Influence of addendum modification coefficient on the gear load capacity

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ABSTRACT

This paper gives the description of the procedure developed for researching the influence of addendum modification coefficient value on the load capacity of cylindrical involute gear. The numerical method – Finite Element Method (FEM) – is used for the determination of stress state for meshed tooth flanks and roots. The model of meshed gears' teeth contact in FEM is made to enable the simultaneously monitoring tooth flanks stress state and tooth roots stress state.

In order to compare the stress states of meshed teeth's flanks and roots during the meshing period for gear pairs with different values of addendum modification coefficients the comparative diagrams are made and shown in this paper. Many conclusions about the influence of addendum modification coefficient at tooth defformation and gear load capacity are obtained from that diagrams.

The diagrams are used for making the special procedure and diagram sets which enable one to make the right choice for the values of the addendum modification coefficients for a particular gear pair in accordance with the general goals – increasing the gear load capacity and reducing vibrations, shocks and noise during the gear pair working. This gives possibility to easily recognise the influence of these values on the all gear mechanical phenomenons. So, the paper opens a completely new access to researching and calculation of gear pairs.

INTRODUCTION

There are many factors that influence at the mechanical behavior of gears. The gear mechanical phenomenon are greatly conditioned by teeth's profile. The addendum modification is one of the most important and influential values for tooth profile. The addendum modification of tooth profile is equal to the product of the addendum modification coefficient value (x_1, x_2) and the gear pair module m.

All diameters of a gear pair with addendum modification are different from the corresponding diameters for similar gear pair without addendum modification, except the reference diameter and the base circle diameter. In accordance, addendum modification gives a possibility for adjustment of tooth profile shape and gear center distance. When gears from one gear pair series are made with the addendum modification and the sum of addendum modification coefficients (x_1+x_2) is not constant value, gear pairs from this series have varying center distance. In accordance with this, one of the most important features of involute gears is to keep a constant transmission ration while center distance is variable, [3], and [4].

Consequently, center distance stays unchangeable while the sum of addendum modification coefficient is constant. Then, the sum division on addendum modification coefficients of meshed gears (x_1, x_2) can be made in accordance with different criteria: to increase load capacity of tooth flanks, to increase load capacity of tooth roots, to increase contact ration, to avoid bad tooth shape or other. The right choice of addendum modification coefficients can directly influence on improvement of gear pair working characteristics. Therefore, it leads to increase of gear load resistance and gear life-time, as well as to reduce of vibrations and shocks during gear pair working.

In accordance with this, first step in choosing of optimum addendum modification coefficients is defining the required aspect. For example, when the tooth flanks have thermic treatment, tooth root load resistance is competent for the gear pair calculation and exist the requirement for approximately uniformly tooth root load resistance of both gears in mesh. On the other hand, the prime goal can be the requirement of the life-time increasing, when usually pinion life-time and wheel life-time should be taken as same. The engineers notice the reducing of noise in the gearings with contact ration value increasing and pressure angle reducing. Therefore, the right choice of values x_1 and x_2 could lead to the noise reducing and vibration reducing during gear transmissions. However, it is clear that different aspects lead to contradictory solutions for addendum modification coefficient values, when the optimal solution can be chosen in accordance with practical knowledge and experience. That is why, the researches and analyses of influence of addendum modification coefficient on the stress and deformation states of meshed gears is very important for advancement of gears calculations.

THE DESCRIPTION OF THE DEVELOPED METHODOLOGY AND THE FEM MODEL

The numerical method – Finite Element Method (FEM) – is used for the determination of deformation and stress state for meshed tooth flanks and roots, which is necessery for the defined research. The model of meshed gears' teeth contact in FEM is made to enable the simultaneously monitoring tooth flanks stress state and tooth roots stress state.

A detailed description of the development procedure for the FEM model intended for the gear teeth deformation and stress state research is presented in [2]. This description contains all details considering the choice and the development of the model with the description of the finite element type and procedure used in modelling the gear teeth contact conditions and the external load. The chosen physical model that was taken for the verification and analysis is a particular gear pair for large transport machines (dredges). Its main characteristics are: number of teeth z_1=20, z_2=96; addendum modification coefficients x_1=0, x_2=0.328; face width b=350mm; module m=m_n=24; pressure angle $\alpha=\alpha_n=20°$; helix angle $\beta=0°$; rotational wheel speed n_2=4.1596 min^{-1}; pinion torque T_1=526.41667 KN·m; wheel torque T_2=2526.8 KN·m; material: E = 206 000 N/mm^2; ν = 0,3. The normal forces for this gear pair of F_{bn} = F_{bn1} = F_{bn2} = 2334.1706 KN have been used as an external load in the gear FEM models, fig.1. Obtained results are compared with corresponds many different aspects. The very high coincidence is obtained.

