FRACTURE CRITERIA OF RUBBER-LIKE MATERIALS UNDER PLANE STRESS CONDITIONS

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ABSTRACT

This work concerns an investigation on fracture criterion of filled and unfilled elastomer vulcanizates and thermoplastics. Firstly, some fracture criteria models reported in the literature have been applied to a set of experimental data obtained from tests including various loading paths (simple tension, equal biaxial tension and biaxial tension) and performed on four materials : a Natural Rubber (NR), a Styrene Butadiene Rubber (SBR), a Polyurethane (PU) and a Thermoplastic Elastomer (TPE).

Then, a new failure criterion based upon an equivalent elongation concept is proposed. This equivalent elongation seems to be linearly dependent on a given biaxiality ratio $n=\frac{\ln(\lambda_{2b})}{\ln(\lambda_{1b})}$, which leads to express the principal elongations at break as function of the biaxiality n and two experimental parameters. A quite good agreement is highlighted when comparing the failure experimental data with the analytical expressions for the tested elastomers.

Introduction

The elastomers are highly deformable materials and their uses are very widespread in industry. Therefore, the establishment of a failure criterion, under monotonic loadings, in order to predict the failure of the rubbery parts in a structure is of a great interest for the engineers when designing and dimensioning these structure components.
The fracture criteria of rubber like materials can be classified in two categories. The first one deals with crack growth of a precracked specimens, whereas, the second one concerns crack nucleation.

Since the work of Griffith [1] who introduced the global approach of the fracture mechanics, characterized by the strain energy release rate G in the case of linear elastic behavior, many authors were interested in the fracture of rubberlike materials. Later, Rice [2] introduced the J integral concept, equivalent to the G parameter, which can be used in the case of non-linear reversible elasticity. The critical value of J integral (or G parameter) allows to determine the critical load (or deformation) corresponding to the beginning of a pre-existing defect propagation.
The determination of this critical value is more easier in the case of linear elastic behaviour than in non-linear one. Many works (see for example Rivlin, Thomas [3]; Andrews [4] and Andrews, Billington [5]), in the case of rubbers, were carried out to evaluate experimentally the J integral. Based upon the η parameter introduced by Turner [6], Aït Hocine et al. [7-10] have proposed in a recent work an alternative expression of J integral, which allows an experimental evaluation of J.

On the other hand, in absence of cracks or defects, several authors have developed some fracture criteria for unfilled rubbers, based on the stresses or elongations at breaking. Smith et al. [11-16] and Bueche, Halpin [17-19] have earlier examined the ultimate properties of elastomers. They defined the concept of "failure envelope" in a stress-strain plan which is built by using the time-temperature equivalence principle. Then, Smith and Rinde [14] have established an experimental equipment to obtain a constrained biaxial strain field (pure shear) in a sample. Therefore, a long thin-walled cylindrical specimen made of an unfilled vulcanizate elastomer (SBR) is stretched in the longitudinal direction, the outside diameter being simultaneously maintained at its initial value by regulating the internal gas pressure. They noticed that the ultimate elongations in simple tension are similar to those in pure shear, whereas the corresponding axial stresses are slightly higher in shear tests. For more details concerning this experimental set up, see Smith and Frederick [20]. Dickie and Smith [15, 16], by inflating a membrane of an unfilled vulcanized elastomer (SBR), have produced an equal biaxial strain field in the region near the pole, as theoretically established by Adkins and Rivlin [21]. Dickie and Smith found that both the failure elongations and the corresponding stresses were more important in equal biaxial tension than in uniaxial tension. These values were also higher than that obtained by Smith and Rinde [14] under a constrained biaxial loading. Later, Kawabata [22] studied the rupture of elastomers under biaxial loadings under a temperature of 30°C and a strain rate of $0.1s^{-1}$. He developed several techniques to obtain various biaxial stretching ratios. The first one consists of pulling thin plates, dimensions of which were $100x100x1mm^3$, in two perpendicular directions, while in the second one, he used a system allowing the inflation of a thin membrane. He noted that the elongations at breaking were independent of the loading path (uniaxial

