RAPID CALCULATION OF STRESS INTENSITY FACTORS

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ABSTRACT

The demand to design evermore efficient, economic and safer structures continues and is set to only increase throughout the twenty-first century and beyond. Thus, the challenges confronting the engineer, concerned with ensuring a state of structural integrity prevails similarly grow evermore demanding. The SIF is widely recognised as the fundamental parameter vital for the assessment of defects, or cracks prone to linear elastic fracture behaviour. Difficulties in computing or measuring SIF are widely accepted especially when the crack is situated in a complex geometry or subject to a non-simple stress state. In addition, with the emergence of structural integrity monitoring systems there is likely to be an increased demand for the rapid availability of accurate SIF solutions for on-line defect assessment. This paper describes the development of a novel weight function methodology, which will potentially permit the determination of SIF solutions for such crack systems.

Recent years have seen the advent of readily available and greatly enhanced computer processing ability leading to exciting developments in finite element and boundary element approaches to complex fracture problems. These require considerable skill and insight from the operator and are, therefore, likely to remain the preserve of specialists. Moreover, implementation by a non-specialist could yield dangerously erroneous solutions. If realised, the weight function methodology described below would present rapid, high quality solutions within the reach of design engineers in a format conducive for incorporation into standards and design codes.

A general need exists in both industry and academia for SIF solutions for engineering components for which currently published solutions represent an idealised and often unrealistic approximation. Further to the limitations stated previously, solutions gained through numerical techniques are developed for specific applications and are generally applicable within restrictive limits of validity. Engineering optimisation and defect assessment of components in service however, often require broad ranging solutions, which can be rapidly calculated. A weight function approach meets these requirements.

Introduction

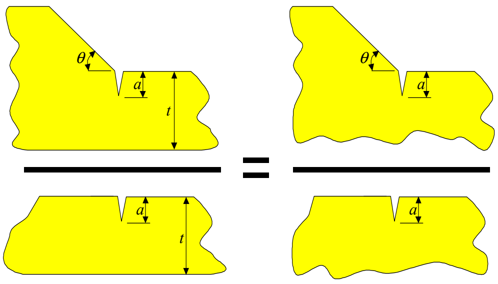
The SIF weight function, '*h(a,x)*', as defined by Rice and Bueckner, is widely recognised as a powerful and efficient means of determining SIF solutions for cracks subject to complex loading configurations. In essence it permits the influence of component geometry, which it represents upon the SIF to be separated from that of applied loading. Once determined, it may be used in conjunction with a crack-line stress, '*σ(x)*', arising from any loading mode to yield new SIF solutions as shown below.

$$K = \int_0^a \sigma(x) h(a,x) dx \qquad (1)$$

By virtue of the definition of the weight function as a unique property of geometry, the influence it represents can be isolated and combined. This characteristic is unique to the weight function. A number of researchers developed similar composition approaches after Impellizzeri and Rich [1] used what they called Geometry Correction Factors for the influence of geometric anomalies on the weight function. These studies [3], [4] represented the beginning of the idea that geometric influences could be analytically separated. Recognition of this and advancements in weight function formulation [2] led to the development of many subsequent engineering solutions by the building of complex geometry weight functions from more simple constitutive geometry weight functions.

Library of Geometric Influences for SIF Weight Functions

Brennan *et al* [9] envisaged a 'library' of solutions for numerous notch types and conducted the most comprehensive study on the composition of weight functions. The 'library' refers to a generic set of constitutive solutions that systematically define a wide range of symmetric notch parameters. In its simplest form the composition principle may be illustrated diagrammatically as in Figure 1. It states that the ratio of SIF weight function of a cracked body having a projection and finite thickness to that of a cracked body without a projection having the same thickness is equal to the ratio of SIF weight function of a cracked body having a projection and infinite thickness to that of a cracked body without a projection having infinite thickness.



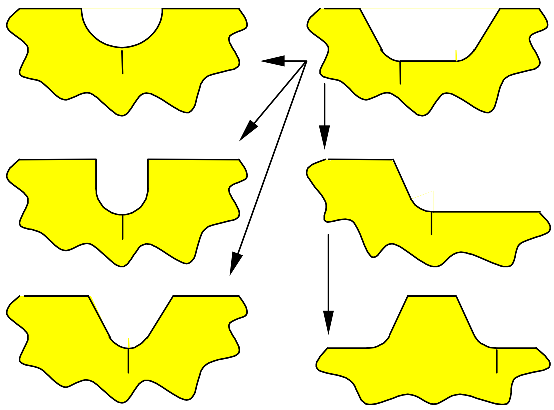


Fig. 1 – (above) Simplified Weight Function Composition Scheme. (right) Inter-Relation between Symmetric and Asymmetric Notches

The weight function composition principle presented and applied to symmetric notches proved the premise that geometric influences, described as weight functions, may be combined, or composed, via a suitable composition scheme to yield new SIF solutions [5]. Extension of the principle to broaden the library database and incorporate symmetric notches, of extreme geometric form and asymmetric notches exposed limitations in the composition scheme in its presented form. Recent developments have addressed these limitations to allow formulation of a universal composition scheme applicable to all inter-related geometries shown above and application to the deepest point of surface cracks (Figure 2).