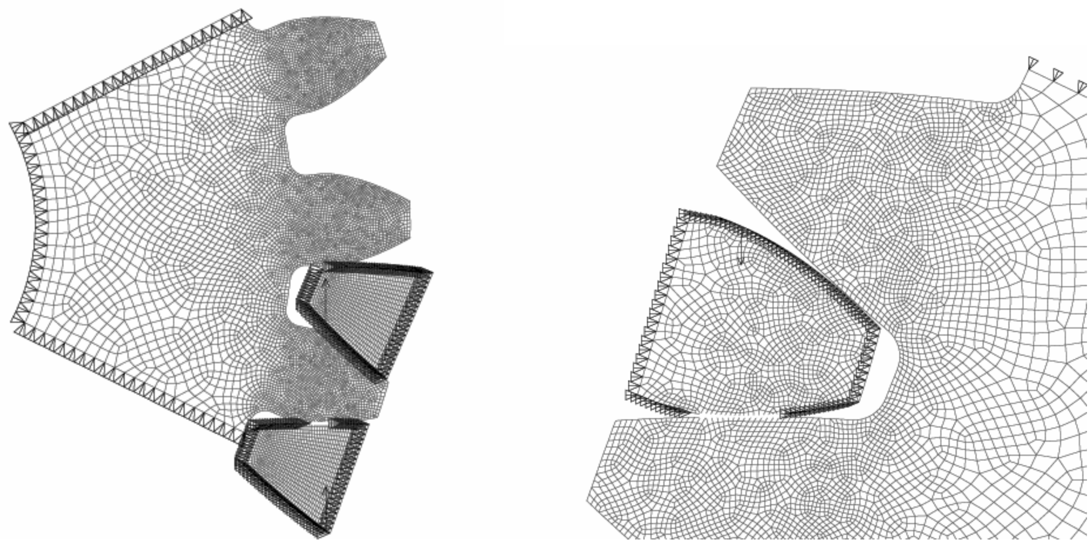

Fig.1. The pinion FEM model (two tooth pairs in contact) and a detail of the wheel FEM model (single meshed period)

In accordance with the described model, two symmetrical FEM models have been developed for each gear pair of studied seria: one for determining pinion deformation and stress state and other for determining wheel deformation and stress state. Some developed models make possible the study of real contact between involute surfaces of meshed teeth flanks and a simultaneous calculation of total teeth deformation and both the flank and the root stress state.

In order to reduce the calculation time, the basic assumptions are taken from the theoretical analysis of the load distribution among meshing tooth pairs, described in [1]. Consequently, the developed methodology described in this paper takes into consideration only the maximum load values (the only values important for the calculations of the tooth flank load resistance and the tooth root load resistance). Maximum load values at the characteristic points of the path of contact are expected. These points correspond to the moments of changing the simultaneously meshed tooth pairs number. This assumptions have been verified through the all-inclusive research performed on the plane FEM model for a certain gear pair. This research is described in details in [7]. Therefore, within the present methodology, the calculations of spur gear pair deformation and stress state are performed only at the characteristic points on the path of contact: A (first contact point), B (point of passing from period with two tooth pairs in contact to single meshed tooth pair period), C (pitch point), D (point of passing from single meshed tooth pair period to period with two tooth pairs in contact), and E (last contact point). A computer program SIGMA, developed in [6], provides the radii that correspond to these points and define their positions on the path of contact.

During the developing of the FEM model for meshed gear pair the special attention is paid to the simulation of contact conditions on meshed teeth's flanks. The "point-to-surface" contact finite elements are chosen for tooth contact simulation. The particular characteristics of the "point-to-surface" contact finite elements are used during the FEM calculations. Very high level of coincidence between the behavior of contact zone obtained by the developed FEM procedure and the behavior of contact zone at real gear pairs.

