SURFACE FATIGUE OF GEAR TEETH FLANKS

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ABSTRACT

Two kinds of teeth damage can occur on gears under cyclic contact loading due to material fatigue - the pitting of gear teeth flanks and tooth breakage in the tooth root. In this study only the pitting phenomenon is addressed and the developed two-dimensional computational model for simulation of contact fatigue of gear teeth flanks is used for determination of pitting resistance, *i.e.* the service life of gear teeth flanks. The fatigue process leading to pitting is divided into crack initiation and crack propagation period that leads to the appearance of small pits on the contact surface.
The model for prediction of identification of critical material areas and the number of loading cycles, required for initial fatigue crack to appear, is based on Coffin-Manson relations between deformations and loading cycles, and comprise characteristic material fatigue parameters. The computational approach is based on continuum mechanics, where homogenous and elastic material model is assumed and results of cyclic loading conditions are obtained using the finite element method analysis.
The short crack theory together with the finite element method is then used for simulation of the fatigue crack growth. The governing factors are the initial crack length, crack angle, contact pressure which takes into account the EHD-lubrication conditions, tangential loading due to friction between contacting surfaces, moving contact conditions, residual stresses due to heat treatment of the material and the hydraulic pressure of trapped fluid acting on the crack surface. The virtual crack extension (VCE) method, implemented in the finite element method, is than used for simulating the fatigue crack propagation from the initial crack up to the formation of the surface pit. The relationship between the stress intensity factor K and crack length a, which is needed for determination of the required number of loading cycles N_p for a crack propagation from the initial to the critical length, is shown.

Introduction

The fatigue process of mechanical elements is a material characteristic and depends upon cyclic plasticity, local deformation, dislocation motion, formation of micro- and macro-cracks and their propagation. Contact fatigue is extremely important for all engineering applications involving localized contacts, such as gears, brakes, clutches, rolling bearings, wheels, rails, screw and riveted joints. Contact fatigue process can be divided into two main parts: (i) initiation of micro-cracks due to local accumulation of dislocations, high stresses in local points, plastic deformation around inhomogeneous inclusions or other imperfections on or under contact surface; (ii) crack propagation, which causes permanent damage to a mechanical element, i.e. exceeding of fracture toughness of the material.

The process of surface pitting can be visualized as the formation of small surface initial cracks, which grow under repeated contact loading. Eventually, the crack becomes large enough for unstable growth to occur, which causes the material surface layer to break away. The resulting void is surface pit [1, 4, 5] (Fig. 1).

The number of stress cycles N required for pitting of a gear teeth flank to occur can be determined from the number of stress cycles N_i required for the appearance of the initial crack in the material and the number of stress cycles N_p required for a crack to propagate from the initial to the critical crack length, when the final failure can be expected to occur [1, 4]

$$N = N_i + N_p \ . \qquad (1)$$

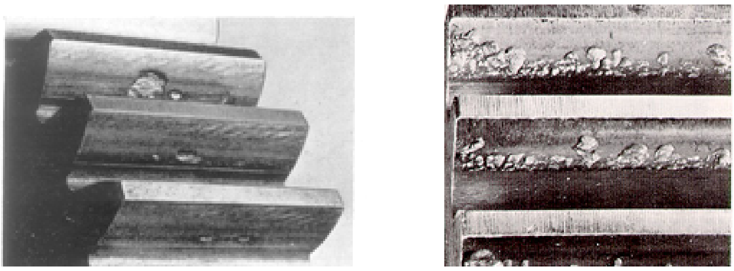

Figure 1. Typical surface pits on gear tooth flank [2]

Computational model of the pitting phenomena

Initial surface cracks leading to surface pitting of gears are, generally, observed to appear in the contact areas, where high normal contact pressure is combined with significant sliding velocities, which result in additional frictional loading of the surface material layer. There are several locations where pitting is apt to occur. However, the most critical contact loading conditions for initial crack formation and propagation are identified for the rolling contact with sliding, where the contact sliding, and with that the effect of friction, is opposite to the direction of the rolling contact motion. Thus, the worst contact loading conditions appear when the gears teeth are in contact at the inner point of single teeth pair engagement (point B on Fig. 2a), where the surface-breaking initial cracks are expected to develop first. This fact is valid for the pinion gear in case of revolutions reduction (reduction gear).

For computational determination of fatigue crack initiation and crack propagation at gear teeth flanks, an equivalent contact model (Hertzian theory) [7, 9, 14] is used (Fig. 2b). The equivalent cylinders have the same radii, as are the curvature radii of gear flanks at the observed point (the inner point of single teeth pair engagement – point B).