or biaxial) for the two investigated unfilled vulcanized elastomers (NR and SBR). However, the stresses at break depend on the loading path, the maximum value being reached under the equal biaxial loading.
Another interesting elongation criterion is the one proposed by Neviere et al. [23] to chraracterize the failure of solid propellants. Some other authors have focused their investigations on criteria based on failure stresses and originally developed for metals (Rankine, Tresca and Von Mises criteria). These criteria were reviewed by Thorkildsen [24] who tried to extend them to rigid polymers. They were also applied by Smith and Rinde [14] to unfilled SBR elastomers.
It must be noted that the most fracture criteria above mentioned have been validated with experimental tests carried out on unfilled elastomers.

In this paper, we will be interested only in the fracture of smooth (no-cracked) specimens. This work consists in seeking a generalized failure criterion at multiaxial quasi-static loadings using industrial unfilled and carbon black filled elastomer vulcanizates and thermoplastics. More precisely, in the first part of this paper, some of the fracture criteria reported in the literature will be examined. In the second part, a criterion based on an equivalent elongation is proposed and its validity is experimentally verified in the case of various loading paths. The most important result is that this criterion requires only two parameters, which can be identified from two loading paths (for example uniaxial tension an equal biaxial tension tests).

Experimental study

Materials

Four kinds of materials have been investigated

- a filled natural rubber (NR) vulcanizate and crystallisable,
- a filled styrene-butadiene rubber (SBR) vulcanizate,
- a thermoplastic elastomer (TPE),
- an unfilled polyurethane (PU).

Specimens and mechanical tests

To obtain a wide range of loading conditions, sets of experiments including uniaxial tension, equal biaxial tension and biaxial tension have been achieved by the LRCCP (Laboratoire de Recherches et de Contrôle du Caoutchouc et des Plastiques)
The tensile tests were carried out using specimens of 60mm height, 10mm width and 2mm thick. The equal biaxial tests were achieved by inflating a thin circular disc, the initial diameter of which was 200mm, its thickness being 2mm. The same device was used to produce biaxial tests on elliptic thin membranes with three ratios R_2/R_1 = 1/2, 1/3 and 1/5 (R_1 and R_2 being the great and the small radii of the ellipse, respectively).
These tests were performed at room temperature and under a strain rate of 100%/min using specimens cut out from plates. These plates have been obtained by moulding compression and their dimensions are 300x 300x2mm^3.

Figures 1a and 1b show a tensile test and the inflation of thin circular membrane respectively. Because the strain field is nonuniform over the surface of the oblate spheroid, the deformation was evaluated using an LVDT extensometer located at the pole, the only region where the strain field is purely equal biaxial.
The different biaxiality ratios are ensured using meniscuses (Figure 1c) with a fixed radius R_1=100mm and various radii R_2={20, 33, 50, 100mm}.

Figure 1. Equipment for (a) tensile test, (b) equal biaxial test, (c) various biaxiality ratios ={1, 1/5, 1/3, 1/2}.

For each loading path, three tests at least are carried out allowing to verify the reproducibility of the experimental data. All the tests are achieved up to total breaking and the ultimate stretches are measured.

Results and discussion

In what follows, we will focus on the identification of a fracture criterion based upon ultimate principal elongations. So, in a first step, we will examine, through the experimental results we have obtained, the fracture criterion proposed by Kawabata [17]. In a second step, the stresses and the strain energy density at breaking will be analysed. Then, a thorough analysis of these experimental results will lead us to propose a new alternative fracture criterion based upon principal stretches.

