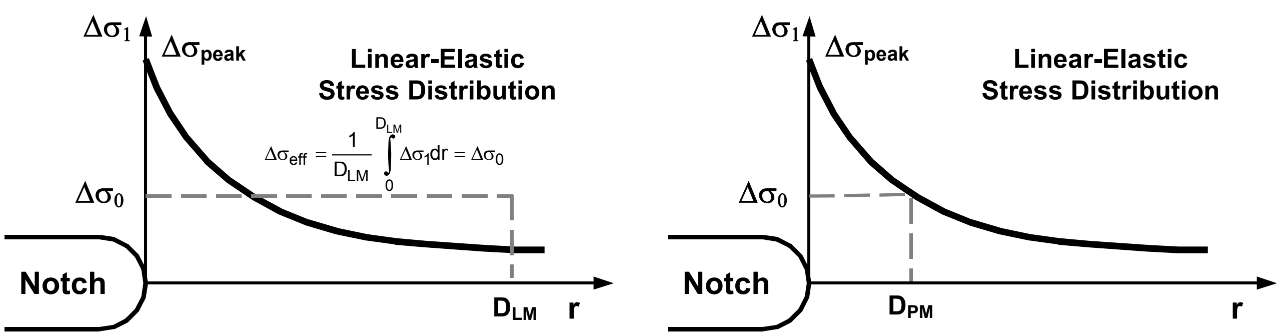
ABSTRACT

This paper attempts to review the most interesting findings in the use of the Theory of Critical Distances (TCD) to predict fatigue strength of notched mechanical components. Initially, the most modern formalizations of the TCD are considered, showing their peculiarities and differences. An ad-hoc section is then focused on the multiaxial high-cycle fatigue problem, considering all the open questions arising in the presence of complex stress fields damaging the fatigue process zone in the vicinity of the stress concentrator apex. Subsequently, the physical idea on the structural volume concept is briefly investigated showing some peculiar results generated in the high-cycle fatigue regime under both uniaxial and biaxial fatigue loading. Finally, our idea to extend the use of the TCD down to the low-medium cycle fatigue regime is briefly explained.
Working in collaboration with Prof. David Taylor, we have spent the last five years investigating this theory both to better understand its physical meaning and to systematically check its accuracy in predicting notched fatigue strength under different loading conditions. After so much work done in this area we feel so confident to proudly and loudly say that the TCD is a powerful engineering tool suitable for assessing real mechanical components in situations of practical interest. Moreover, the fact that the TCD formalization we believe in is based on the use of linear-elastic stresses suggests that our theory can successfully be used to post-process simple linear-elastic Finite Element (FE) Models reducing time and costs of the design process.

Introduction

As far as the author is aware, the first attempt of using the TCD to predict fatigue strength of notched mechanical components was made by Neuber [1, 2] in Germany at the beginning of the last century. In order to formalise this theory he took as starting point the idea that the elastic-stress in the vicinity of stress risers does not reach those values predicted according to the continuum mechanics theory: in order to calculate an engineering quantity representative of the real stress damaging the process zone, the stress close to the stress concentrator apex has to be averaged over materials units (that is, crystals or structural particles). In other words, Neuber suggested calculating a reference stress to be used for assessing real components considering finite volumes and not infinitesimal volumes as postulated by the classical theory. From a practical point of view, Neuber formalised this idea in terms of the so-called Line Method (LM) making this theory suitable for predicting high-cycle fatigue strength of components weakened by different geometrical features. In particular, he suggested that a real component is in its fatigue limit condition when the stress averaged over a line of length equal to the crystal size equals the plain fatigue limit, $\Delta\sigma_0$, of the material (Fig. 1a).



(a) (b)

Figure 1. Formalisation of the Line Method (a) and of the Point Method (b).

A few years later, Peterson [3] suggested a simplified way of applying Neuber's idea: he observed that the reference stress to compare to the material plain fatigue limit could simply be calculated at a given distance from the apex of the stress raiser (Fig. 1b). This method is known as the Point Method (PM). In Fig. 1, we denoted the critical distance to apply the LM as D_{LM} and the one to apply the PM as D_{PM}. In the next sections the different methods of calculating these quantities will be more deeply investigated and discussed: Figures 1a and 1b just schematise the way the LM and the PM work when used to predict the high-cycle fatigue strength of notched components.
To conclude this section, it is worth noticing that the TCD can be formalised also averaging the stress either over an area or over a volume: these two methods are called the Area Method (AM) and the Volume Method (VM), respectively, and, again, the reference length defining the size of the integration domains is a material property.

The TCD to predict high-cycle fatigue strength of notched components

The TCD applied to fatigue problems assumes that fatigue damage depends on the stress field distribution in the vicinity of the stress concentrator. In other words, it assumes that fatigue damage can correctly be estimated only if the entire stress field damaging the fatigue process zone is correctly accounted for.
According to the TCD, notched components are in their fatigue limit condition when the effective stress, $\Delta\sigma_{eff}$, which depends on the entire stress field distribution ahead of the tip of the stress concentrator, equals the material plain fatigue limit, $\Delta\sigma_0$:

$$\Delta\sigma_{eff} = \Delta\sigma_0 \quad (1)$$

As briefly said above, the PM was first formalised by Peterson [3], whereas the LM was proposed by Neuber [1,2]. Both of them were subsequently revisited by Tanaka [4], Lazzarin et al. [5] and Taylor [6]. It is interesting to observe that all these researchers independently developed their theories coming to similar conclusions.
Initially, they defined the critical distance using both classic and linear elastic fracture mechanics (LEFM) arguments. According to the linear-elastic TCD, the material characteristic length, L, turns out to be:

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_0}\right)^2 \quad (2)$$

where ΔK_{th} is the range of the threshold value of the stress intensity factor and $\Delta\sigma_0$ is the plain fatigue limit (both determined under the same load ratio, R, as the one damaging the mechanical component to be assessed). Observing that the material characteristic length, L, depends on two material properties, it is evident that L is in turn a material property which is different for different materials and different load ratios. Table 1 summarises the L values for four materials tested under different load ratios: this table should better highlight the evident influence of R on L

Material	σ_{UTS} [MPa]	σ_Y [MPa]	**R**	ΔK_{th} [MPa m$^{1/2}$]	$\Delta\sigma_0$ [MPa]	**L** [mm]	**Ref.**
SM41B	423	194	-1	12.4	326	0.458	[7]
			0	8.4	274	0.299	
			0.4	6.4	244	0.218	
SAE 1045	720	390	-1	9	606	0.070	[8]
			0	6.9	448	0.076	
Grey Cast Iron	249	202	-1	15.9	160	3.150	[9]
			0.1	11.2	99	4.070	
			0.5	8	68	4.410	
			0.7	5.2	48	3.740	
2024-T351	-	360	-1	4	248	0.100	[8]
			0	4.4	172	0.172	

Table 1. Influence of the load ratio R on the values of the material characteristic length L.