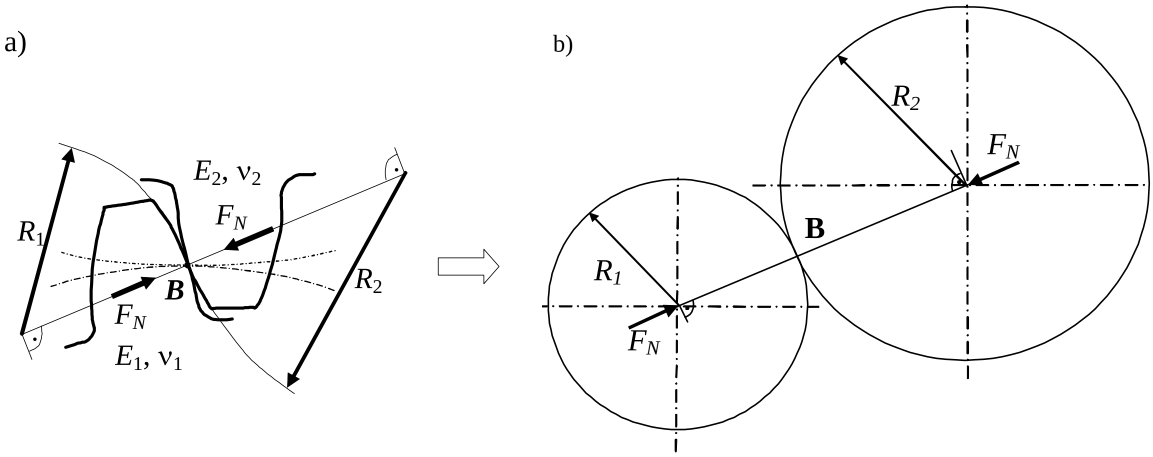

Figure 2. Model transformation of meshing gears (a) to the equivalent model of two cylinders for the meshing point *B* (b)

In analysing real mechanical components, some partial sliding occurs during time depended contact loading, which can originate from different effects (complex loading conditions, geometry, surface etc.) and it is often modelled by traction force due to the pure Coulomb friction law [9]. In the analysed case frictional contact loading $q(x)$ is a result of the traction force action (tangential loads) due to relative sliding of the contact bodies and is determined here by utilizing Coulomb friction law [9]

$$q(x) = \mu \cdot p(x), \tag{2}$$

where μ is the local coefficient of friction between contacting bodies. For general case of elastic contact between two deformable bodies in a standstill situation, the analytical solutions are well known. However, using general Hertzian equations [9] it is difficult to provide the loading cycle history and/or simulation of a contact pressure distribution of moving contact in the analytical manner. Therefore, the finite element method (FEM) is used for simulating two-dimensional friction contact loading in this case and the same procedure is usually used when dealing with complex contact loading conditions (i.e. in case of gears analysis).

Fatigue crack initiation analysis

Standards for gears determination and design, such as American Gear Manufacturers Association (AGMA) standards or German DIN standards, indicate the possibility of dimensioning the so-called "time gearing". However, all standard models are rough and give not accurate enough results since they do not take into account the actual operational conditions. Therefore, our research group decided to develop models and procedures of calculations, that will give more reliable and, in particular, more accurate results [7, 14].

When the stress loading cycles are determined, the fatigue analysis for each observed material point can be performed. The methods for fatigue analysis are most frequently based on the relation between deformations, stresses and the number of loading cycles and are usually modified to fit the nature of the stress cycle, i.e. repeated or reversed stress cycle [15]. The number of stress cycles required for fatigue crack to appear, can be determined iteratively with the strain-life method ε-N, where the relationship between the specific deformation increment $\Delta\varepsilon$, and the number of loading cycles N_f, is fully characterized with the following equation [15]

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_a}{E} + \frac{\Delta\varepsilon_p}{2} = \frac{\sigma'_f}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c, \tag{3}$$

where σ_f' is the fatigue strength coefficient, b the strength exponent, ε_f' the fatigue ductility coefficient and c the fatigue ductility exponent. Generally, the following modified approaches of strain-life method are most often used for fatigue calculations: Coffin-Manson's hypothesis (ε-N method), Morrow's analysis, Smith-Watson-Topper (SWT) method [15].

Fatigue crack propagation analysis

By considering initiation of micro crack in the contact area of gear teeth flank of meshing gears (Fig. 3), the short crack theory can be used for describing crack propagation from the initial to the critical crack length, when the pit occur on the gear teeth flank.