Fracture criterion of Kawabata

The fracture criterion of Kawabata [17] is similar to a Rankine's criterion applied to the principal elongations. More precisely, he postulated that the rupture under any loading path would take place when one of the two principal stretches reaches the one of uniaxial tension at breaking

$$\lambda_b = Max\left[\lambda_i\right] = \lambda_{b(UT)} \tag{1}$$

where i ={1, 2} defines the principal direction. This criterion suggests that the fracture of a given elastomer needs only a simple tensile test to be identified. Let us remind that, Kawabata has studied unfilled elastomers vulcanizate (NR and SBR).

To verify the validity of Kawabata criterion, breaking stretches λ_1 (principal direction 1) are reported in the Figures 2a and 2b versus λ_2 (principal direction 2) (only the results of SBR and TPE materials are repoted in this section). The representation is given in terms of a normalized stretch by dividing the elongations at breaking by that obtained in uniaxial tension λ_{UT} (λ_i/λ_{UT}, λ_j/λ_{UT}). The theoretical prediction of Kawabata criterion is also plotted in these graphs and represented by the broken lines. Because of the symmetry (i.e. λ_1=λ_2), each fracture data is twice plotted in the plan (λ_i/λ_{UT}, λ_j/λ_{UT}).

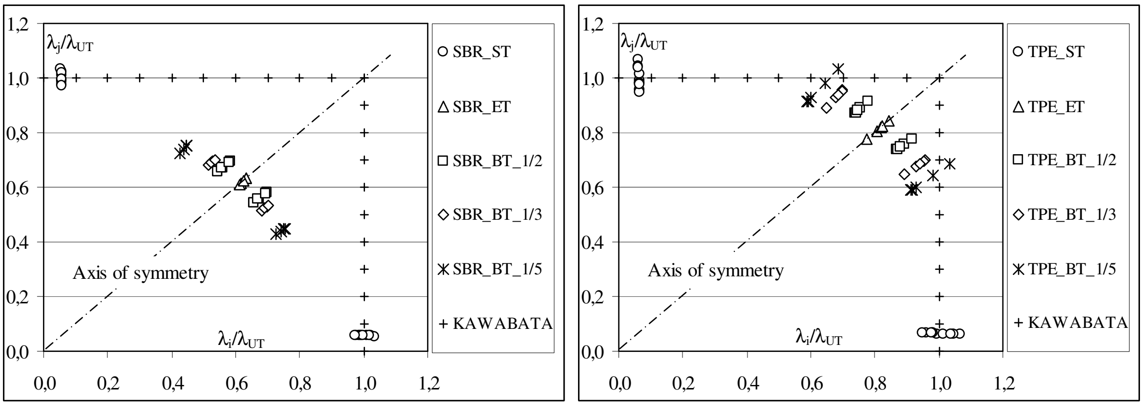

Figure 2. Ultimate normalized stretches : (a) SBR, (b) TPE

This Figure show that the fracture stretches of the TPE can be approximately described by the Kawabata criterion. However, it is clearly shown that this criterion is unable to predict the fracture of the SBR material (the same conclution is obtained with the NR and PU materials).

Strain energy density (S.E.D.) at breaking

The strain energy densities at breaking have been estimated through constitutive laws defined in terms of strain energy density functions. Only two materials are concerned in this section : NR and SBR. The parameters of the corresponding functions are issued from four kinds of loadings (uniaxial tension, uniaxial compression, equal biaxial tension and pure shear). The NR data are fitted using a 3-coefficient Mooney-Rivlin function while an Ogden formulation with two parameters has been retained to represent the SBR behaviour. In Figure 3, the failure strain energy density values reduced by that of simple tension are plotted as a function of the biaxiality ratio $n = \ln(\lambda_{2b})/\ln(\lambda_{1b})$.