All the formalisations of the TCD mentioned above postulate that, according to the LM, a component is in the fatigue limit condition when the following condition is assured (see Fig. 2 for the adopted symbolism):

$$\Delta\sigma_{eff} = \frac{1}{2L}\int_0^{2L}\Delta\sigma_1(\theta = 0, r)dr = \Delta\sigma_0 \quad (3)$$

where, in the above identity, $\Delta\sigma_1$ is the range of the maximum principal stress.

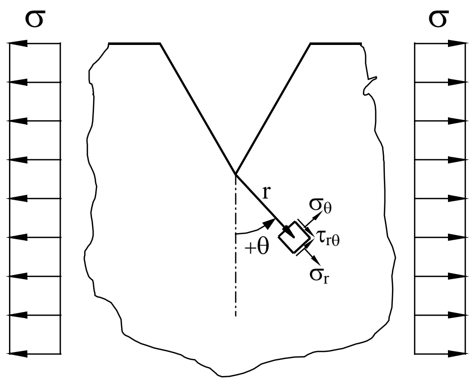


Figure 2. Notched specimen subjected to a remote uniaxial fatigue loading and frame of reference adopted to describe the linear-elastic stress field ahead of the notch tip.

The way the LM works, as expressed by Eq. (3), is summarised in Fig. 1a, where D_{LM}=2L.

Tanaka [4] and Taylor [6] suggested formalising the PM as follows:

$$\Delta\sigma_{eff} = \Delta\sigma_1\left(\theta = 0, r = \frac{L}{2}\right) = \Delta\sigma_0 \quad (4)$$

in other words, according to the PM, a notched component is in its fatigue limit condition when the range of the maximum stress at a distance from the notch tip of L/2 equals the plain fatigue limit (Fig. 1b where D_{PM}=L/2).

On the contrary, Lazzarin, Tovo and Meneghetti [5] argued that, in order to correctly apply the PM, the range of the maximum principal stress at the point having coordinates (θ=0, r=L) must be corrected by using an adimensional function depending on both L and the notch root radius, r_n. In more detail, they formulated the PM as follows:

$$\Delta\sigma_{eff} = \Delta\sigma_1(\theta = 0, r = L)\frac{1+\sqrt{2}\,L/r_n}{1+L/r_n} \quad (5)$$

The peculiarities related to these different ways of formalising the PM were subsequently discussed by Atzori, Lazzarin and Filippi in a exhaustive Technical Note [10].

Finally, Taylor [6] observed that the range of the effective stress could be calculated also by averaging the range of the maximum principal stress over a semicircular area, formalising the so-called area method (AM):

$$\Delta\sigma_{eff} = \frac{4}{\pi L}\int_{-\pi/2}^{\pi/2}\int_0^{L}\Delta\sigma_1(\theta, r)drd\theta \cong \Delta\sigma_0 \quad (6)$$

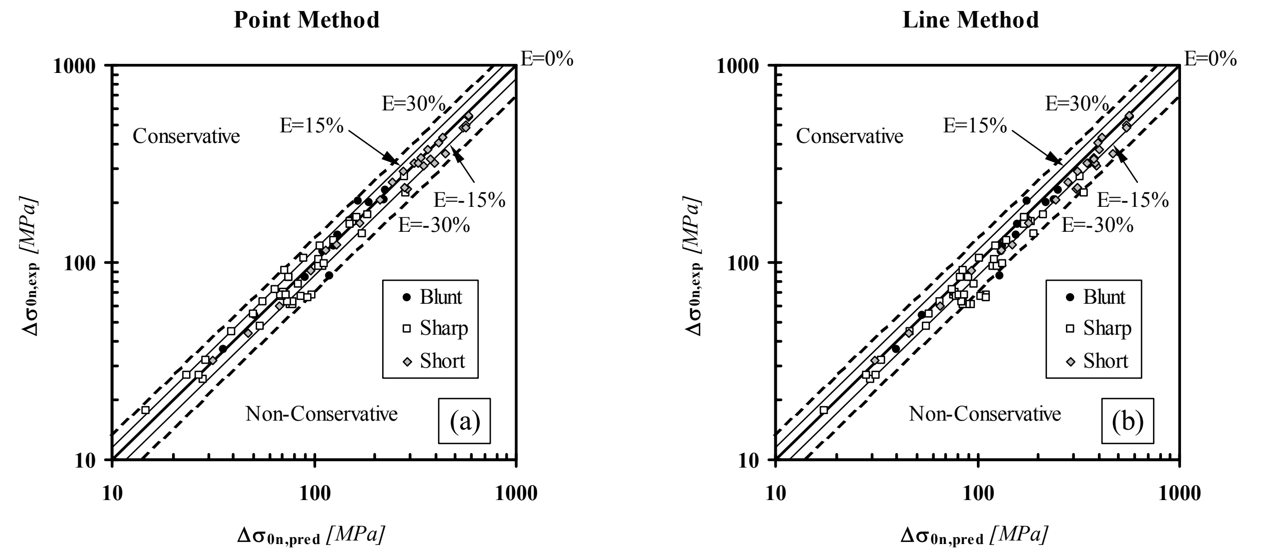


Figure 3. PM and LM accuracy in predicting high-cycle fatigue strength of standard notches - PM applied according to Eq. (4) [12].

We extensively checked the accuracy of the above formalisations of the TCD both considering standard notches [11, 12] and real components [13]: this systematic investigation highlighted that the TCD is successful in estimating high-cycle fatigue strength of notched components, giving predictions falling within an error interval of about 20% (and it holds true independently of both material and geometrical feature weakening the considered component). Figure 3 summarises the results we obtained by applying the PM, Eq. (4), and the LM, Eq. (3), to fatigue results generated by testing notched specimens weakened by different types of notches (blunt, sharp and short) [12] (In Fig. 3 $\Delta\sigma_{0n,pred}$ and $\Delta\sigma_{0n,exp}$.are the predicted and the experimental value of the nominal notch fatigue limit calculated with respect to the net section).

It is interesting to observe that recently Wiersma and Taylor [14] successfully applied the TCD to predict the high-cycle fatigue strength of micro- notched specimens of AISI 316L having net width of about 100μm (that is, of about 10 grains). In particular, they observed that the micro-dimensions of the tested specimens resulted in a decrease of the threshold value of the stress intensity factor. This experimental evidence was ascribed to two different causes: the lack of crack closure and the reduced length of non-propagating cracks. The author's opinion is that these results are extremely interesting and innovative especially in light of the fact that a high accuracy level was obtained by using the average grain size to define the material characteristic length, L: in this case the existing link between the TCD and the physical reality is evident.

