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The role of fracture toughness in the cutting of wood

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ABSTRACT Offcuts in cutting are formed in one of two modes, viz: shear or bending, depending on conditions. The former tends to be produced by tools having large included angles (that is, small or negative rake angles) and is familiar in workshop machining and sawing. The latter occurs with knife-like tools having small included angles (large rake angles), and is found in thin slicing operations such as microtoming and in the process of making wood veneer. Recent improvements in modelling to predict cutting forces are reviewed, in particular incorporation of significant work of surface separation (fracture toughness) which provides answers unresolved questions, especially for offcut deformation by shear. The treatment is applied to offcut formation by bending for both elastic and irreversible chip deformation and compared with experimental results from veneer peeling.

1.Introduction The cutting of solids is a branch of fracture mechanics. Indeed the so-called wedge-opening-loading test, eg [1] to determine fracture toughness of materials is mechanically the same as the simple process of chopping wood along the grain with an axe, and the same as the process of veneer peeling in wood cutting with an inclined wedge. In a test to determine the toughness of adhesives, a wedge is driven along the interface between two ductile sheets glued together [2].

The difference between these two tests is in the behaviour of the rest of the solid away from the separated surfaces. In splitting concrete or wood the material is removed by combined elastic bending and fracture, and the broken pieces (offcuts) remain undistorted after cutting: the process can be analysed by elastic fracture mechanics. In that particular adhesive test (intended to study the behaviour of adhesives during extensive plastic deformation of the adherands) the material is bent plastically before and during separation, and is permanently curled afterwards: analysis requires elastoplastic fracture mechanics.

In these sorts of process, where the angle of the wedge is about 20°, the principal mode of deformation accompanying separation is *bending*; the strains may be elastic or plastic depending on circumstances. At larger inclined wedge angles (say > 45°, but it depends upon the conditions), a change in mode of deformation occurs and it is energetically more favourable for deformation to take place by *shear*, primarily in a narrow band emanating from the tip of the wedge to the free surface, together with secondary shear along the tool face which gives curl to the offcut/chip. In this case, most of the deformation is irreversible, the offcut is permanently deformed and the process is one of elastoplastic fracture. Often the shear deformation is so great that elasticity may be neglected and the process analysed using rigid-plastic fracture mechanics. Large-angle wedge inclined tools are typical in metalcutting operations and are found in the geometry of saw blading. (In workshop parlance, it is the *rake angle* of the cutting tool that is customarily employed rather than the included angle of the wedge/tool).

Analysis of all such processes may be performed using fracture mechanics but with the special feature that the location of the application of the load is moving towards any starter crack tip before cracking commences, and subsequently continues to move as separation proceeds, rather than remain fixed in relation to the starter crack tip. That is, in offcut bending cases, the wedge/tool is driven along the starter crack, widening the gap between the ‘arm’ and the cut surface until cracking begins. If subsequent cutting is steady-state, the surfaces separate at the same speed as that with which the tool moves forward, but unsteady cutting can occur in which the load initially falls because the crack jumps ahead and arrests, after which the load increases until the process is repeated all over again, producing a saw-tooth, cyclical, load vs. wedge position behaviour; (the stiffness of tool holders in practical cutting machines will also affect oscillations in cutting forces). Similar arguments about crack stability apply to cutting in the alternative mode of irreversible shear deformation to the free surface.

Of course, in practical cutting, there are no starter cracks in which the wedge/tool is inserted. Rather, the tool at first indents the material before cutting initiates. In the non-uniform stress and strain field surrounding the tip of the tool, cracks are initiated and the process of cutting starts up. The sharper the tool, the ‘more intense’ the stress and strain patterns, and the easier it is to initiate a crack, thus starting the separation process. Nevertheless, as we shall show later in this paper, modelling how a wedge proceeds along a starter crack, building up, or dissipating, strain energy in an 'arm' until the transition to splitting takes place, is informative in understanding the mechanics of cutting.