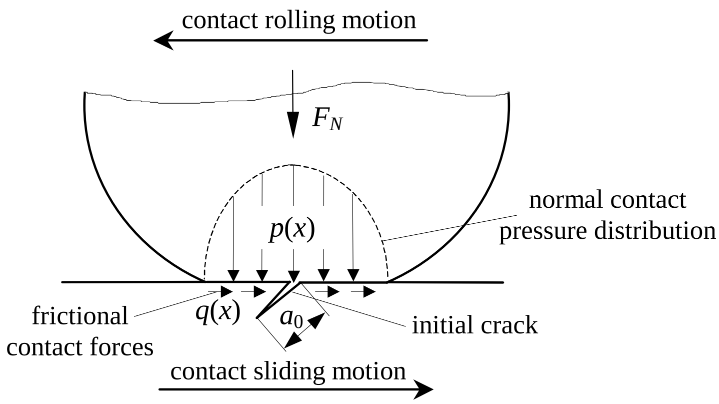

Figure 3: Orientation of the initial surface-breaking crack and contact loading.

The short crack growth is characterised by a successive blocking of persistent slip bands (PSB) with grain boundaries, which implies the discontinuous character of the crack growth process. Navarro and Rios [11] and Sun *et al.* [12] proposed the model where the crack growth rate da/dN is assumed to be proportional to the crack tip plastic displacement $\Delta\delta_{pl}$

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C_o\left(\delta_{pl}\right)^{m_o}, \tag{4}$$

where C_o and m_o are material constants that are determined experimentally. In view of the numerical simulation, it is beneficial to express the crack tip plastic displacement δ_{pl} in terms of the stress intensity range ΔK. This relationship has been provided in the form [11]

$$\Delta\delta_{pl} = \frac{2\kappa}{G\sqrt{\pi}} \cdot \frac{\sqrt{1-n^2}}{n}\Delta K\sqrt{a}, \tag{5}$$

where G is the shear modulus, and κ=1 or $\kappa=1-\nu$ depending on whether screw or edge dislocations are being considered, with ν being Poisson's ratio. Parameter n describes the relative position of the crack tip to the grain boundary. The number of stress cycles required for a crack to propagate through each crystal grain is obtained with integration of Eq. (4)

$$N_j = \int_{a_{j-1}}^{a_j} \frac{da}{C_o\left(\delta_{pl}\right)^{m_o}}; \quad j = 1, 2, 3, \ldots, z \quad , \tag{6}$$

in which z is the number of grains transverse by the crack ($z=a/D$). The total number of stress cycles N required for a short crack to propagate from the initial crack length a_o to any crack length a can then be determined as

$$N = \sum_{j=1}^{z} N_j \; . \tag{7}$$

As the crack extends through ten or more grains, the influence of the material structure on the crack growth becomes negligible and linear elastic fracture mechanic (LEFM) theory can be applied thereafter [6].

Expressing the plastic displacement $\Delta\delta_{pl}$ in Eq. (5) in terms of the stress intensity range ΔK enables treatment of short and long cracks in a similar fashion. If the relationship between the stress intensity range and the crack length $\Delta K=f(a)$ can be derived in some way, the remaining service life of a mechanical element with the crack can be estimated with appropriate integration of rate Eq. (4). Considering small crack lengths observed during pitting in the contact area of mechanical elements, only the theory of short cracks is usually needed for describing crack propagation from the initial to the critical crack length.

For simple cases, the relationship between the stress intensity factor and the crack length $K=f(a)$ is available in the technical literature in analytical form [6]. However, for cases with complicated geometry and boundary conditions, it is necessary to use alternative methods for its determination. Here, the numerical procedure based on the virtual crack extension method (VCE) in the framework of the finite element analysis is used for this purpose [8].

Application of the computational model

The proposed computational model is applied to the analysis of the fatigue crack initiation and crack propagation on the tooth flank of a real gear pair, with material and geometric data set given in Table 1.

The determination of loading cycles at the contact region of meshing gears (at teeth flanks) appears to be difficult task. Indeed, the worst case of pitting phenomena, i.e. the area around the inner point of a single teeth pair engagement, is often simulated with maximum value of contact pressure via point force put at this point B. However, this is way out of the fact that the loading cycles are competent for evaluating fatigue process. For the treated gear pair the contact point B can be identified with equivalent radius, as well as other characteristics points (see Fig. 4). Following, the prediction of time depended loading cycles and evaluation of number of critical loading cycles of real meshing gears are presented [13].