Lastly, it is worth noticing that the TCD was also employed to predict the high-cycle fatigue strength of welded components, obtaining very accurate predictions [15].

The main advantage of the above formalisations of the TCD is that these methods are based on the use of linear-elastic stresses: this makes them suitable for being used in situations of practical interest by post-processing linear-elastic FE analysis, reducing time and costs of the design process.

It is important to highlight that there exist other formulations of the TCD based on the assumption that stress fields in the vicinity of stress raisers have to be determined by considering the real elasto-plastic material behaviour. For instance, Pluvinage and co-workers [16] proposed to calculate the effective stress according to the following relationship:

$$\Delta\sigma_{eff} = \frac{1}{L_{eff}} \int_0^{L_{eff}} \Delta\sigma_1(\theta = 0, r) \cdot [1 - \chi \cdot r] dr = \Delta\sigma_0 \quad (7)$$

where χ is the stress gradient of the elasto-plastic stress field [16]:

$$\chi = \frac{1}{\sigma_1(\theta = 0, r)} \cdot \frac{d\sigma_1(\theta = 0, r)}{dr} \quad (8)$$

It is evident that Eq. (7) is just a particular formalisation of the LM. In more detail, Eq. (7) defines the effective stress using a weighted average of the elasto-plastic stress field damaging the fatigue process zone over a line having length equal to L_{eff}, which is defined as the distance, from the notch tip, at which the stress gradient, χ, reaches its first relative minimum. This makes it evident that the length of the integration path, L_{eff}, is assumed to be dependent on the geometry of the notch weakening the considered component.

We never extensively investigated the accuracy of Pluvinage's LM as done for the TCD formalisations reviewed above. In any case, whenever we tried to apply Eq. (7) to notched components we obtained, in general, good results, even though characterised by a lower precision level than the one obtained by the simpler linear-elastic LM. In any case, we believe that this method is extremely appealing from a philosophical point of view, because based on the assumption that plasticity plays a fundamental role in damaging the fatigue process zone (which is true!). Unfortunately, correctly modelling plasticity is a complicated task, which requires a lot of experience (especially when addressed by using numerical methods). Probably, this is the reason why elasto-plastic approaches are never well thought of by engineers engaged in assessing real components: it is believed that such methods increase time and cost of the design process (which is probably not true!).

On the use of TCD to predict fatigue limits of notched components subjected to multiaxial fatigue loading

In light of the good accuracy shown by the TCD when employed to predict uniaxial fatigue limits of notched components, in the recent past, we tried to extend the use of such a method to multiaxial fatigue situations. In particular, due to the complexity of the stress fields (complexity that can be increased more and more by the presence of non-proportional loading) we initially attempted to verify if the critical distance could be assumed constant independently of the type of loading damaging the component. By using a large amount of data taken from the literature, we systematically compared the critical distance value under torsion to its value under uniaxial fatigue loading [17]. This investigation allowed us to come to two interesting conclusions: (i) from a scientific point of view, L under Mode I loading is different from its value under Mode III torsion; (ii) from an engineering point of view, assuming L under torsion equal to its value under uniaxial fatigue loading results in predictions characterised by a maximum error of about 25%. Thanks to the latter conclusion we could greatly simply the problem assuming that to address real components in situations of practical interest, L can be assumed constant (independently of the degree of multiaxiality of the stress field damaging the fatigue process zone) and equal to its value generated under uniaxial fatigue loading.

Using the above outcome, we attempted to extend the use of the TCD to multiaxial fatigue situations taking as starting point a very simple question: "What is the most important stage to be modelled in the presence of multiaxial fatigue loading?". To answer this question we investigated two different strategies, trying to re-formulate the TCD in order to predict either the formation of a Stage 1 crack or of a Stage 2 crack [18]. It is useful to remember here that according to Miller's schematisation [19], Stage 1 is mainly Mode II governed, whereas Stage 2 is Mode I dominated.

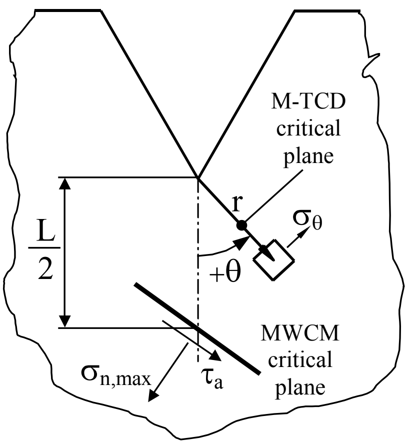


Figure 4. Different strategies to apply the TCD to multiaxial fatigue situations.

Assuming that Stage 1 is the most important stage when attempting to predict multiaxial notch fatigue limits, we argued that a critical plane approach had to be used. In particular, we re-interpreted the Modified Wöhler Curve Method (MWCM) [20, 21] in terms of the TCD. In order to apply such a procedure to a real component, initially the linear elastic stress state has to be determined at a distance from the assumed crack initiation point equal to L/2. This point is positioned along a line emanating from the hot spot and perpendicular to the component surface (Fig. 4). A mechanical component then is in its fatigue limit condition when [20, 22]:

$$\tau_a + \left(\tau_0 - \frac{\sigma_0}{2}\right)\frac{\sigma_{n,max}}{\tau_a} \leq \tau_0 \tag{9}$$

where τ_a is the shear stress amplitude relative to the plane of maximum shear stress amplitude, $\sigma_{n,max}$ is the maximum stress perpendicular to such a plane and, finally, σ_0 and τ_0 are the amplitudes of the fully-reversed plain fatigue limit under uniaxial and under torsional fatigue loading, respectively.

On the contrary, if Stage 2 is assumed to be the most important aspect to account for to correctly predict multiaxial notch fatigue limits, then it is logical to assume that the critical plane is the one emanating from the hot spot and experiencing the maximum Mode I loading (Fig. 4).

If the orientation of such a plane is defined by the angle $\theta=\theta^*$ (see Fig. 4 for the symbolism), then the Multiaxial Theory of Critical Distances (M-TCD) can be formalised in terms of PM and LM, respectively, as follows:

(Multiaxial-Point Method, M-PM)
$$\Delta\sigma_{eff} = \Delta\sigma_\theta\left(\theta = \theta^*, r = \frac{L}{2}\right) = \Delta\sigma_0 \tag{10}$$

(Multiaxial-Line Method, M-LM)
$$\Delta\sigma_{eff} = \frac{1}{2L}\int_0^{2L}\Delta\sigma_\theta(\theta = \theta^*, r)dr = \Delta\sigma_0 \tag{11}$$

We checked the accuracy of the two above strategies using experimental results generated testing sharply V-notched specimens subjected to in-phase Mode I and II loading [18]: both methods were seen to be extremely accurate, giving predictions falling within an error interval of about ±15%.