Materials scientists have, for some time, employed cutting as a means of determining fracture toughness, particularly for soft and squidgy natural materials for which standard tests are difficult or impossible to apply, and in which work of separation predominates. In these tests, microtomes instrumented for forces are often employed. Metal cutters, on the other hand, have traditionally argued that fracture has no part to play in continuous chip formation because cracks are rarely seen at the tips of tools in the machining of ductile solids. Traditional metalcutting analyses therefore concern plasticity and friction only. It is strange that one body of workers considers that fracture is a necessary part of cutting mechanics and another body says not. The situation has been rationalised in a series of recent papers [3-7] where it was shown that incorporation of significant work of surface separation in even simple single shear plane models of orthogonal metal cutting answered many questions for which 'plasticity and friction only' analyses had no explanation. In particular, the inclination of the primary shear plane became material dependent which was long-known experimentally. Again, the role of the *separation criterion* in finite element modelling of cutting (which does not appear in algebraic analyses) was made clear.

The important material parameter in cutting in the shear mode was shown to be the toughness (R)/shear strength (τ_y) ratio, which may be combined with the uncut chip thickness (i.e. depth of cut, t) to give the non-dimensional parameter $Z = (R/\tau_y t)$. The new analysis predicts that for given tool angle, the cutting force $F_{x\ cut}$ should vary linearly with t when Z is greater than about 0.1; and that the plot has a positive force intercept, over and above any contribution from rubbing on the rake face of the tool. For greater Z (smaller t in a given material) the plots curve downwards but still do not pass through the origin. Both intercepts are measures of the toughness, and the slope of the plots indicates the shear yield strength of the material. Friction may be estimated from the ratio of F_x/F_y. Application to results on beech wood is given in [8]. It was also shown that the different types of chip/offcut that can occur depend upon the value of Z [5]. Thus, for a given material and tool angle, increase in depth of cut leading to a reduction in the value of Z, can lead to a transition, from chip formation by shear, to splits in bending running far ahead of the tool tip. Alternatively, at fixed t and fixed tool angle, 'more brittle' materials having smaller (R/τ_y) and hence smaller Z, are more likely to split, but 'more ductile' materials having greater (R/τ_y) are more likely to be machinable with continuous chips/offcuts. Various authors have published combinations of tool angle and depth of cut at which different types of chip are produced in metals [9], plastics [10] and wood [11,12]. These patterns of behaviour may be explained in terms of the particular Z values [5], and include the formation of discontinuous chips/offcuts where, in formation by intense shear, partial or complete fracture also occurs within the chip along the shear plane.

The present paper investigates the conditions under which offcuts are formed in bending.

2. Forces on a Wedge-shaped Tool

Central to all analyses are the forces on the tool. We follow Williams [2]– but with a change of symbols – where an inclined wedge simulates a sharp cutting tool forming a flat surface. The effective wedge angle (including the clearance angle beneath the wedge) is θ. The rake angle of the tool is $\alpha = (90 - \theta)°$. When the offcut is formed in bending, there is a contact force K normal to the wedge face at the tangent point to the beam; when the offcut is formed in shear, there are stresses over the region of contact which have a resultant force K acting on the wedge. In both cases, there will be a friction force μK opposing the motion of the offcut as it passes up the wedge, Figure 1a.

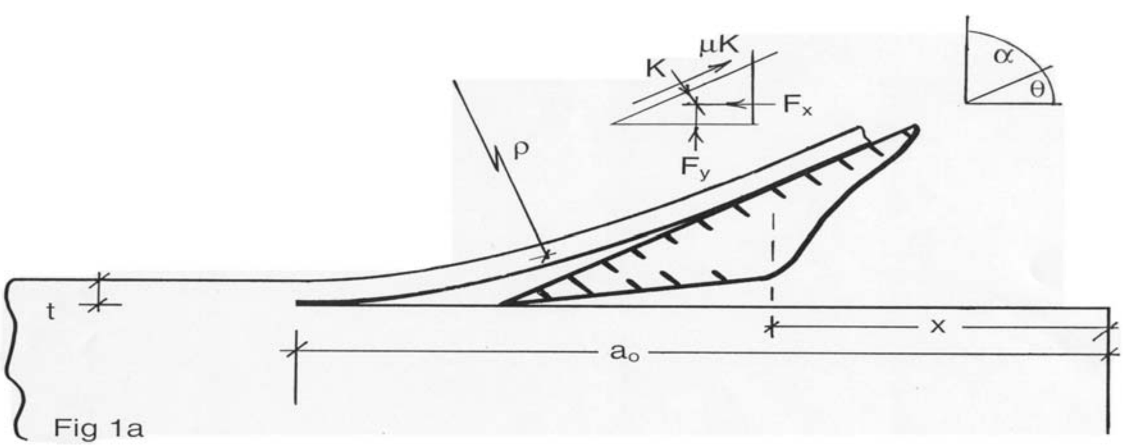


Fig 1a

The forces on the wedge parallel and perpendicular to the cut surface are F_x and F_y. Equilibrium gives

$$F_x = K (\sin\theta + \mu\cos\theta)$$
$$F_y = K (\cos\theta - \mu\sin\theta) \tag{1}$$