Table 1. Data set for the gear pair

Parameter	Pinion		Gear
Normal module		m_n = 4,5 mm	
Number of teeth	z_1 = 16		z_2 = 24
Pressure angle on pitch line		α_n = 20°	
Helical angle		β_0 = 0°	
Coefficient of profile displacement	x_1 = 0,182		x_2 = 0,171
Centre distance		a = 91,5 mm	
Tooth width		$b_1 = b_2$ = 14 mm	
Pitch diameter	d_1= 72 mm		d_2= 108 mm
Material of gear pair		42CrMo4, case hardened	
Torque of a pinion: M_1=183,4 Nm; rotational velocity of pinion n_1=2175 min^{-1}			

Moving contact loading of meshing gears

Comprehensive model for contact fatigue life prediction of single mechanical elements should consider time history of applied contact loads, regarding both their magnitude and shape. For more realistic simulation of the fatigue crack initiation and crack propagation at teeth flanks it is necessary to consider the influence of moving contact in the vicinity of expected initial crack(s). The moving contact can be simulated with different loading configurations, as it is shown in Fig. 4.

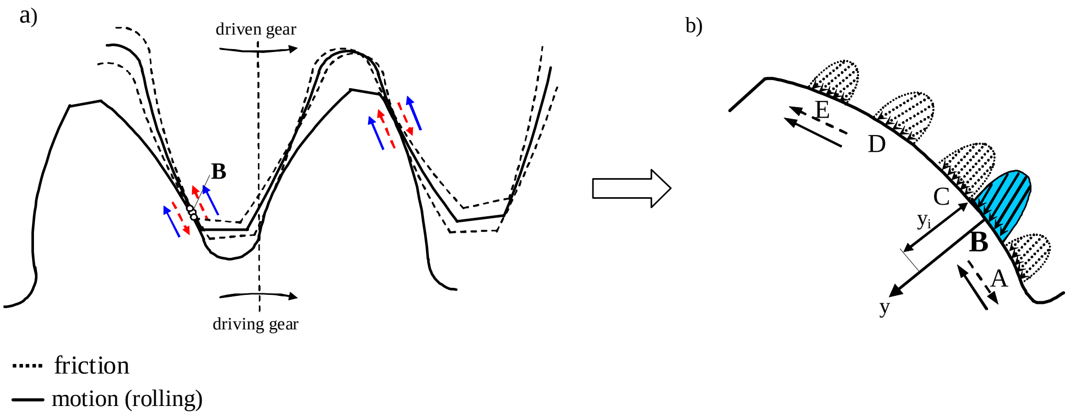

Figure 4. Moving contact loading configurations in respect to the initial crack position: meshing of gear pair (a), equivalent contact model of meshing gears (b)

Characteristic contact points (*A*, *A-B*, *B*, *C*, *D* and *E* – Fig. 4) of meshing gears have been considered, each with the specific normal $p(x)$ and tangential $q(x)$ contact loading distributions. The values of basic contact parameters have been considered using Hertzian theory and DIN 3990 standard procedure. On the basis of discussed facts and restrictions, the contact analysis, using FEM (Fig. 4b) was performed for evaluating loading cycles. Generally, obtained times are very short, since in presented case, the rotational velocity of the pinion is 2175 rpm [13]. In the case of Tresca stress comparative cycles, the time dependent normalised distribution of loading cycles, are presented in Fig. 5. Obtained loading cycle (considering point *B*), corresponds well with the fact that, in most cases, the compressive stress on spur gear teeth occurred at the lowest point on a pinion tooth at which full load is carried by a single pair of teeth. Theoretically, if one pair of teeth carries full load and the position of loading is at considered point *B*, there is the worst-load condition for Hertzian contact stress corresponding to the worst-load position [4] for strength. The values of normalised stress cycles are higher under contact surface, which is influenced particularly by the coefficient of friction and this fact is also in agreement with basic contact theory. Anyway, in critical area, the high contact pressure is combined with significant amount of gear sliding velocities, which results in the frictional loading of the surface layer. On the basis of the defined contact loading cycles of meshing gears, considering teeth flanks loading, the fatigue analyses can then be performed.