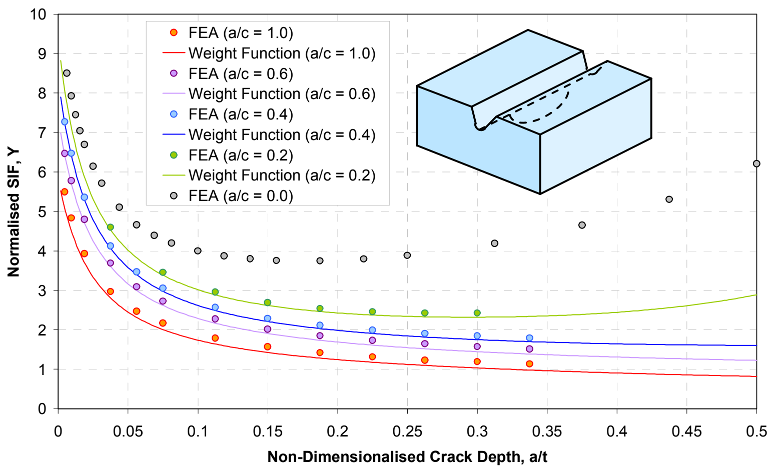


Figure 2: Deepest Point SIF Solutions for a Surface Cracked, Grooved Plate Subject to Tensile Loading

SIF solutions obtained via the composition of weight functions have been shown to be of high accuracy, having wide ranging limits of validity in a manner that is both rapid and of relative mathematical simplicity. The composition of weight functions is therefore, ideally suited to numerous engineering applications for which full numerical modelling maybe unwarranted or inappropriate.

The Composition of SIF Weight Functions

By isolating the effects of stresses, the constitutive SIFs can be composed geometrically in terms of weight functions to give the composed weight function. The mathematical representation of weight function composition can be expressed as [5]:

$$\frac{m(a,x)_f^{\theta}}{m(a,x)_f} = \frac{m(a,x)_s^{\theta}}{m(a,x)_s} \quad \Rightarrow \quad m(a,x)_f^{\theta} = m(a,x)_f \cdot \frac{m(a,x)_s^{\theta}}{m(a,x)_s} \tag{2}$$

Where the subscripts *f* and *s* represent finite thickness and infinite thickness, respectively and θ refers to a geometry with a projection having an angle θ. The constitutive SIF weight functions represented on the right hand side of *Equation (2)* can be relatively easily calculated using a multiple reference states weight functions approach [5].

Figure 3 below illustrates the three constitutive solutions, Edge Crack in a Finite Strip, Edge Crack in an Infinite Strip and an Edge Crack in a Semi Infinite Notched Strip [7] composed using equation (2) above to give the solution for an Edge Crack in a Finite Strip.