First of all, in order to perform the defined research and in accordance with the developed methodology, the plane FEM models for meshed teeth are developed. These models correspond to the middle tooth between the modelled teeth for one of the characteristic points, while each FEM model is a model of a gear segment consisting of three teeth. The models enable the successful calculation of all important variables (deformation, stiffness, load distribution, flank stress state, root stress state), as well as the determination of variable changes along the path of contact for a tooth pair. Having positioned the developed FEM model onto one of the calculated positions, the boundary conditions and the external loads are defined for every finite element model. The external load is simulated by the unit load, which is uniformly distributed along the tooth pair contact line and acts in the direction of the path of contact (the direction normal to the contact involute surfaces of meshed teeth flanks). This is in accordance with the assumptions of the uniform load distribution along the contact line. Fig.1 represents discrete models (finite element meshes) for two cases of FEM contact models: the first for the pinion calculations in point A (period with two tooth pairs in contact) and the second for wheel calculations in point B (single meshed tooth pair period).

During the developing of gears' meshing simulation this paper pays special attention to the developing of the procedure for load distribution calculation and for obtaining its influence on the load capacity of meshed gear pair. This is in accordance with the fact that the teeth stiffness is the most influential value for load distribution among simultaneously meshing tooth pairs. Also, this is in accordance with the fact that the load distribution is the most influential mechanical phenomenon for the deformation and stress state of gear pairs as well as for theirs load capacity. The stiffness of tooth pair directly depend on the values of the loads tresferred by the tooth pairs that is simultaneously in contact. As a consequence, for period with two tooth pairs in contact the developed procedure used iterations for obtaining the values of stiffness and load for each tooth pair in contact.

Some papers, [2] and [8], give the detailed researchs of the mathematical models developed for analytical calculation of load distribution while the cylindrical involute gear pair transmits some external load. In accordance with them: in the first iteration, for the period with several tooth pairs in contact, the uniform distribution of normal load F_{bn} among simultaneously meshing tooth pairs is assumed. For such load case, the FEM calculation with the developed meshed gears models gave the results for total pinion tooth deformation and total wheel tooth deformation (u_{z1} i u_{z2}).

The sum of these values is the total deformation for a tooth pair in mesh u'. Then, the formulas:

$$c = \bar{q}/u\,;\; c' = \frac{c_1 \cdot c_2}{c_1 + c_2}\,;\; c_0 = c_1' + c_2' \tag{1}$$

give the values of the tooth pair stiffness c_i' for all simultaneously meshed tooth pairs, as well as the total mesh stiffness c_0. When the obtained values are inserted in the following equations:

$$\bar{q}_1 = \frac{c_1'}{c_1' + c_2'} \cdot \frac{F_{bn}}{b} = \frac{c_1'}{c_0} \cdot \frac{F_{bn}}{b}\,; \qquad \bar{q}_2 = \frac{c_2'}{c_1' + c_2'} \cdot \frac{F_{bn}}{b} = \frac{c_2'}{c_0} \cdot \frac{F_{bn}}{b} \tag{2}$$

the unit loads q_i, get corrected values for all simultaneously meshing tooth pairs. Then new FEM calculations with similar gears models, but corrected loading, give the results for the determination of the next iteration unit load values. The number of iterations depends on the individually required accuracy.

In the last iteration, the developed methodology and FEM calculations give all the values necessary for the determination of monitored variables: tooth deformations, tooth stiffness, load distribution, maximum equivalent stress in tooth roots and maximum equivalent stress on the meshing tooth flanks.

THE ANALYSIS OF THE CALCULATION RESULTS

In this paper, the requirement for constant center distance is respected during use the developed methodology to choose the optimum tooth profile shape for a gear pair. In engineering practice, this is the most frequently requirement. The defined center distance and defined values of module and number of teeth mean that sum of addendum modification coefficients for meshed gears will be constant. One particular gear pair is taken for the analysis presented in this paper. Its main characteristics are: z_1=20; z_2=96; b=175mm; m=m_n=24; $\alpha=\alpha_n=20°$; $\beta=0°$; n_2=4.1596 min^{-1}; T_2=1263 KN·m; E=206 000 N/mm^2; ν=0,3, pressure angle and radius of root curvature belong to standard involute profile. In accordance with ISO/DIN standard recommendation (detail presented in [3] and [4]) the addendum modification coefficient sum of $x_1+x_2=0.5$ is chosen, as well as five cases of sum distribution on the addendum modification coefficient for pinion teeth x_1 and the addendum modification coefficient for wheel teeth x_2, tab.1.