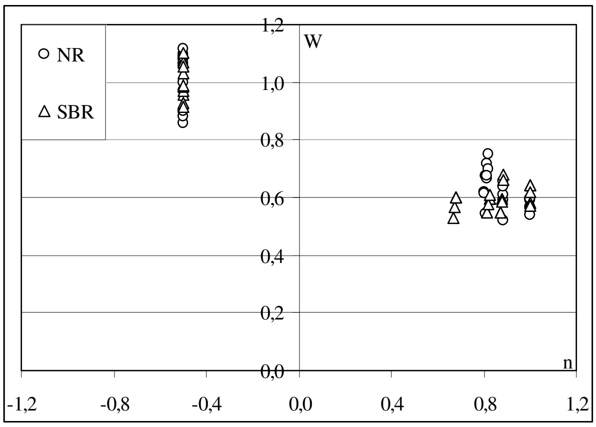

Figure 3. S.E.D. at breaking function of biaxiality ratio n

The strain energy density at breaking seems to decrease with respect to the biaxiality ratio n. However, data for pure shear loading (n=0) are required to confirm this trend and to build a criterion based upon the S.E.D.

Stresses at breaking

The Cauchy stress tensor $\overline{\overline{\sigma}}$ is derived from the strain energy density function. In the principal directions, the Cauchy stress components are expressed as follows

$$\sigma_k = \lambda_k \frac{\partial W}{\partial \lambda_k} - p \quad \text{(no sum on k)} \tag{2}$$

for $k=\{1,2,3\}$, where the pressure p is induced by the incompressibility constraint $\lambda_1\lambda_2\lambda_3 = 1$.

For each material, the previous fracture data are plotted in a normalized plan of corresponding stresses (Figures 4a and 4b). The representation is given in terms of normalized Cauchy stresses (i.e. the values are divided by the breaking stress under uniaxial tension).

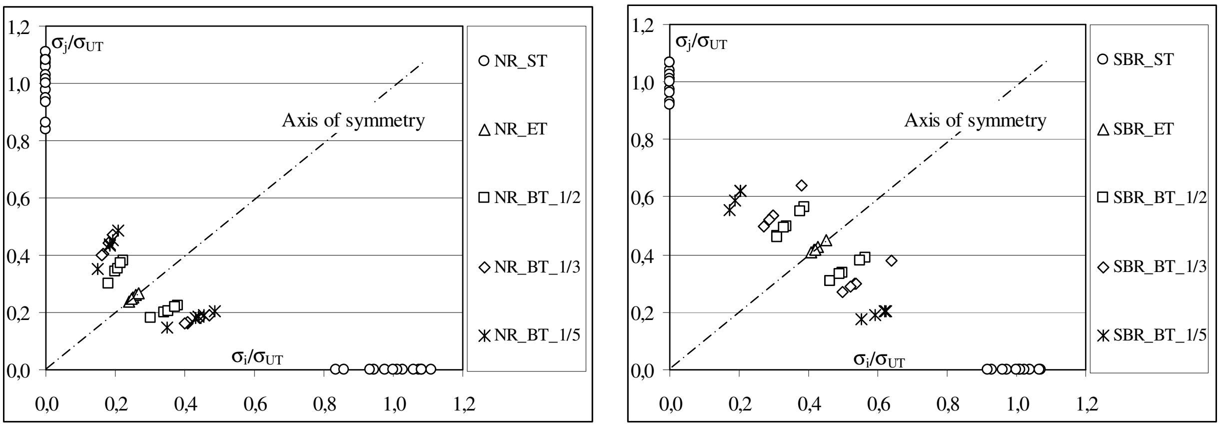

Figure 4. Failure normalized Cauchy stresses : (a) NR, (b) SBR.

It can be clearly seen that the evolution, which is pointed out, is clearly different from that obtained by Kawabata [17]. Indeed, the failure stresses he obtained increase from that of simple tension to reach a maximum under equal biaxial tension. On the other hand, the rupture points of the four materials do not exhibit the same evolution in the stresses plan. From a material to another, the evolutions, which are pointed out, are quite different, hyperbolic for NR and SBR and parabolic for PU and TPE. Thus, the definition of a unified criterion only based upon the representation of the Cauchy stresses seems to be compromized.