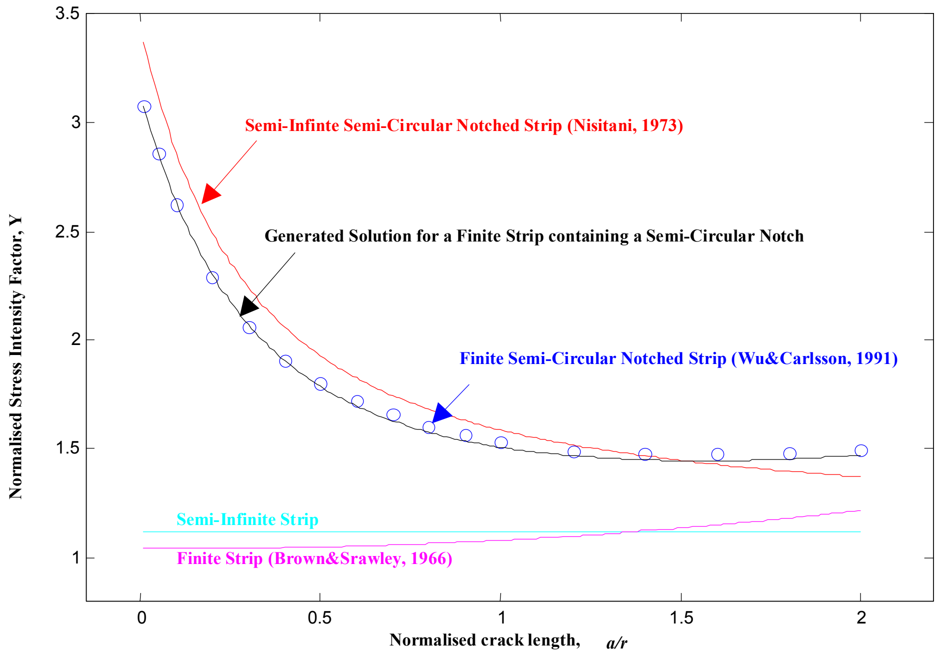


Figure 3: Normalised SIFs (Y-Factors) for an edge crack emanating from a semicircular notch in a finite thickness strip with r/t=0.125 and those of the constitutive geometries under pure bending.

The result is shown compared with published finite element solutions by others [6] and demonstrates that if the crack solution for a geometric anomaly is known on a body of infinite thickness then the solution for that geometric form on a body of finite thickness can be rapidly computed without further experimentation, finite element or boundary element analyses.

Figure 4 [5] below shows further verification of the process against published results for a number of semi circular notched bodies.

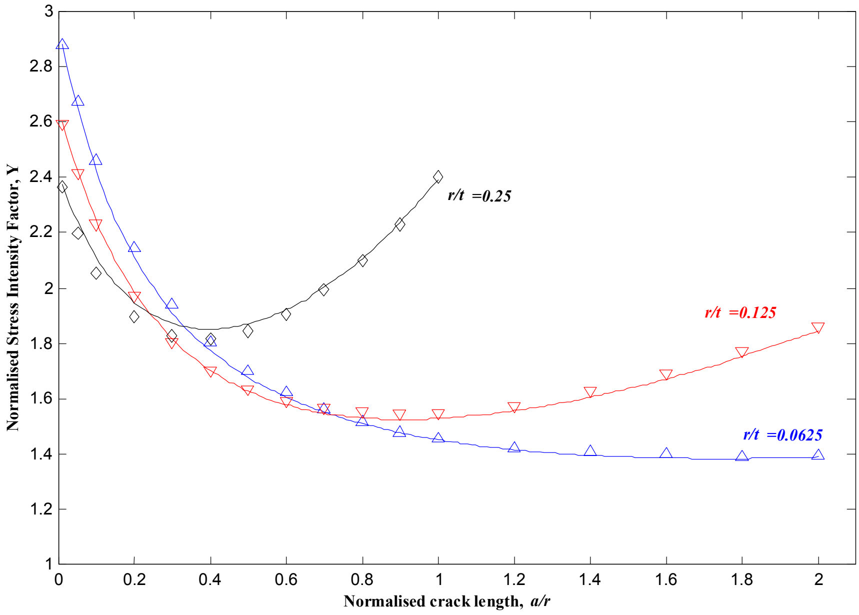


Figure 4: Verification of the normalised SIFs (Y-Factors), for an edge crack emanating from a semicircular notch in a finite thickness strip determined by the composition model (represented as lines) with the Wu and Carlsson (1991) SIF (represented as symbols) under pure tension.

Rapidly generated SIF solutions from simple geometric constituents

Following from the previous section, Figure 5 below shows a broad range of rapidly generated edge crack solutions from semicircular notches under pure bending.

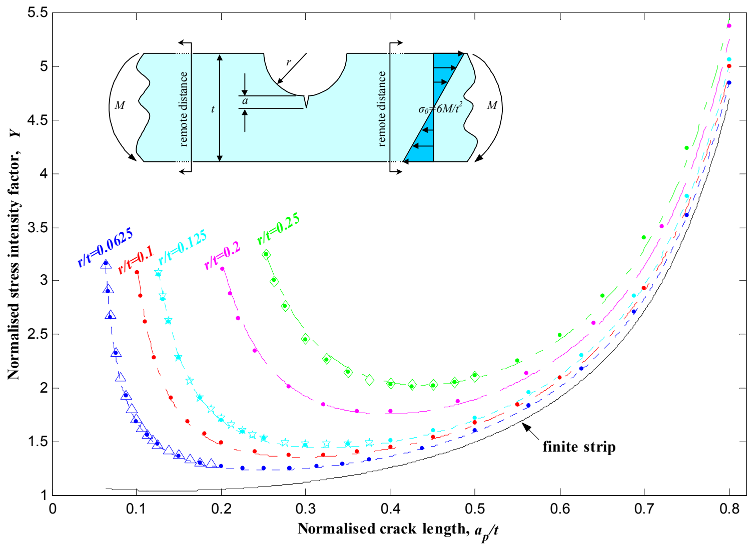


Figure 5: Normalised SIFs (Y-Factors) for an edge crack emanating from a semicircular notch in a finite thickness strip with r/t=0.0625, 0.125, 0.25, 0.1, and 0.2 pure bending and those of finite strip [13].