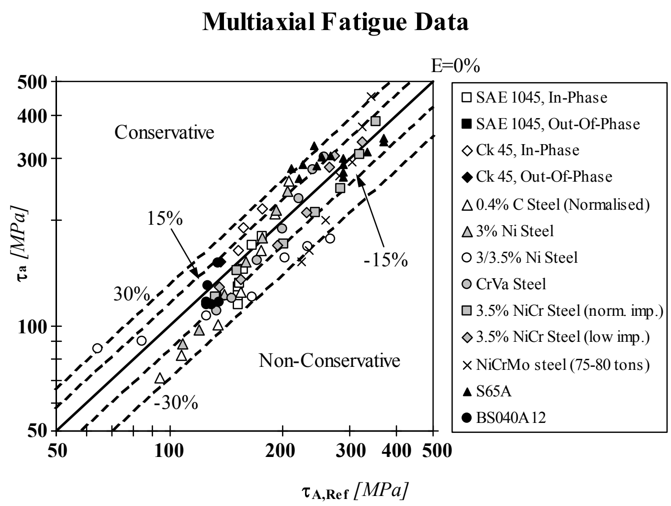


Figure 5. Accuracy of the MWCM reinterpreted in terms of the TCD in predicting multiaxial notch fatigue limits [22].

Subsequently, we more deeply investigated the accuracy of the MWCM re-interpreted in terms of the TCD in predicting multiaxial notch fatigue limits. We considered data generated by testing specimens weakened by different geometrical features, made of different materials and subjected to in-phase and out-of-phase biaxial fatigue loading [22]. This systematic investigation showed that our method is definitely promising, giving predictions falling within the typical error interval pertinent to the TCD (Fig. 5). It is worth noticing that, in Ref. [22] both the material fatigue properties and the material characteristic length were determined under fully-reversed loading, but our method was systematically applied to predict multiaxial fatigue strength of notched specimens in the presence of non-zero mean stresses as well as non-zero out-of-phase angles: the sound agreement between predictions and estimations seems to strongly support the idea that Eq. (9) is capable of correctly estimating fatigue strength independently of the complexity of the stress field damaging the fatigue process zone.

Lastly, it can be highlighted that the MWCM reformulated in terms of the TCD was seen to be successful also in predicting Fretting Fatigue Strength of Cylindrical Contacts [23].

It is the author's opinion that the main advantage of the proposed extensions of the TCD to multiaxial fatigue situations is that they are based on the use of linear-elastic stresses. This aspect is very important, because, when a component is subjected to a complex system of forces, the contribution of every applied force can be computed separately. The resulting stress tensor needed to apply Equations (9), (10) and (11) can then be calculated by superposing the contribution of every considered force, paying attention not to lose the synchronism among the different forces: this simple procedure allows the presence of non-zero out-of-phase angles to be correctly taken into account, making our approach suitable for being used to post-process linear-elastic FE models.

Following an approach similar to the one adopted in the uniaxial fatigue field, Pluvinage and co-workers have recently proposed an extension of his method to multiaxial fatigue situations [24]. In particular, he argued that fatigue damage in the presence of complex stress states depends on the effective shear stress as well as on the hydrostatic pressure averaged over the fatigue process zone. These two elasto-plastic stress components have to be calculated as follows [24]:

$$\tau_{eff} = \frac{1}{L_{eff}} \int_0^{L_{eff}} \tau_{max}(\theta, r) \cdot [1 - \chi_t \cdot r] dr \tag{12}$$

$$\sigma_{H,eff} = \frac{1}{L_{eff}} \int_0^{L_{eff}} \sigma_H(\theta, r) dr \tag{13}$$

where

$$\tau_{max}(\theta, r) = \sigma_1(\theta, r) - \sigma_3(\theta, r) \tag{14}$$

$$\sigma_H(\theta, r) = \frac{\sigma_1(\theta, r) + \sigma_2(\theta, r) + \sigma_3(\theta, r)}{3} \tag{15}$$

Equations (12) and (13) show that the multiaxial method proposed by Pluvinage is an extension to multiaxial fatigue situations of his elasto-plastic LM. Again, for a given material, the effective distance, L_{eff}, depends on the notch geometry and it is equal to the distance from the notch tip at which the relative stress gradient, χ_t, defined as:

$$\chi_t = \frac{1}{\tau_{max}(\theta, r)} \cdot \frac{d\tau_{max}(\theta, r)}{dr} \tag{16}$$

reaches its first relative minimum. The number of cycles to failure can then be estimated by using an elliptical relationship formalised in terms of τ_{eff} and $\sigma_{H,eff}$.

It has to be admitted that the method proposed by Pluvinage is very elegant and sophisticated, but, unfortunately, its in field application hides some problems. First, it is well-known that correctly modelling the material plastic behaviour in the presence of multiaxial stress gradients is a very complicated task and the complexity of the problem increases even more when non-proportional loadings are involved. Secondly, it is not clear the orientation of the path to be used for determining τ_{eff} and $\sigma_{H,eff}$ in the presence of stress raiser having complex geometry. In our opinion, these two reasons should justify the fact that the accuracy of the above method has not been checked in the presence of out-of-phase fatigue loading yet. In any case, apart from the above considerations, Pluvinage's multiaxial elasto-plastic LM doubtless represents another interesting attempt to use the TCD to address problems of practical interest.

The Structural Volume Concept

The above sections clearly show that the TCD is successful in predicting the high-cycle fatigue strength of real mechanical components weakened by different geometrical features and subjected to both uniaxial and multiaxial fatigue loading. In light of these accurate results, a simple and cruel question arises: "Why does the TCD work?". Unfortunately, it is very difficult to answer this question expressing a definitive verdict about this aspect of the problem. But, some interesting considerations can be made observing the metallic material cracking behaviour in the high-cycle fatigue regime.

Initially, it is possible to remember that the fatigue limit condition results in the formation of non-propagating cracks (NPCs). The length of these NPCs is closely related to the stress gradient of the stress field damaging the fatigue process zone. In particular, in plain specimens NPCs have length equal to the material grain size, because the propagation of such cracks is arrested by the first grain boundary (or by the first micro-structural barrier) [25]. On the contrary, in the presence of sharp notches, their length is of the order of L [26, 27]. In particular, it is worth noticing that, using different arguments in conjunction with the LM, Taylor [27] suggested that the TCD might work because it is able to predict the propagation (or non-propagation) of cracks emanating from the apex of the stress concentrator and having length equal to 2L: this would imply that NPCs have a length equal to 2L when initiated at the tip of crack-like notches. Unfortunately, even if this idea is very interesting from a scientific point of view and it is supported by different experimental evidences, it can not be used to explain the reason why the TCD is successful in predicting fatigue limits of blunt notches.