Equivalent elongation criterion "I" : preliminary study

The parameter I is defined as the second invariant of the deviatoric components of the deformation gradient tensor $\overline{\overline{F}}$. In the principal directions, $\overline{\overline{F}}$ is written as

$$\overline{\overline{F}} = diag\left[\lambda_1, \lambda_2, \lambda_3\right] \tag{3}$$

Which lead to the following expression of I

$$I = \left[(\lambda_1 - \lambda_2)^2 + (\lambda_1 - \lambda_3)^2 + (\lambda_2 - \lambda_3)^2 \right]^{1/2} \tag{4}$$

For an incompressible material ($J = \lambda_1\lambda_2\lambda_3 = 1$), the deformation gradient tensor can be written in the principal base as the following diagonal matrix

$$\overline{\overline{F}} = diag\left[\lambda, \lambda^n, \lambda^{-(1+n)}\right] \tag{5}$$

The combination of the Equations (4) and (5) leads to the expression of I as function of the principal elongation λ and the biaxiality n

$$I = \left[\left(\lambda - \lambda^n\right)^2 + \left(\lambda - \lambda^{-(n+1)}\right)^2 + \left(\lambda^n - \lambda^{-(n+1)}\right)^2 \right]^{1/2} \tag{6}$$

where n represents the above mentioned biaxiality ratio. The values of n we obtained at failure in our experiments, are summarized in the table 2.

Note that, under biaxial loading obtained by inflating a sheet, the values of n reported at fracture strongly deviates from that theoretically expected when designing the devices. This is because under high extension, the state of deformation at the pole progressively varies and, at the onset of fracture, is closer to equal biaxial tension.

Type of loadings	NR	PU	SBR	TPE
Simple tension	-0.5	-0.5	-0.5	-0.5
Equal biaxial tension	1	1	1	1
Biaxial tension : 1/2	0.88	0.94	0.88	0.90
Biaxial tension : 1/3	0.81	0.89	0.82	0.82
Biaxial tension : 1/5	0.82	-	0.68	0.76

Table 2. Biaxiality ratio n at rupture.

The values of I evaluated at breaking, are plotted as a function of the biaxiality ratio n in Figure 5a. Note that for the commonly used specimen, the loading path corresponds to a constant value of n, i.e. a straight vertical line in Figure 5.

In order to compare the four materials, is also given in Figure 5b the evolution of this parameter when normalised to the value obtained in uniaxial tension (i.e. $I/I_{(UT)}$).
Representing the fracture data in such a plan lead to a reduction of the discrepancy of the data even it remains quite important. The evolution which is obtained seems to be quasi linear but a lack of data in the middle region (around n=0 corresponding to pure shear) make this trend to be confirmed.