The above defined geometric dimensions of studied gear pair series and described procedure for numerical calculation have been use for numerical experiments, whose results are presented in this paper. Graphic display is the best way for presenting these results. In order to compare the stress and deformation states of meshed teeth's flanks and roots during the meshing period for gear pairs with different values of addendum modification coefficients the comparative diagrams are made and shown in this paper. All values useful for the researching of mechanical phenomenon at gear pairs are monitored through the comparative diagrams.

Figure 2 presents the comparative diagrams for total pinion tooth deformation u_{z1}, total wheel tooth deformation u_{z2}, total deformation for a tooth pair in mesh u', figure 3 presents the comparative diagrams for tooth pair stiffness c', total mesh stiffness c_0 and unit load distribution q and figure 4 presents corresponding diagrams for comparing the values of equivalent stresses on the meshed teeth flanks σ_H, as well as the values of equivalent stresses in the critical cross sections of pinion tooth root and wheel tooth root (σ_{F1}, σ_{F2}).

Tab. 1

Gear pair	1	2	3	4	5
coefficient x_1	-0.1	0	0.3	0.5	0.7
coefficient x_2	0.6	0.5	0.2	0	-0.2
contact ration ε_α	1.700695	1.636393	1.5755	1.5238	1.46091

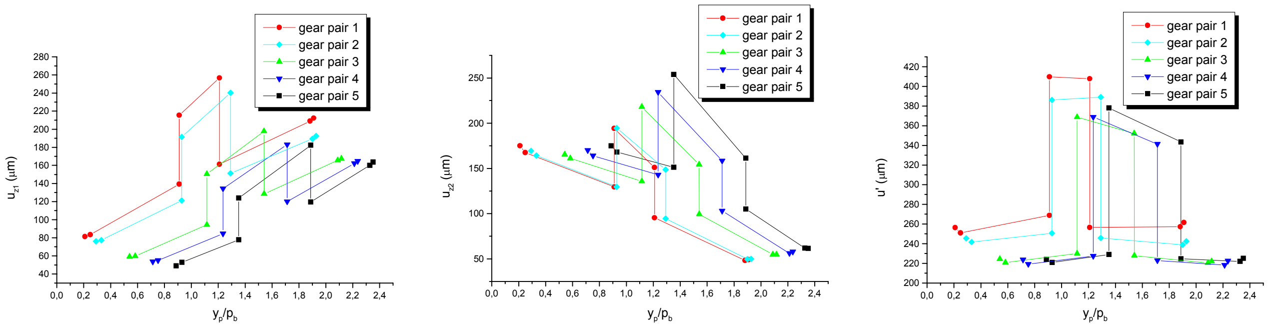

Fig.2. Comparative diagrams for monitoring the tooth deformations for the gear pair series

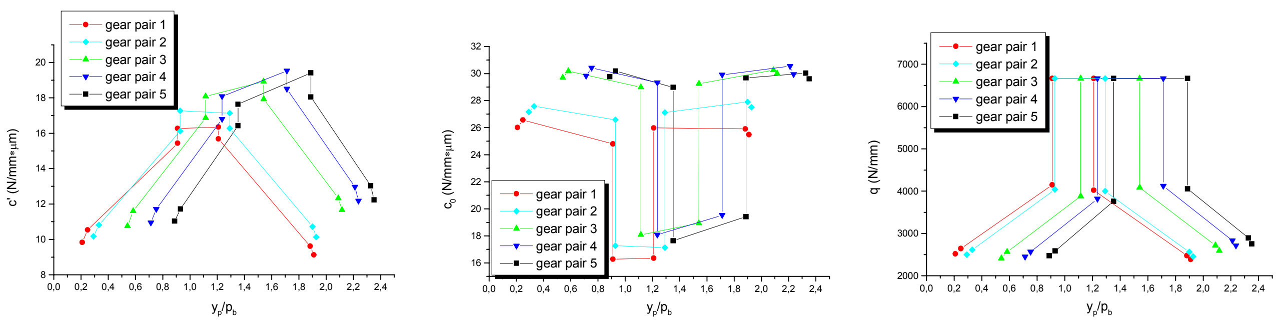

Fig.3. Comparative diagrams for monitoring the tooth stiffness and load distribution for the gear pair series