According to Neuber's theory [1, 2] fatigue limits of notched components can successfully be predicted only if the stress close to the stress concentrator is averaged over material units: in particular, he suggested that the size of the integration domains should correspond to the size of the crystals (or of the structural particles). In Table 2 the values of L are compared to the grain size: these results were generated testing different materials both in Trinity College and in the University of Ferrara. The above Table highlights that the L value is somehow related to the grain size, but, in general, its value is of an order of magnitude higher than the average dimension of the grains. This experimental finding suggested to us that we use the structural volume concept to interpret the material cracking behaviour in the fatigue process zone, trying to create a link between the linear elastic stress distribution within such a volume and the initial crack path [22].

Material	R	L [μm]	Grain Size [μm]
EN3B	-1	472	13
	0.1	483	
C10	0.1	80	18
AISI 316L	0.1	112	11

Table 2. Comparison between grain size and material characteristic length, L.

The structural volume idea takes as its starting point the assumption that all the physical processes resulting in the formation of fatigue cracks are confined within a finite volume. The size of this volume is assumed to be constant (but different for different materials) and it depends on neither the geometrical feature nor the degree of multiaxiality of the stress field damaging the fatigue process zone.

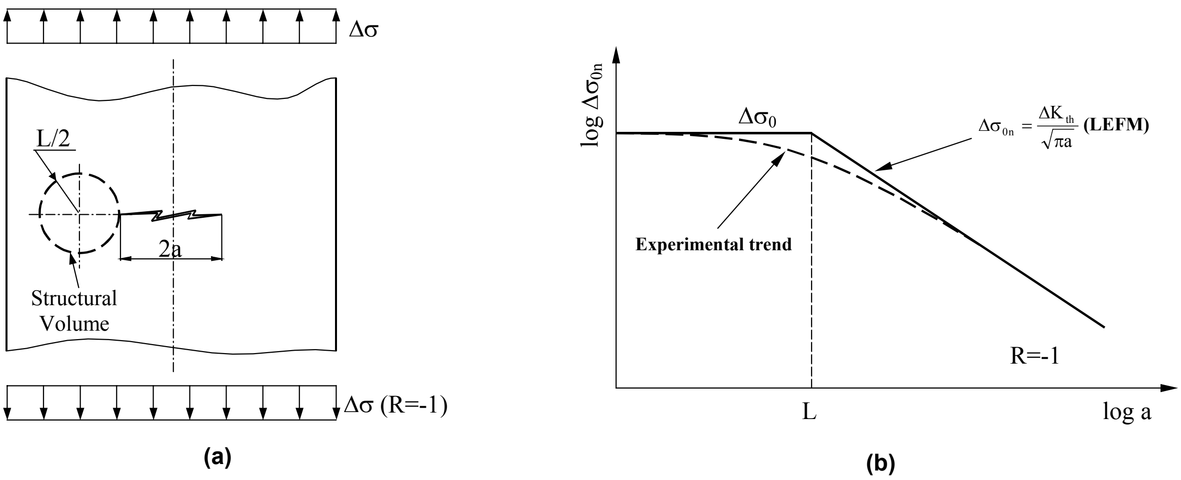


Figure 6. Central through-crack in an infinite plate subjected to a remote uniaxial load (a) and corresponding Kitagawa and Takhashi's diagram (b).

Now the question is: "What size is the structural volume?". To answer this question consider an infinite plate weakened by a central through-crack which is perpendicular to the applied fully-reversed uniaxial remote loading (Fig. 6a). According to the fact that, in the considered reference configuration, geometry, nominal stress gradients and non-zero mean stress do not affect the crack growth, this particular cracking behaviour can be assumed to be capable of describing the "pure" crack propagation phenomenon. It is well-known that the reduction of the nominal fatigue limit due to the presence of a crack in a plate can successfully be described by using the classical Kitagawa-Takahashi curve (Fig. 6b). In particular, the profile of the experimental curve (the dashed one in Fig. 6b) can be approximated by considering two straight asymptotic lines: the horizontal one corresponds to the plain fatigue limit and the sloping one to the threshold condition calculated according to the LEFM. The intersection point of these two straight lines corresponds to a crack of half-length equal to L. If, for the sake of simplicity, one refers just to these two straight asymptotic lines, it is possible to assume that, as long as the half-length of the crack is lower than L, no reduction of the nominal fatigue limit occurs. It is logical then to form the hypothesis that the size of the structural volume is directly related to the material characteristic length, L, because a reduction of the nominal fatigue limit is avoided, provided that, all the cracking processes are confined within this material portion. In Ref. [22] we assumed that the structural volume has circular shape in 2D bodies (Fig. 6a) and spherical shape in 3D components, but it is important to highlight that this assumption is arbitrary and not supported by any experimental evidence.

As said above, the fatigue limit condition results in the formation of NPCs and such cracks reach their maximum length when the considered component is weakened by a crack-like notch. According to Yates and Brown [26] the maximum NPC length is equal to:

$$a_0 = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{F \cdot \Delta\sigma_0}\right)^2 \tag{17}$$

In the above equation, F is the geometrical correction factor for the LEFM stress intensity factor and, in real components, this factor is always larger than unity: it is evident now that NPCs should be always confined within the structural volume as defined above.

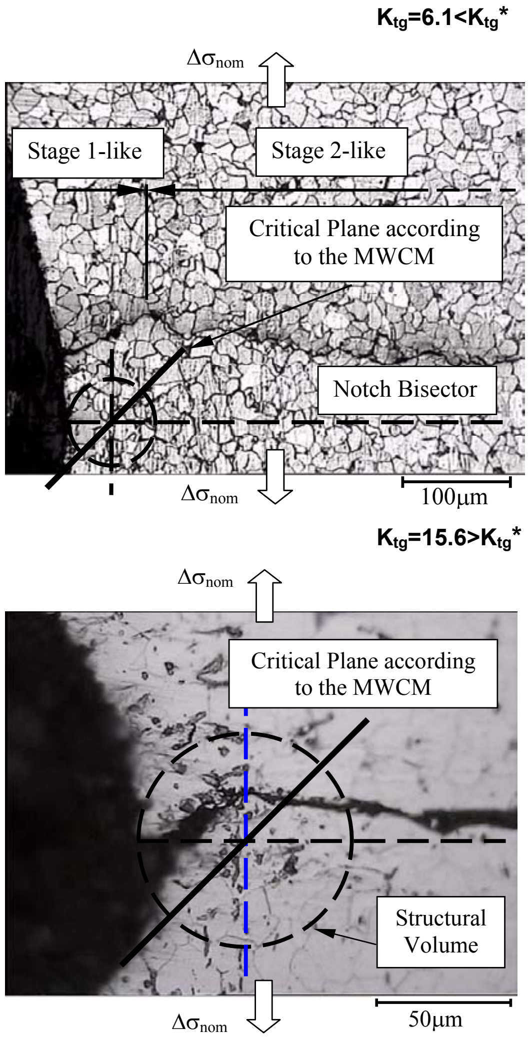


Figure 7. Crack paths in U-notched specimens under uniaxial fatigue loading [28]

Having defined size and shape of the fatigue process zone, another important question arises: "Can we predict the crack path orientation within the structural volume?". Over the last few years, working in collaboration with Prof. David Taylor, Prof. Roberto Tovo and Dr. Giovanni Meneghetti, we extensively investigated the material cracking behaviour in the fatigue process zone when failures occur in the high cycle fatigue regime both under uniaxial and under biaxial fatigue loading [28-30].