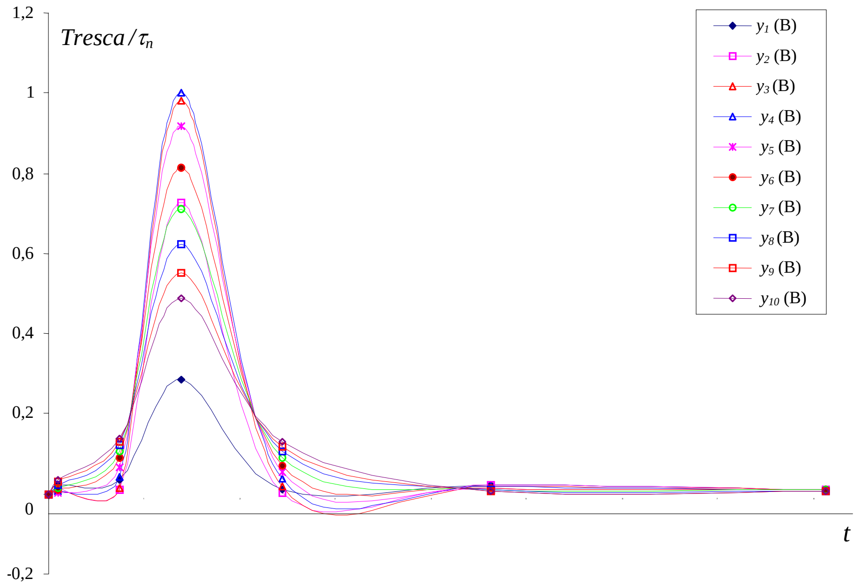

Figure 5. Normalised Tresca comparative stress cycles at meshing gears, on and under contact point *B* (see Figure 4b)

Fatigue crack initiation

The strain-life method (ε-N) and its modified forms, present above, was used to determine the number of stress cycles N_i required for the fatigue crack initiation (starting of pitting damage process), in the framework of the FEM program package MSC/FATIGUE [10]. Fatigue material parameters for the generalised contact model used in analysis are: modulus of elasticity E = 2,06·10^5 MPa, Poisson ratio ν =0,3, fatigue strength coefficient σ_f' = 1820 MPa, fatigue ductility coefficient ε_f' = 0,65, exponent of strength b = -0,08, fatigue ductility exponent c = -0,76, hardening exponent n' = 0,14, strength coefficient K' = 2259 MPa and surface finish factor C_{surf} = 0,6÷0,9. Additional factors, influencing fatigue life, are particularly: machine component size C_{size}, the type of loading C_{load} and effect of surface finish and treatment C_{surf}. Practically all fatigue failures start at the surface. So, it is evident, that fatigue properties are very sensitive to surface condition [4]. The usual way to account for these effects is through the calculation and application of specific modifying factors, influencing actual fatigue (endurance) limit S_e for a real component (spur gears)

$$S_e = S_e^{'} \cdot C_{surf} \cdot C_{size} \cdot C_{load}, \qquad (8)$$

where S_e' is measured material endurance limit. The influence of surface finishing on fatigue strength was considered with the surface finishing factor C_{sur}, for different surface roughness R_a. Surface finishing is categorized by means of qualitative terms such as polished, machined and surface treatment is categorized by means of nitrided and shot peened, which provide the state of residual stresses in a compressive layer that have the effect of decreasing the likelihood of fatigue failure.

Table 2. Case studied: quality and material treatment description

Case studied (Examples)	Case description
Ex 1	Averaged machined and "shot peened"
Ex 2	Averaged machined
Ex 3	Good machined and nitride
Ex 4	Polished
Ex 5	Averaged machined and nitride
Ex 6	Polished and "shot peened"
Ex 7	"Shot peened"
Ex 8	Poor machined

Results of number of loading cycles for crack initiation are presented in Fig. 6. Dealing with Tresca based loading cycles and ε-N method for determination of initiation damage, first damage appears at (10^4 ÷ 3·10^7) loading cycles (at approximately 0,12 mm under observed inner point of single teeth pair engagement - point B).