We assume that all forces meet in a point, so that moment equilibrium is automatically satisfied. Dividing the first and second equations in (1) we obtain

$$F_x/F_y = H \tan\theta \tag{2}$$

$$\text{where } H = (\tan\theta + \mu) / \tan\theta(1 - \mu\tan\theta) \tag{3}$$

In books on metalcutting where formation of a chip in shear is considered, this analysis is done slightly differently: (i) instead of F_x, F_y and K, the resultant force F_{res} acting on the face of the wedge in contact with the chip/offcut is used with $F_{res} = \sqrt{(1 + \mu^2)}K = \sqrt{(F_x^2 + F_y^2)}$; (ii) instead of μ, the angle of friction β is employed, where $\tan\beta = \mu$; and (iii) instead of the wedge angle θ, the rake angle α is employed. It is easily shown that the 'force circle' of metalcutting theory is identical with these basic force equilibrium relations.

3. Offcuts formed in Bending

3.1 Elastic formation of offcut

A block of material has a starter crack of length a_o situated at a depth t below the surface, Figure 1a. A wedge is driven into the starter crack as shown, the location of the wedge before cracking commences is at some position distant x from the corner of the block. The length of the bent cantilever is thus $a = (a_o - x)$ and its elastic stiffness is $F_y/u = 3EI/a^3$ where u is the deflexion of the beam at the contact point with the wedge, E is Young's modulus and I is the second moment of area of the beam ($I = wt^3/12$). Using (2) we have

$$F_x (a_o - x)^2 = 3EIH\tan\theta (1-\cos\theta) / \theta \tag{4}$$

since $\rho + (t/2) = u / (1- \cos\theta) = (a_o - x) / \theta$ where for simplicity, we assume that ρ the radius of curvature of the beam is constant at any instant; this is not strictly correct for a cantilever, but the error will not be great for small deflexions. Equation (4) gives the force (F_x) ~ displacement (x) relation for pushing the wedge into the starter crack. Clearly greater force is required the further the wedge is inserted and the stiffness is non-linear, increasing with x, as shown in Figure 1b, but partially elastically reversible on unloading.

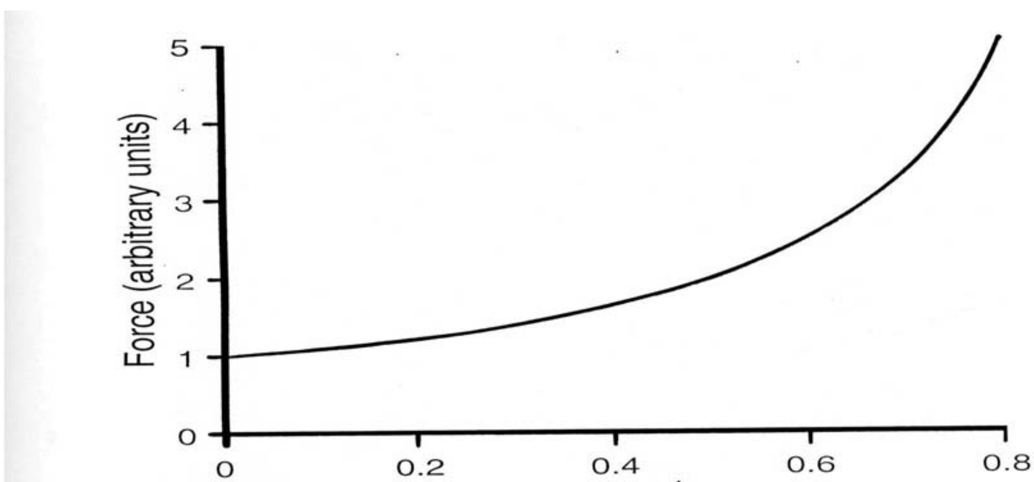


Fig 1b

Before any cracking commences, work is done as the wedge is driven in, given by

$$\text{external work done} = \int F_x dx = [3E\,I\,H\tan\theta(1-\cos\theta)/\theta]\int dx/(a_o - x)^2$$
$$= [3E\,I\,H\tan\theta(1-\cos\theta)/\theta]\,[\{1/(a_o - x)\} - \{1/(a_o)\}] \quad (5)$$

using (4). This work is partly dissipated in friction and also provides stored elastic strain energy in the bent beam.