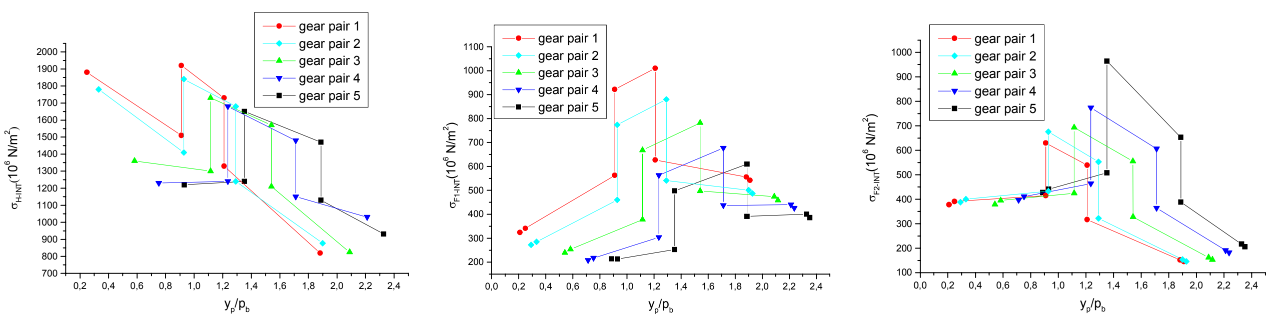

Fig.4. Comparative diagrams for monitoring the stress states of teeth flanks and teeth roots for the gear pair series

Many conclusions about the influence of addendum modification coefficient on tooth defformation and gear load capacity are obtained from these comparative diagrams:

- The addendum modification coefficient increase leads to the tooth deformations reduction;
- The reduction of contact ration ε_α, while sum of addendum modification coefficients of pinion and wheel (x_1+x_2) stays constant, leads to single meshing period extend, as well as to total tooth deformations reduction and tooth stiffness increasing. This characteristics make gearing work very quiet;
- When addendum modification coefficiens of meshed gears have variable character, the contact ration value and the lengths of periods with different number of tooth pairs in mesh also became variable, but the load distribution values stay unchanged;
- In all cases stadyied in this paper, the maximum contact stresses exist in the point of passing from period with two tooth pairs in contact to single meshed tooth pair period (point B on the path of contact);
- The contact ration reduction, that appropriate with increasing of addendum modification coefficient for pinion x_1 and reducing of addendum modification coefficient for wheel x_2, leads to the contact stresses reduction;
- The most uniformly contact stress variation in the periods with two tooth pairs in contact exists when the addendum modification coefficients have the values of x_1=0.5 and x_2=0. That is why, the gear pair with x_1=0.5 and x_2=0 can be the exelent choise when the reduction of impact load in moments of changing of meshed tooth pair number is required;
- The maximum stresses in tooth roots always exist at the point on the path of contact that define external point of single meshed period (point D for pinion and point B for wheel).

The describe comparative diagrams are used for making the procedure and diagram sets which enable one to make the right choice for the values of the addendum modification coefficients for a particular gear pair in accordance with the chosen goal. These diagram sets gives possibility for easy monitoring the changing of tooth flanks stress and tooth roots stress during meshing period for a tooth pair. In accordance with the conclusions for the points with maximum stress values, which are described above, diagram sets monitor following values:

- Equivalent stress in the pinion tooth root in point D on the path of contact (point of passing from single meshed tooth pair period to period with two tooth pairs in contact) - $\sigma_{F1\text{-}INT\text{-}D}$,
- Equivalent stress in the wheel tooth root in point B on the path of contact (point of passing from period with two tooth pairs in contact to single meshed tooth pair period) - $\sigma_{F2\text{-}INT\text{-}B}$ and
- Equivalent contact stress on tooth flanks B on the path of contact (point of passing from period with two tooth pairs in contact to single meshed tooth pair period) - $\sigma_{H\text{-}INT\text{-}B}$.