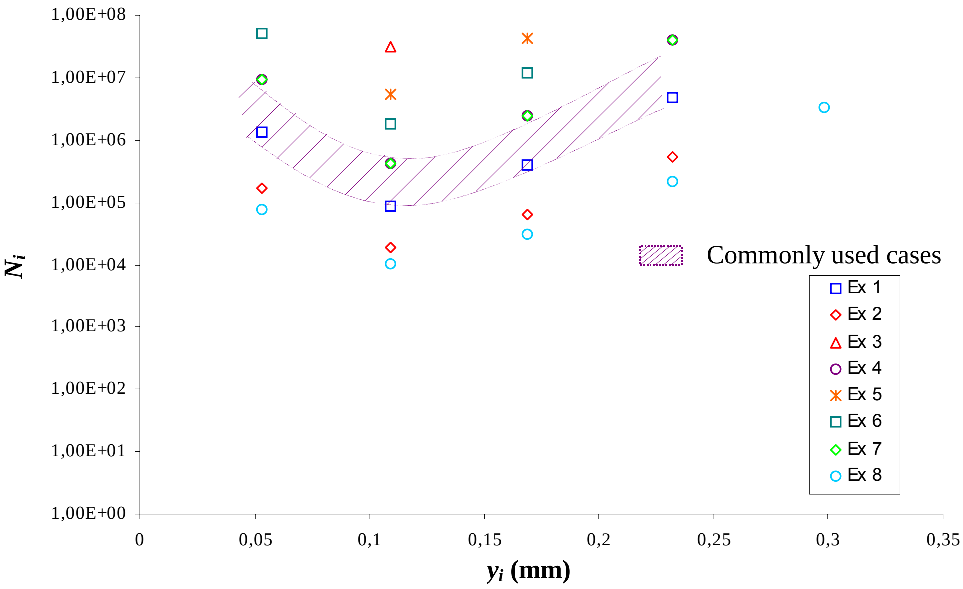

Figure 6. Results: number of loading cycles, when initial crack appear at the observed material points (y_i) – Tresca comparative stresses, ε-N analysis method

Modified Coffin-Manson methods for determination of crack initiation are used, regarding different nature of loading cycles. The treated problems of mechanical treatment (Examples 1-8 in Table 2) result in different numbers of critical loading cycles for crack initiation. Hatched areas on presented diagram (Fig. 6) are valid for most frequently used gear pairs in practice. Thus, average values of critical number of loading cycles are in the range of ($1{,}2 \cdot 10^4 \div 7{,}1 \cdot 10^7$) loading cycles, defined in the proposed model.

Fatigue crack propagation

With regard to the results of crack initiation model the initial crack is presume to be located at the inner point of single teeth pair engagement (point *B*). The configuration of initial crack follows from metallographic investigation, see Chapter below. After the crack is initiated at the surface of gear teeth flank the crack propagation model has been used for simulation of surface crack propagation on a gear teeth flank.

The gear pair is made of case hardened steel 42CrMo4 with Young's modulus E=2.06·10^5 N/mm^2 and Poisson ratio ν=0.3. The maximum contact pressure acting at point *B* is p_0=1453 N/mm^2, with the equivalent radius of gear teeth flanks R_B^*=7,2562 mm. Using Hertzian theory [5], the half-length of the contact area is equal to b=0,1778 mm. The Hertzian normal loading distribution $p(x)$ along the entire contact width of the gear flanks has then been determined using Eq. (1). For all computations, the coefficient of friction μ_B = 0,1075 has been used. Therefore, the tangential loading $q(x)$ has been determined using Eq. (2).

Moving contact and the influence of fluid lubricant on crack propagation

For a more realistic simulation of fatigue crack growth, it is necessary to consider the influence of moving contact in the vicinity of initial crack. The moving contact can be simulated with different loading configurations as shown in Fig. 7. Five contact loading configurations have been considered, each with the same normal $p(x)$ and tangential $q(x)$ contact loading distributions, but acting at different positions in respect to the initial crack.