Our understanding of the problem is that initiation and initial growth of micro/meso-cracks is always mixed-mode governed (and it holds true independently of notch sharpness and complexity of the stress field damaging the fatigue process zone) resulting in crack initiation phenomena similar to the classical Stage 1 taking place in plain specimens [19]. Another interesting outcome of our experimental investigations is that the transition from Stage 1-like to Stage 2-like process seems always to occur at a distance from the notch tip equal to about L/2: when the crack length is larger than this threshold value, cracks begin to change their direction showing a propagation occurring along planes experiencing the maximum Mode I loading (Stage 2-like process).

Figures 7 and 8 report some examples showing the correspondence between the observed Stage 1-like and Stage 2-like planes and the critical plane orientation determined by using the MWCM as well as the M-TCD. In more detail, Fig. 7 shows that under uniaxial fatigue loading the orientation of the predicted Stage 1 plane is quite close to the orientation of the observed one and it holds true in the presence of both blunt and sharp notches. It can be highlighted here that, by testing under uniaxial fatigue loading U-notched specimens characterised by K_{tg} values ranging from 3.8 up to about 25, we observed that crack initiation occurred on planes at about 24° to the applied load, when the orientation of the critical plane determined by means of to the MWCM was equal to 45° [28]. Even though trivial, it has to be highlighted that in Fig. 7 the critical plane predicted by the M-TCD (that is, the plane experiencing the maximum Mode I loading) coincides with the notch bisector: under uniaxial fatigue loading the M-TCD perfectly describes the experimental reality, correctly modelling the Stage II propagation.

Figure 8 instead shows the material cracking behaviour within the structural volume observed in sharp V-notches under in-phase Mode I and II loading [30].

The above figures seem to suggest that there always exists an acceptable agreement between predicted and observed Stage 1-like and Stage 2-like directions. In any case, it is not superfluous to highlight that the fact estimations are not so accurate is not surprising at all. It is well-known that predicting the orientation of the initial propagation planes is mainly a short crack problem, so that, crack path within the structural volume could be rigorously estimated only if both the real material morphology in the vicinity of crack initiation sites and the elasto-plastic grain behaviour were taken into account. On the contrary, our different formalisations of the TCD maximise the fatigue damage due to either an ideal Stage 1-like or an ideal Stage 2-like process taking place in homogeneous and isotropic linear-elastic materials. In our opinion, the simplifications introduced by the above hypotheses could explain the reason why there exists a certain level of inaccuracy in predicting the crack path orientation within the structural volume.

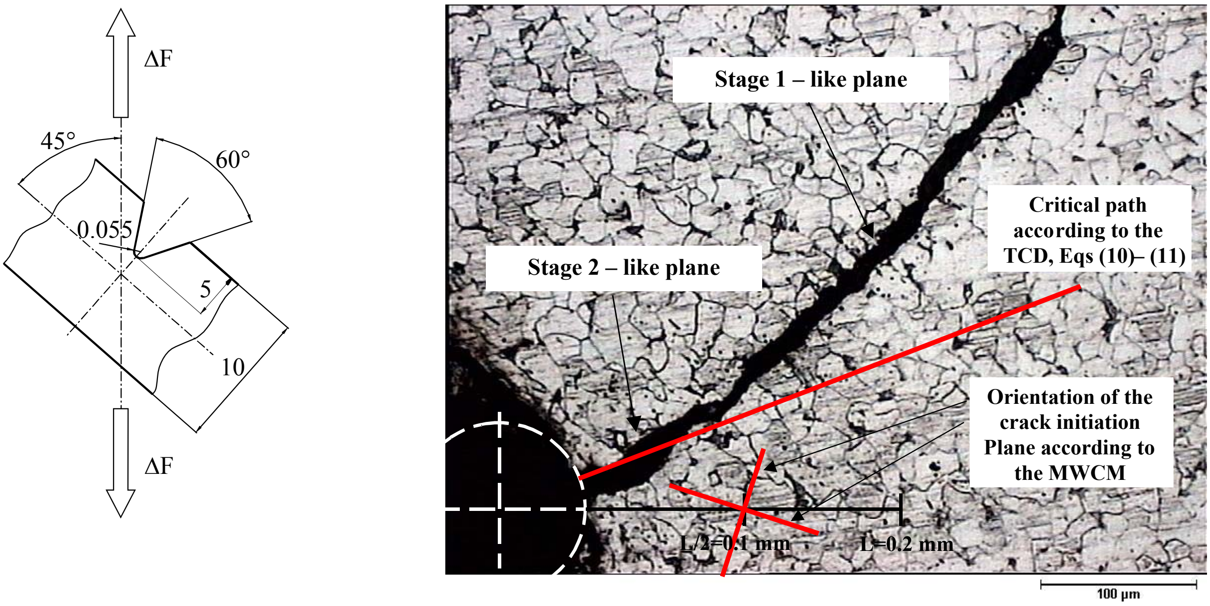


Figure 8. Material cracking behaviour under in-phase Mode I and II fatigue loading and crack initiation plane predicted by the MWCM and by the M-TCD [30].

What we have summarised above is an attempt to understand the reason why the TCD is successful in predicting fatigue limits of notched components. We believe that the TCD just represents a pragmatic compromise between the need for rigorous modelling of the physical phenomena taking place within the structural volume and the need for having a simple engineering tool suitable for addressing problems of practical interest. In any case, even if our methods have been formulated by introducing some simplifications, Figures 7 and 8 should better clarify that our simplified theories are in any case closely related to the physical reality.

To conclude this section, it is possible to say that if crack initiation is assumed to be the most important stage in determining fatigue limits, then critical plane approaches can successfully be used in conjunction with the TCD to describe the physical reality. On the other hand, if the Stage 1-like process is assumed to be the most important one because it results in the formation of NPCs, then the M-TCD is the more adequate one because it is based on the definition of the plane experiencing the maximum Mode I loading. It is the author’s opinion that these two strategies are equally successful because they simply model two equally important phenomena damaging the structural volume: the crack initiation process and the initial Mode I propagation. In other words, even though based on the calculation of linear-elastic stresses, the TCD is capable of capturing the essence of the two most important phenomena resulting in the formation of micro/meso crack within the structural volume, which is the reference volume defining the material fatigue strength.