The friction force μK moves through dx/cosθ when the wedge advances by dx and the increment of friction work in terms of F_x becomes

$$d(\text{friction}) = \mu F_x dx / \cos\theta\,(\sin\theta + \mu\cos\theta) \quad (6)$$

using (1). The total work done against friction when the wedge has advanced a distance x is given by

$$\text{friction work} = \int \mu F_x dx / \cos\theta\,(\sin\theta + \mu\cos\theta) = \{3E\,I\,H\tan\theta(1-\cos\theta)/\theta\,\cos\theta\,(\sin\theta + \mu\cos\theta)\} \quad X$$
$$[\{1/(a_o - x)\} - \{1/(a_o)\}] \quad (7)$$

substituting for F_x from (4).

The difference in magnitude between (5) and (7) is the strain energy Λ stored in the beam which is thus

$$\Lambda = [3E\,I\,H\tan\theta(1-\cos\theta)/\theta] \quad X$$
$$[1 - \mu/\cos\theta\,(\sin\theta + \mu\cos\theta)]\,\,[\{1/(a_o - x)\} - \{1/(a_o)\}] \quad (8)$$

Elastic cracking will commence when $R = -(1/w)\,\partial\Lambda/\partial x\,|_u$ where R is the fracture toughness and w is the width of the block [13]. Hence

$$R = (1/w)\,[3EIH\tan\theta(1-\cos\theta)/\theta]\,[1 - \mu/\cos\theta\,(\sin\theta + \mu\cos\theta)]\,[1/(a_o - x)^2] \quad (9)$$

which may be written

$$R = (F_x / w)[1 - \mu/\cos\theta\,(\sin\theta + \mu\cos\theta)] \quad (10)$$

using (4). That is, cracking/cutting takes place at constant load given by

$$F_x = Rw / [1 - \mu/\cos\theta\,(\sin\theta + \mu\cos\theta)] = Rw / Q_{bend} \quad (11)$$

where

$$Q_{bend} = [1 - \mu/\cos\theta\,(\sin\theta + \mu\cos\theta)] \quad (12)$$

Under frictionless conditions $F_x = Rw$ (an expression not involving θ) which is the same as that for the simple cutting of floppy materials which neither store nor dissipate energy. The reason that the strain energy of the beam seems to disappear in the above calculations is that in steady cutting, the incremental bending energy put in as the crack advances is matched exactly by the increment of bending energy recovered as the beam flattens out when the wedge moves forward (ρ is assumed constant), so that it is only external work that provides the fracture and friction work. The wedge angle θ appears in (11) since it determines K and hence the friction work. The reduced force and energy available for fracture caused by friction is given by the factor Q_{bend}.

For given fracture toughness R, Equation (9) gives the length $a_{trans} = (a_o - x_{trans})$ of the crack at the transition to cutting. Knowing a_{trans} enables ρ_{trans} to be calculated for given wedge angle θ and hence the gap $(a_o - x_{trans}) / (1 + \cos\theta)$ that exists between the tip of the tool and the crack tip. It is never zero but can be very small. It implies that, once started, a cut could just as well be progressed by a blunt tool (truncated wedge) and that sharpness does not matter. Sharpness would only be important at the commencement of the cut. However, surface damage at the start of a cut caused by a blunt tool may not be acceptable so tools are kept sharp. The allowable truncation of a wedge tip is $u_{trans} = [a_{trans} - \text{gap}]\tan\theta = a_{trans}\sin\theta$.