The exponential functions which describe how the maximum teeth's flanks stress values and the maximum teeth's root stress values depend on the values of addendum modification coefficients and contact ration for a particular gear pair are defined using numerical methods with chi-square minimization. The chosen exponential functions describe these dependences with minimum declinations:

$$\sigma_{F-INT}(x) = a \cdot e^{b \cdot x} \tag{3}$$

$$\sigma_{H-INT}(x) = y_0 + A_1 \cdot e^{x/t_1} + A_2 \cdot e^{x/t_2} \tag{4}$$

$$\sigma_{F-INT}(\varepsilon_\alpha) = y_0 + A_1 \cdot e^{\varepsilon_\alpha/t_1} + A_2 \cdot e^{\varepsilon_\alpha/t_2} \tag{5}$$

$$\sigma_{H-INT}(\varepsilon_\alpha) = y_0 + A_1 \cdot e^{\varepsilon_\alpha/t_1} + A_2 \cdot e^{\varepsilon_\alpha/t_2} \tag{6}$$

In these expressions e is natural logarithm basis and a, b, y_0, A_1, A_2, t_1 and t_2 – constants.

The function which describes how the maximum teeth's root stress values depend on the values of addendum modification coefficient is presented with the expression (3). The same exponential function is result obtained in paper [5] thought numerical experiments especially adapted for tooth root stress investigation. This confirms obtained relations.

Figure 5 presents the diagram sets obtained for the studied gear pair series. In this case, the exponential functions which describe researched relations are:

$$\begin{aligned}
\sigma_{F1-INT-D} &= (921.87814 \cdot e^{-0.60367 \cdot x_1}) \cdot 10^6 N/m^2 \\
\sigma_{F2-INT-B} &= (651.99516 \cdot e^{0.37026 \cdot x_1}) \cdot 10^6 N/m^2 \\
\sigma_{H-INT-B} &= (1575.5721 + 264.61127 \cdot e^{-x_1/0.54562}) \cdot 10^6 N/m^2
\end{aligned} \tag{7}$$

$$\begin{aligned}
\sigma_{F1-INT-D} &= (682.51642 \cdot e^{0.60367 \cdot x_2}) \cdot 10^6 N/m^2 \\
\sigma_{F2-INT-B} &= (784.01507 \cdot e^{-0.37026 \cdot x_2}) \cdot 10^6 N/m^2 \\
\sigma_{H-INT-B} &= (1575.53626 + 3.1527 \cdot e^{x_2/0.01449} + 106.25982 \cdot e^{x_2/0.54581}) \cdot 10^6 N/m^2
\end{aligned} \tag{8}$$

$$\sigma_{F1-INT-D} = (37.25227 + 21.14178 \cdot e^{\varepsilon_\alpha / 0.44403}) \cdot 10^6 N/m^2$$
$$\sigma_{F2-INT-B} = (577.37069 + 6238387.8 \cdot e^{-\varepsilon_\alpha / 0.14634}) \cdot 10^6 N/m^2 \quad (9)$$
$$\sigma_{H-INT-B} = (1481.59144 + 0.30842 \cdot e^{\varepsilon_\alpha / 0.23379}) \cdot 10^6 N/m^2$$