A strategy to extend the use of the TCD down to the medium-cycle fatigue regime

Recently, we have attempted to extend the use of TCD to the medium-cycle fatigue regime [31, 32]. This attempt takes as its starting point the assumption that the critical distance value depends on the number of cycles to failure, N_f. According to this idea, fatigue lifetime of a notched component can be estimated provided that the L vs. N_f relationship is known for the material of which the component to be assessed is made. We observed that many different strategies could have been adopted to define such a relationship, but, in order to simplify the addressed problem, we initially decided to employ a pragmatic engineering procedure based on the use of two calibration curves: a fatigue curve generated testing plain specimens and a fatigue curve generated testing notched specimens having known geometry. In more detail, by using the PM argument, it is possible to determine the critical distance for two different values of the number of cycles to failure. In fact, for a given value of N_f, that is, $N_f=N_i$, it is trivial to determine the distance from the notch tip, $L(N_f)/2$, at which the linear-elastic maximum principal stress equals the stress applied to the plain specimens to generate a failure at $N_f=N_i$ cycles to failure. By calculating the L value both in the medium-low and in the high-cycle fatigue regime, it is possible then to determine constants A and B of the following L vs. N_f relationship:

$$L(N_f) = A \cdot N_f^B \quad (18)$$

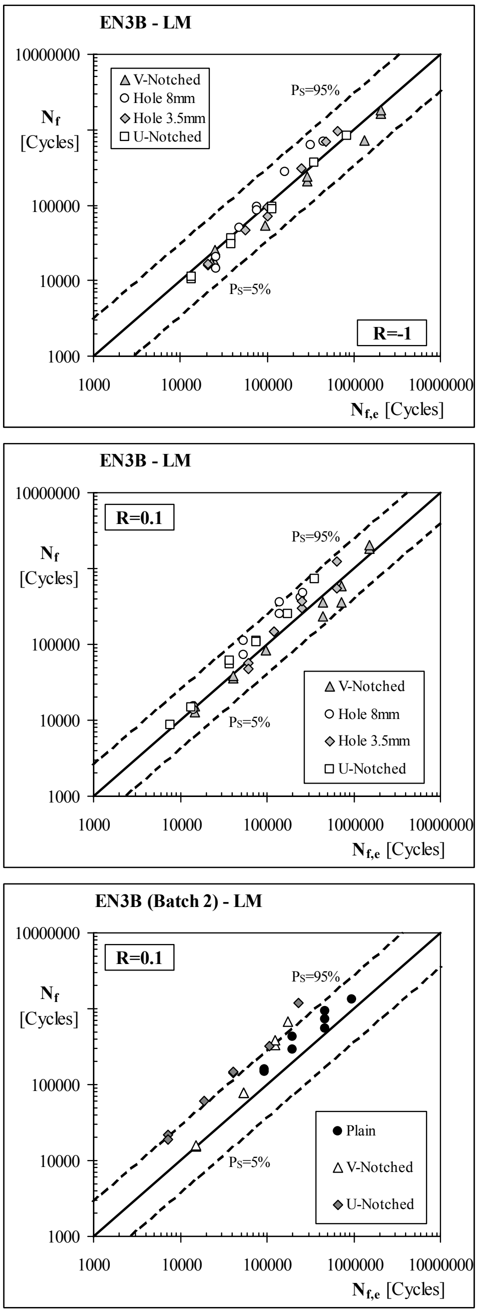


Figure 9. LM method accuracy in predicting fatigue lifetime of notched components in the medium-cycle fatigue regime [31, 32].

Remembering that the critical distance in our theory is always assumed to be a material property [6], the constants of the above equation are expected to be different for different materials, but their values do not depend on the notch geometry. Equation (18), applied by using a recursive procedure [31], can then be used to estimate fatigue lifetime of notched components (made of the same material for which the constants A and B have previously been determined) and weakened by any kind of geometrical feature.

In order to check the accuracy of the proposed extension, we tested specimens of a commercial low carbon steel weakened by different geometrical features [31, 32]. Constants A and B were determined by using the plain fatigue curve as well as a fatigue curve generated testing sharp V-notches. Figure 9 summarises the accuracy obtained by applying the above procedure, in terms of the LM, to the generated results. These diagrams make it evident that our method was capable of predictions falling within the scatter band of the parent material, confirming reliability and accuracy of the TCD even when employed to address this particular problem. It is interesting to observe that the predictions summarised in the last diagram of Fig. 9 were done considering plain and notched specimens tested under 3-point bending, but constants A and B were calculated using two calibration curves generated under tension-tension with a load ratio, R, equal to 0.1. Moreover, the calibration specimens had thickness equal to 6mm, whereas in the 3-point bending specimens it was 25mm. These results are definitively promising, because they strongly support the idea that the TCD is capable of correctly accounting for the presence of stress gradients generated by the geometry itself superimposed on the stress gradients generated by the applied loading.

Discussion

This paper summarises the most important results which have been obtained by applying the TCD to predict fatigue strength of notched components. This review should highlight that a lot of work has already be done in this area, so that, now we can certainly say that we have a powerful engineering tool which can be used to assess, with a high accuracy level, problems of practical interest.

In any case, we believe that more effort has to be made in order to better explore all the potentialities of the TCD. In particular, there are still many open problems which might successfully be addressed by using our theory. In our opinion, the main open problems can be summarised as follows:

1) The use of the TCD in the presence of three-dimensional stress concentrators resulting in three-dimensional stress states damaging the fatigue process zone (including constraint);
2) Predicting medium-cycle fatigue lifetime of notched components subjected to multiaxial cyclic loading, considering also the problem of assessing welded structures;
3) The use of the TCD under both uniaxial and multiaxial variable amplitude loading, checking its capability of predicting the effect of overloading.

It is evident that to efficiently address all the above problems, researchers should create sounds links amongst different fields of the structural engineering by a continuous exchange of knowledge. Moreover, observing that the TCD was already employed to assess real components in different ambits of the structural integrity problem [33], it would be an important goal if we could create a transversal correlation amongst the different classical engineering disciplines (practically overcoming the historical difficulties in transferring knowledge amongst mechanical engineering, civil engineering and material science): due to its appealing peculiarities the TCD might be the right theory to create a transversal correlation amongst areas of interest which at first appear to be so different. If we were able to do that, engineers engaged in assessing real components could reach a higher level of flexibility, because they could use similar engineering tools to address problems which are nominally different and, for this reason, classically addressed using different techniques.

Conclusion

The TCD was seen to be a powerful engineering tool suitable for predicting fatigue strength of real components characterised by complex geometries resulting in stress concentration phenomena. A lot of work has already been done in this area, but in light of the good results obtained so far, the peculiarities of the TCD deserve to be better explored in order to check accuracy and reliability of this theory in different fields of the structural integrity problem.