3.2 Plastic or other irreversible formation of offcut

For simplicity, the plastic strains are assumed to be much greater than the elastic strains, and again the neutral axis of the arm is assumed to be bent into a circular arc of radius ρ. We assume that the stress (σ) - strain (ε) curve of the material is given by $\sigma = \sigma_o \varepsilon^n$ where σ_o is constant, so that the bending stress at distance y from the neutral axis is $\sigma_{bend} = \sigma_o(y/\rho)^n$. An element at y has a moment about the neutral axis and, in the usual way [14], integration gives the plastic bending moment M_P on a section of the beam as

$$M_P = [w\, \sigma_o\, t^2/4]\, [2/(n + 2)]\, [t/2\rho]^n \qquad (13)$$

The increment of total work done on the element at y is given by

$$[\sigma_o(y/\rho)^{n+1}/(n+1)]w\, dy \qquad (14)$$

and integration of this gives

$$\sigma_o w t^{n+2} / (n+1)(n + 2)\, (2\rho)^{n+1} \qquad (15)$$

for the total plastic work done on the section. Dividing by wt we obtain the plastic work done per volume in bending W as

$$W = [\sigma_o/ (n+1)(n + 2)]\, [t/2\rho]^{n+1} \qquad (16)$$

from which

$$dW = -\, [\sigma_o/ (n + 2)]\, [t/2]^{n+1}[d\rho/\rho^{n+2}] = +\, [\sigma_o/ (n + 2)]\, [t/2]^{n+1}\, dx / \theta[(a/\theta) - (t/2)]^{n+2} \qquad (17)$$

since $a = (a_o - x) = [\rho + (t/2)]\theta$.

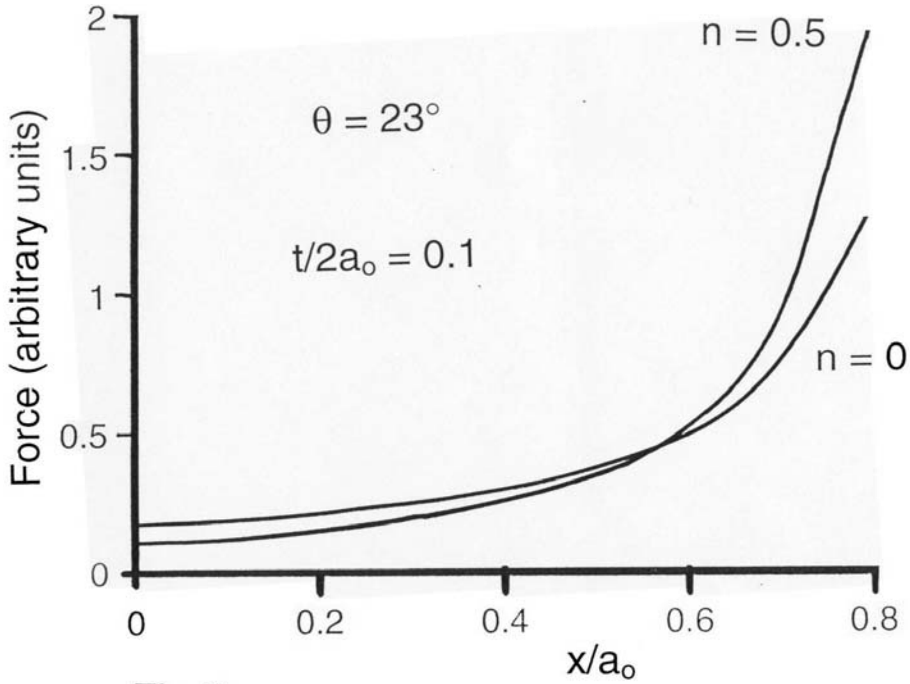
Before cutting occurs, as the wedge/tool is inserted down the starter crack of length a_o, the work done $F_x dx$ goes to plastic bending and friction, both of which are irreversible. The increment of plastic bending $d\Gamma$ is given by

$$d\Gamma = d(WV) = WdV + VdW = 0 + VdW \qquad (18)$$

where V (= awt) is the volume of material being bent. The WdV term is zero because, as x increases dV is negative, which implies a recovery of work. There can be no recovery in plastic bending: there would be if $\sigma = \sigma_o \varepsilon^n$ represented *non-linear elasticity* and it is situations like this that care has to be taken when thinking that Hencky total strain plasticity and non-linear elasticity are identical (see Section 3.3). The dW term is positive because, as the wedge advances, ρ decreases and ε increases. Thus before cutting