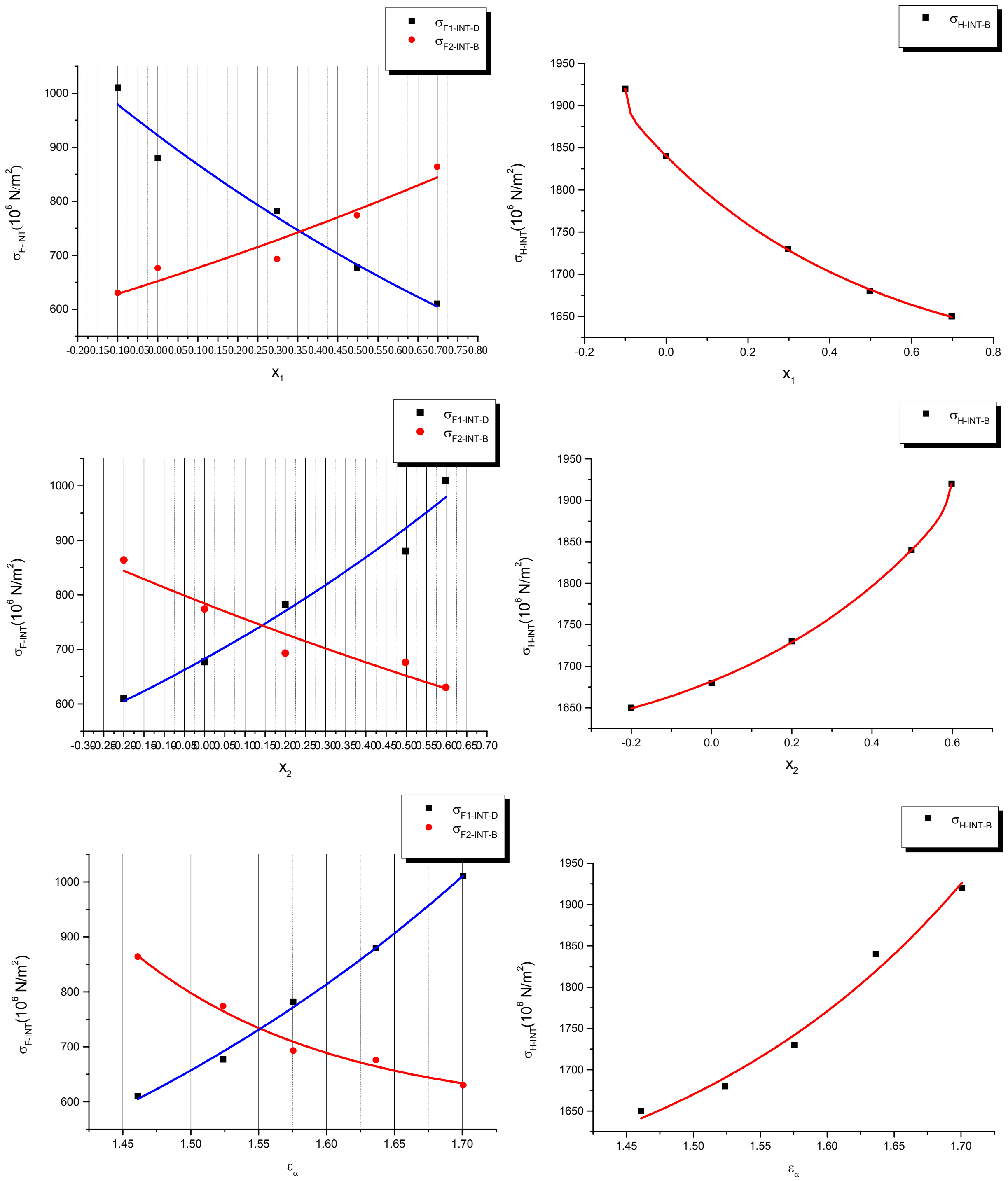

Fig.5. Diagram sets for choosing the optimal values of the addendum modification coefficients for defined gear pair series

These diagrams are used for making the right choice for the values of the addendum modification coefficients for a particular gear pair in accordance with the general accepted aspect – the requirement for approximately uniformly tooth root load resistance of both gears in mesh, which are made from same material in this case. The obtained optimal values for addendum modification coefficients are x_1=0.35 and x_2=0.15. It is important to notice that the same values would be obtained with the procedure defined by DIN 3992 Standard (presented in [4]), which is mostly used procedure in engineering practice today. This is one more confirmation for the precision and convenience of developed procedure. Even more, the procedure and diagram sets developed and described in this paper are more explicit and simpler than the mentioned DIN diagrams.

CONCLUSION

The procedure developed for researching the influence of addendum modification coefficient values on the load capacity of cylindrical involute gears are described and verified in this paper. The special attention is paid to the developing of such Finite Element Model which enables the simultaneously monitoring tooth flanks stress state and tooth roots stress state. Also, the FEM model simulate the contact conditions on meshed teeth's flanks in the optimum way. That gear pair model is a very important improvement of existing gear pair models in FEM and gives a possibility for detailed researchings presented in this paper.

The large researching is performed for one gear pair series. Graphic display is the best way for presenting these results. In order to compare the stress and deformation states of meshed teeth's flanks and roots during the meshing period for gear pairs with different values of addendum modification coefficients the comparative diagrams are made and shown in this paper. All values useful for the researching of mechanical phenomenon at gear pairs are monitored through the comparative diagrams. Many important conclusions about the influence of addendum modification coefficient on tooth defformation and gear load capacity are obtained from that comparative diagrams. Also, that diagrams are used for making the special procedure and diagram sets which enable one to make the right choice for the values of the addendum modification coefficients for a particular gear pair in accordance with the general goal – increasing the gear's load capacity and reducing vibrations, shocks and noise during the gear pair working. This gives possibility to easily recognise the influence of these values on the all gear mechanical phenomenons. So, the paper opens a completely new access to researching and calculation of gear pairs.

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