These are shown compared with finite element results and because the composition solution is in the form of a weight function, solutions under tension or any other stress state can be equally easily generated knowing only the crack-line stress distribution in the uncracked body.

The authors have also demonstrated the principle with U-Notches [11] and V-Notches [12]. Figure 6 below shows Edge Crack solutions for a large number of U-Notch cracked geometries under remote tension. One single r/D solution with infinite t can be used to compose an infinite number of solutions in any thickness strip. The calculation is preformed in seconds using a basic PC.

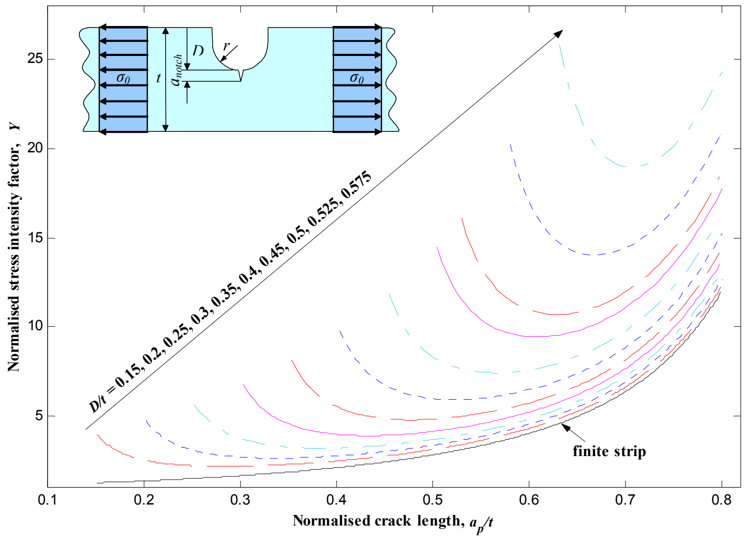


Figure 6: Normalised SIFs (Y-Factors) for an edge crack emanating from a U-notch in a finite thickness strip: r/t=0.125 and various r/D ratios, under remote uniform tension and those of a finite strip (i.e. D/t=0) [13].

Future Work

Ongoing development of the composition theme at UCL has identified the possibility of combining geometric influences of two or more basic notch forms to build SIF solutions for more intricate geometric profiles in a manner that is both conceptually and mathematically simple (Figure 7). Realisation of this observation allows the rapid determination of an almost limitless number of SIF solutions from a small 'library' of constituent geometry solutions. Similarly, is thought that the current 'library' of solutions for basic notch types may contain sufficient information to allow solutions to be derived for a number of differing notch types. Internal notches and axis-symmetric notches are given as representative examples in Figure 7. In all cases the precise form of a suitable composition scheme and constituent solutions required are at present not validated.

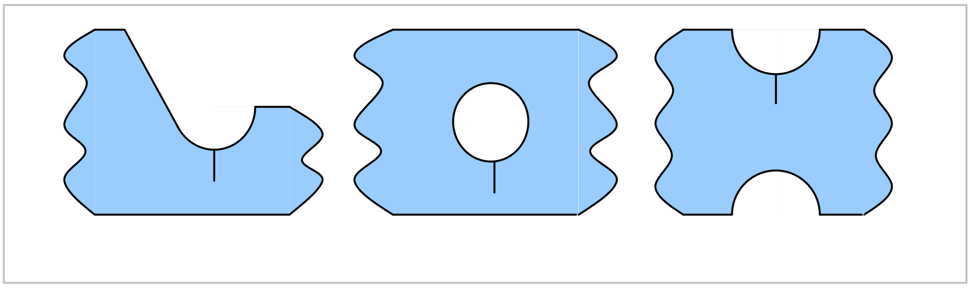


Figure 7: Examples of geometrical shapes that can be built by combining basic notch types.

Components of the above form are common to numerous engineering applications. Successful determination of a weight function solution of the accuracy and flexibility, already achieved for the more basic notch types, would provide design engineers with a powerful analysis tool.

Cracks at external notches in real three-dimensional engineering components invariably develop as two-dimensional surface flaws. An extension to the existing methodology envisages application of the composition scheme to such crack systems. Reference [5] describes a general procedure to compose weight functions to determine SIF solutions for surface cracks in complex three-dimensional geometries. This is achieved, in essence, via the composition of two-dimensional geometry solutions with three-dimensional 'base' geometry solutions. At present the analysis is restricted to the deepest point of the crack utilising weight functions of the form employed for the two-dimensional analyses. Preliminary solutions, (Figure 2) validated against full three-dimensional FE solutions, indicate that the fundamental concept is applicable and may be used to generate new solutions for notched plate, pipes and shells.

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