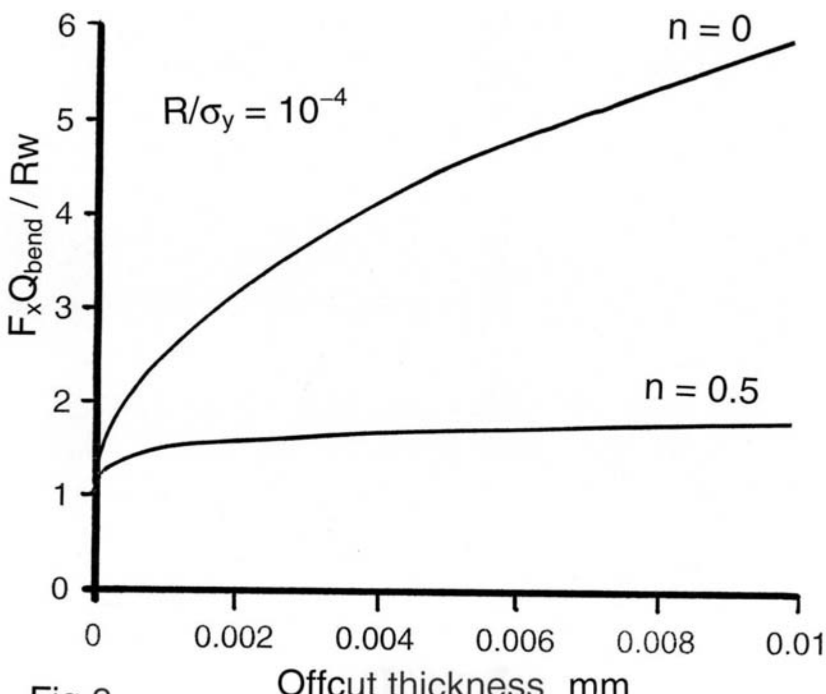
$$F_x dx = VdW + d(\text{friction}) \qquad (19)$$

and using $V = (a_o - x)wt$ with dW from (17) and d(friction) from (6) gives the $F_x \sim x$ relation for the wedge before cutting commences. It is non-linear and irreversible on unloading, Figure 2.



n = 0.5
θ = 23°
t/2a$_o$ = 0.1
n = 0
Force (arbitrary units)
0
0.5
1
1.5
2
0
0.2
0.4
0.6
0.8
x/a$_o$

Fig 2



n = 0
R/σ$_y$ = 10^{-4}
n = 0.5
F$_x$Q$_{bend}$ / Rw
0
1
2
3
4
5
6
0
0.002
0.004
0.006
0.008
0.01
Offcut thickness mm

Fig 3

At some x, it will be energetically favourable for cutting to occur instead of bending to a smaller radius. Then

$$F_x dx = d\Gamma + Rwda + d(\text{friction}) = WdV + Rwda + d(\text{friction}) \quad (20)$$

and using W from (16) and dV = wtda, with d(friction) from (6), (20) gives the $F_x \sim x$ relation for the wedge during cutting.

The transition to cutting takes place when (19) and (20) are equal, that is when

$$VdW = WdV + wRdx \quad (21)$$

and the transition is therefore characterised by a radius ρ_{trans} which satisfies

$$(t/2\rho_{trans})^{n+2} + [n/(n + 1)]\ (t/2\rho_{trans})^{n+1} = (n + 2)R / \sigma_o t \quad = (n + 2)\ Z / 2 \quad (22)$$

where $Z = (R/\tau_y t)$ and where $\tau_y = (\sigma_o/2)$ according to the Tresca yield criterion. Note that while here, for offcut formation by beam bending, we employ the same symbol Z as in cutting by shear band formation to represent the non-dimensional toughness/strength ratio, it may very well be that the two R's are not the same for the different modes. The transition radius of curvature is carried along in steady bending, so fixing the bending strain during cutting. The value of F_x for cutting is obtained from (20) by substituting ρ_{trans} from (22) in (16).

The gap between the tip of the tool and the crack front is obtained as before from the crack length a_{trans}.

Table 1 gives values of (ρ_{trans}/a) for different n and different Z from solution of (22).