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References

1. Neuber, H., Forschg Ing-Wes, **7**, 271-281 (1936).
2. Neuber, H., Theory of notch stresses: principles for exact calculation of strength with reference to structural form and material, II Ed., Springer Verlag, Berlin (1958).
3. Peterson, R. E. “Notch Sensitivity”. In: Sines G., Waisman J. L., Editors. Metal Fatigue. New York. McGraw Hill, 293-306 (1959).
4. Tanaka, K., “Engineering formulae for fatigue strength reduction due to crack-like notches”. Int. J Fracture, **22**, R39-R45 (1983).
5. Lazzarin, P., Tovo, R. and Meneghetti, G., “Fatigue crack initiation and propagation phases near notches in metals with low notch sensitivity”. Int. J Fatigue, **19**, 647-657 (1997).
6. Taylor, D., “Geometrical effects in fatigue: a unifying theoretical model”. Int. J. Fatigue, **21**, 413-420 (1999).
7. Tanaka, K., Nakai, Y., “Propagation and non propagation of short fatigue cracks at a sharp notch”. Fatigue Fract. Engng Mater. Struct., **6**, 315-327 (1983).
8. DuQuesnay, D. L., et alt. “The effect of notch radius on the fatigue notch factor and the propagation of short cracks”. In: The behaviour of short fatigue cracks, EGF Pub. 1, Ed. by K. J. Miller and E. R. de los Rios, 323-335 (1986).
9. Taylor, D., Hughest, M., Allen, D., “Notch fatigue behaviour in cast irons explained using a fracture mechanics approach”. Int. J. Fatigue, **18**, 439-445 (1996).
10. Atzori, B., Lazzarin, P., Filippi, S., “Cracks and notches: analogies and differences of the relevant stress distributions and practical consequences in fatigue limit predictions”. Int. J. Fatigue, **23**, 355-362 (2001).
11. Taylor, D., Wang, G., “The validation of some methods of notch fatigue analysis”. Fatigue Fract. Engng. Mater. Struct., **23**, 387–394 (2000).
12. Susmel, L., Taylor, D. “Fatigue Design in the Presence of Stress Concentrations”. Int. J. of Strain Analysis for Eng. Components, **38**, 443-452 (2003).
13. Taylor, D., Bologna, P., Bel Knani, K., “Prediction of fatigue failure location on a component using a critical distance method”. Int. J. Fatigue, **22**, 735-742 (2000).
14. Wiersma, S., Taylor, D., “Fatigue of materials used in microscopic components“. Fatigue Fract. Engng. Mater. Struct., in press (2005).
15. Taylor, D., Barrett, N., Lucano, G., “Some new methods for predicting fatigue in welded joints”. Int. J. Fatigue, **24**, 509-518 (2002).
16. Qylafku, G., Azari, Z., Kadi, N., Gjonaj, M., Pluvinage, G., “Application of a new model proposal for fatigue life prediction on notches and key-seats”. Int. J. Fatigue, **21**, 753-760 (1999).
17. Susmel, L., Taylor, D., “A simplified approach to apply the theory of critical distances to notched components under torsional fatigue loading”. Int. J. Fatigue, *in press* (2006).
18. Susmel, L., Taylor, D., “Two methods for predicting the multiaxial fatigue limits of sharp notches”. Fatigue Fract. Engng. Mater. Struct., **26**, 821-833 (2003).
19. Miller, K. J., “The two thresholds of fatigue behaviour”. Fatigue Fract. Engng. Mater. Struct., **16**, 931-939 (1993).
20. Susmel, L., Lazzarin, P., “A Bi-Parametric Modified Wöhler Curve for High Cycle Multiaxial Fatigue Assessment”. Fatigue Fract. Engng. Mater. Struct., **25**, 63-78 (2002).

21. Lazzarin, P., Susmel, L., “A Stress-Based Method to Predict Lifetime under Multiaxial Fatigue Loadings”. Fatigue Fract. Engng. Mater. Struct **26**, 1171-1187 (2003).
22. Susmel, L., “A unifying approach to estimate the high-cycle fatigue strength of notched components subjected to both uniaxial and multiaxial cyclic loadings”. Fatigue Fract. Engng. Mater. Struct., **27**, 391-411 (2004).
23. Araújo, J. A., Susmel, L., Taylor, D., Ferro, J. C. T., Mamiya, E. N., “On the Use of The Theory of Critical Distances and the Modified Wöhler Curve Method to estimate Fretting Fatigue Strength of Cylindrical Contacts”. Submitted to: Int. J. Fatigue (2006).
24. Quilafku, G. , Kadi, N., Dobranski, J., Azari, Z., Gjonaj, M., Pluvinage G., “Fatigue specimens subjected to combined loading. Role of hydrostatic pressure”. Int. J. Fatigue, **23**, 689-701 (2001).
25. Akiniwa, Y., Tanaka, K., Rimura, H., “Microstructural effects on crack closure and propagation thresholds of small fatigue cracks”. Fatigue Fract. Engng. Mater. Struct., 24, 817-829 (2001).
26. Yates, J. R., Brown M. W., “Prediction of the length of non-propagating fatigue cracks”. Fatigue Fract. Engng. Mater. Struct. 10, 187-201 (1987).
27. Taylor, D., “A mechanistic approach to critical-distance methods in notch fatigue. Fatigue Fract. Engng. Mater. Struct., 24, 215-224 (2001).
28. Menghetti, G.; Susmel, L.; Tovo, R., “High cycle fatigue crack paths in specimens having different stress concentration features”. In: F. Nilsson (Ed.), Proc. of 15th European Conference of Fracture, Stockholm (Sweden) (2004).
29. Susmel, L.; Taylor, D.; Tovo, R., “Crack paths in sharply notched specimens under in-phase biaxial loadings”. In: A. Carpinteri and L. Pook (Ed.), Int. Conf. on Crack Paths, Parma, Italy (2003).
30. Susmel, L.; Taylor, D., “Non-propagating cracks under in-phase mode I and II loadings”. In: F. Nilsson (Ed.), Proc. of 15th European Conference of Fracture, Stockholm (Sweden) (2004).
31. Susmel, L., Taylor, D., “The Theory of Critical Distances to Predict Fatigue Lifetime of Notched Components”. In: Proc. of FDM 2005, Edited by F. M. Aliabady et al., Majorca (Spain), 411-416 (2005).
32. Susmel, L., Taylor, D., “On the Use of The Theory of Critical Distances to Estimate Fatigue Strength of Notched Components in the Medium-Cycle Fatigue Regime”. In: Proc. of FATIGUE 2006, Atlanta, USA (2006).
33. Taylor, D., “The Theory of Critical Distances”, In: Proc. of ECF XVI, Alexandroupolis, Greece (2006).