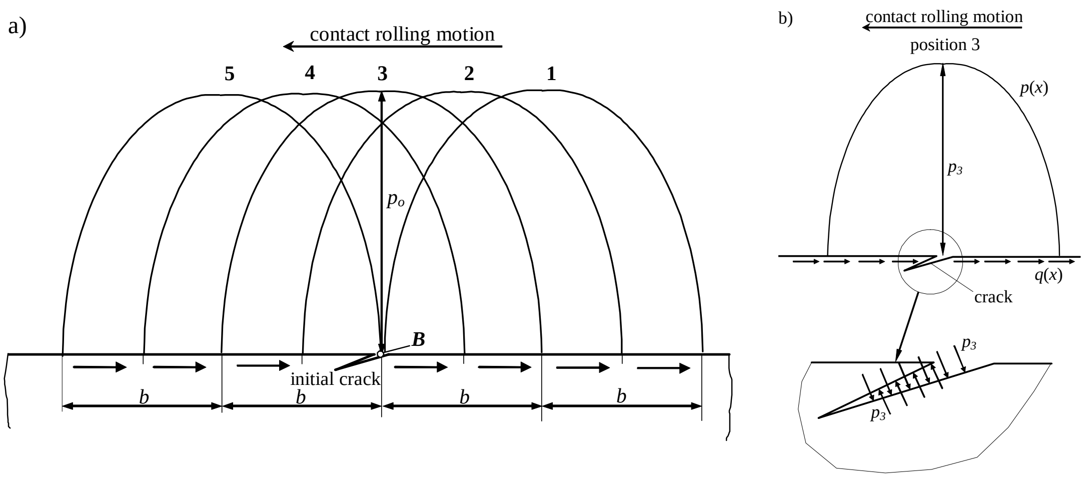

Figure 7. Moving contact loading configurations in respect to the initial crack position a) and modelling of the lubricant driven in the crack by the hydraulic pressure mechanism b)

The influence of fluid lubricant on crack propagation

The simulation of the surface initiated crack propagation should also consider the influence of the lubricant on crack propagation. Several mechanisms and models have been proposed in the past [3]. In the present computational procedure the hydraulic pressure mechanism has been adopted, where the lubricant pressure inside the crack is simply approximated with a uniform pressure distribution along the crack faces [3]. The level of lubricant pressure is equal to the current pressure determined at the crack mouth, i.e. the pressure depends on the contact pressure distribution position in respect to the crack mouth. Fig. 7b illustrates crack face pressure determination and distribution for two consecutive contact loading configurations. The finite element mesh shown in Fig. 8, and the boundary conditions as described above, have been used in subsequent analyses. For the configuration of the initial crack on the surface, located at point *B*, it was assumed that the initial length of the crack is equal to a_o=15 μm, with the initial inclination angle towards the contact surface equal to α_o = 22°. It is recognised, that the predicted crack growth heavily depends on the size and orientation of the initial crack. However, the used configuration follows from metallographic investigations of initial cracks appearing on gears [5, 16]. A thorough investigation of the crack growth dependence on position and size of various initial defects is to be performed in future investigations with parametric simulations based on the proposed model.

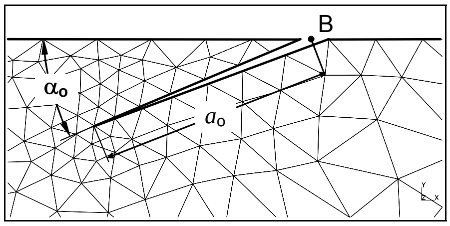

Figure 8. FE discretisation and configuration of the initial crack

In this study, the FE analysis program BERSAFE [2], based on VCE method, has been used for computational estimation of the stress intensity factor *K* and subsequent incremental crack growth simulation. Five different loading configurations have been considered in each computation for the purpose of simulating the effect of the moving contact of the gear flanks (see Fig. 7). For each crack increment, the crack was actually extended in the direction of the recorded K_{max} from all calculated load cases. Figure 9 shows the relationship between stress intensity factor *K* and the crack length *a*, and also the shape and magnitude of the surface pit. It can be noticed that the computed stress intensity factor *K* is very small at the beginning, but later increases as the crack propagates towards the contact surface.

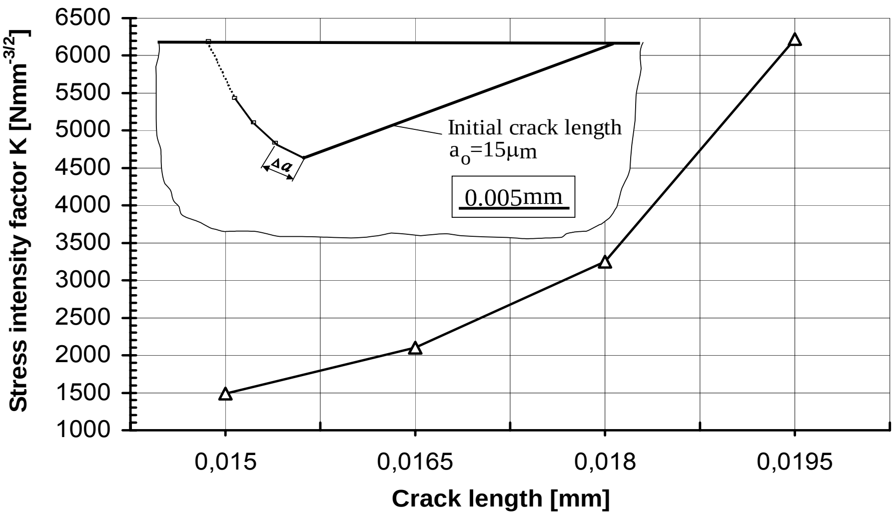

Figure 9. Stress intensity factor for initial surface micro crack propagation

Numerical simulations have shown that at the moment when the crack reaches the vicinity of the contact surface, the stress intensity factor is extremely high. At that moment it can be expected that the material surface layer breaks away and the pit occurs on the surface. Because of the very small dimensions of surface pits, they can be termed “micro-pitting”.