TABLE 1

(ρ_{trans}/t)

Z	n=0.5	n=0.3	n=0.1	n=0.05	n=0
0.0001	38.9	38.9	38.9	45	50
0.001	20.1	20.1	24.1	24.1	15.8
0.01	5.3	6.2	6.3	5.8	5.2
0.1	1.5	1.6	1.7	1.6	1.6
1	0.5	0.52	0.51	0.51	0.5
10	0.2	0.18	0.17	0.16	0.16

Since ρ is supposed to be much greater than t in the assumptions of engineers' beam theory, the Table shows that transitions to plastic bending occur only at small $Z = (R/\tau_y t)$. Insofar as what will happen at large Z, it would appear that the shear mode of offcut formation will supervene when the transition radius is forced to levels smaller than the thickness of the beam. Since the mean bending strain is some $(t/4\rho_{trans})$, we observe that $\varepsilon \sim 0.01$ for small Z and about unity for Z = 1.

The cutting force is given by (20) using $(t/2\rho)_{trans}$ (=P, say) which takes the constant values given in Table 1 for given Z. We have

$$F_x/Rw = (1/Q_{bend})\ \{ [1/Z\ (n+1)(n+2)]\ P + 1\} \quad (23)$$

which at first sight implies a linear relation between F_x and t (since $(1/Z) = \tau_y t/R$) with an intercept of Rw/Q_{bend}. However, different t give different Z for fixed τ_y and R, and therefore different P for every t. For bigger t, Z is smaller and P goes down as illustrated in Table 1. This means that the local slope of the plot decreases at greater t and so the plot is non-linear. In the case of n = 0, Equation (22) gives $(t/2\rho_{trans})^2 = Z$, so that $P = \sqrt{Z}$ and

$$F_x/Rw = (1/Q_{bend})\ \{ (1/2\sqrt{Z}\ ) + 1\} \quad (24)$$

which gives $F_x \propto \sqrt{t}$ with an intercept of Rw/Q_{bend} [2]. Representative F_x vs t behaviour is shown in Figure 3 and is typical of that found experimentally in veneer peeling, wood planing and so on.

We also note that in " $[\rho + (t/2)]\theta = a$ ", if ρ alone is employed from the outset instead of $[\rho + (t/2)]$ in the analysis (in the sense of $t << \rho$ for beam theory to apply), we predict a constant cutting force, independent of t, for plastic beam bending, given by $F_x/Rw = (1 + n)/nQ_{bend}$.

3.3 Invalid non-linear elastic solution for plastic beam bending.

We note that an *invalid* solution for cutting with offcut formation in plastic bending is obtained if $\sigma = \sigma_o\varepsilon^n$ represents non-linear elasticity which, in turn, is supposed to represent plasticity. The non-linear stiffness of a cantilever following $\sigma = \sigma_o\varepsilon^n$ is given in [13] and following the procedures in §3.1, the *same result* for F_x as given by (11) is obtained (the algebra is not given here). It is wrong because it presumes that bending energy is recovered from those parts of the arm overtaken by the wedge as it is driven down the starter crack. But in plasticity, no energy is recovered on unloading, so more external work has to be done to achieve cracking. Had we permitted dV to be negative in (19) instead of zero, we would have obtained (11) in place of the solution in §3.2. Also, if we put n = 1 in every step of the solution obtained from the Phillips [14] expression, we demonstrate that the transition line of attack employed in §3.2 applies just as well for linear beams in §3.1.

4. Conclusions

The paper has briefly reviewed cutting mechanics in general, and then concentrated on the formation of offcuts in bending by small-angle wedge tools. The treatment is similar to that in[2] but the line of attack is different, in that the body from which a slice is to be taken already has a starter crack present, and the analysis concerns what happens as the wedge cutting tool is driven down the starter crack. The conditions for the transition from wedge insertion at constant crack length to steady cutting are solved.

In elastic cases, the cutting force is constant and independent of offcut thickness t which, in the absence of friction, would simply be $F_x = Rw$ as already shown in [2].

In plastic or other irreversible cases, the present analysis does not assume the same radius of curvature for all uncut chip thicknesses (unlike [2]) and solves for the different transition radii at different t. It is predicted that F_x varies with uncut chip thickness t in a non-linear manner, and that in the particular case of a rigid-perfectly plastic material where n = 0, F_x varies with the square root of uncut chip thickness (cf. [2]).

Experiments on small samples to mimic veneer peeling show a variation in F_x with t similar to that predicted for irreversible bending offcut formation, and the offcuts do display residual curvature, eg [15]. It might have been thought that the reversible elastic analysis would apply. This is being investigated further together with quantitative comparison between theory and experiment.

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