Micro pitting as shown in Fig. 9 is not the final and most critical surface failure. Further operation of the gear pair results in the formation of larger pits, and consequently progressive pitting. Following the above procedure, one can numerically determine the functional relationship $K=f(a)$, which is needed for determining the required number of loading cycles N_p for a crack propagating from the initial to the critical length.

Figure 10 shows that the shape and magnitude of the numerically determined pits correspond well with available experimental data, as determined by experimental testing of a spur gear pair using the FZG - pitting test machine according to DIN 51354 standard. The tested gears have been subjected to the same operating conditions and loading parameters as used in the numerical computations.

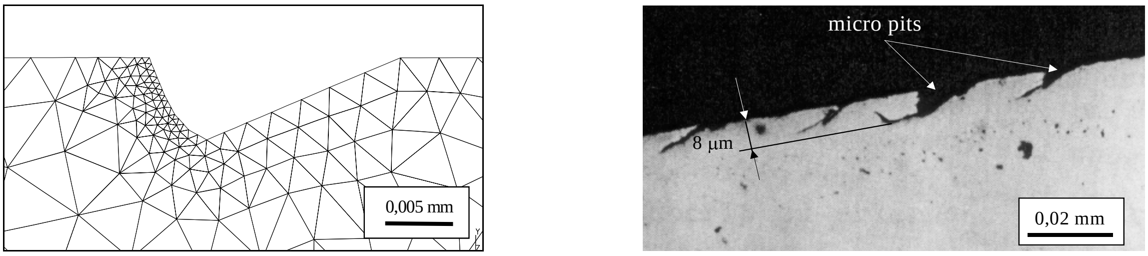

Figure 10. Numerically (left) and experimentally (right) determined pit shapes [2]

Concluding remarks

The paper presents a computational model for determining the service life of gears with regard to pitting on the gear teeth flanks. The process of surface pitting can be visualized as the formations of small surface initial cracks grow under repeated contact loading. Eventually, the crack becomes large enough for unstable growth to occur, which causes the material surface layer to break away. The resulting void is a surface pit. The fatigue process leading to pitting of gear teeth flank is divided into crack initiation (N_i) and crack propagation (N_p) periods, which enables the determination of total service life as $N = N_i+N_p$.
An equivalent contact model of a cylinder and flat surface is used for the simulation of the contact fatigue crack initiation and the crack propagation under conditions of rolling and sliding contact loading of meshing gears.
A general computational model for the determination of initiation fatigue loading cycles in meshing gears is based on material model which is assumed to be homogenous, without imperfections and/or inclusions and elastic shakedown of the material model is considered. The modified Coffin-Manson method, in the framework of finite element analysis (FEA), is used for iterative analyses of contact fatigue crack initiation. The number of loading cycles and places (on/under the contact surface) for contact fatigue are presented. Generally, regardless of select stress component, the number of loading cycles, required for initial fatigue damages, is in the range of $N_i = (10^4 \div 10^7)$ and where (on the contact surface or subsurface) the contact fatigue damage first occur mostly depends on the coefficient of friction, material parameters and contact geometry.
On the basis of results of fatigue crack initiation analysis the initial micro crack on gear teeth flank is assumed. A two-dimensional finite element approach is applied together with the short fatigue crack growth theories to predict the surface initiated crack propagation. The numerical model attempts to account for the different parameters influencing the crack propagation process (Hertzian contact pressure, friction between contacting surfaces, hydraulic pressure in the crack, meshing of gears, etc.) leading to pitting, starting from the initial surface fatigue crack to the critical crack length, when the occurrence of a surface pit is expected. The virtual crack extension (VCE) method in the framework of FEA is used for two-dimensional simulation of fatigue crack propagation from the initial crack up to the formation of the surface pit. Consequent computational determination of the functional relationship $K= f(a)$ from Fig. 9 enables the estimation of the crack propagation period in regard to the surface pitting, if combined with the previously developed model [6, 7], short fatigue crack growth theory and with consideration of some particular material parameters. Comparison of numerically predicted and experimentally recorded pit shapes show that they are in a very good agreement.

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