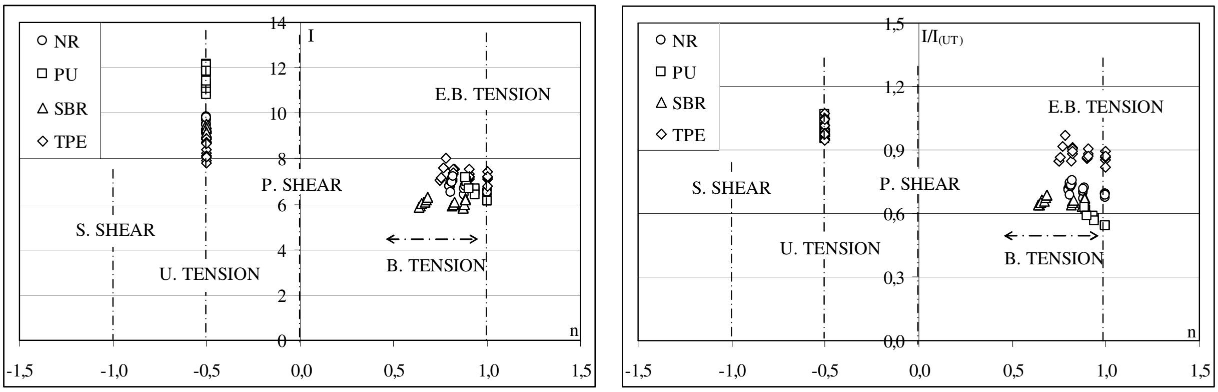

Figure 5. Fracture data : (a) plan (n, I), (b) reduced plan (n, I/I(UT))

Modified equivalent elongation criterion : " I^{ln} "

Elongations expressed in term of logarithmic quantities allow to linearize the deformation tensor $\overline{\overline{F}}$ witch is written in the principal directions as follows

$$\ln(\overline{\overline{F}}) = diag\left[\ln(\lambda), n\ln(\lambda), -(1+n)\ln(\lambda)\right] \tag{7}$$

Substituting in the Equation (4) the components of the logarithmic gradient tensor (Eq. 7) leads to introduce the new parameter

$$I^{ln} = \sqrt{6}\sqrt{1+n+n^2}\,\ln(\lambda) \tag{8}$$

At breaking, λ is equal to the ultimate elongation λ_b.

The values of the parameter I^{ln} corresponding to the onset of breaking are plotted in the Figure 6a as a function of the biaxiality ratio n. The experimental data points seem to be nicely fitted by a straight line the parameters of which are material dependant.

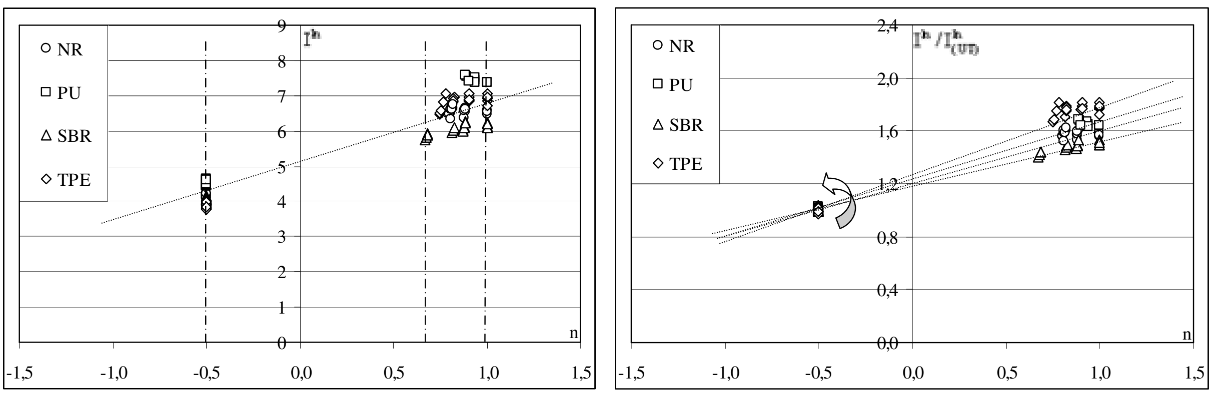

Figure 6. Fracture data : (a) plan (n, I^{ln}), (b) reduced plan (n, $I^{ln}/I^{ln}_{(UT)}$).

Reducing the ordinate axis by dividing the values by that corresponding to uniaxial tension leads to the representation given in the Figure 6b. In this new representation, the uniaxial tension corresponds to a single point of coordinates (-0.5, 1) independently of the material. The failure data can be fitted by straight lines equations of which can be written as follows

$$\left(\frac{I^{\ln}}{I^{\ln}_{(UT)}}\right)_b = \alpha.n + \beta \quad (9)$$

where α and β are the material constants obtained by a linear regression.

Thus, if the term on the right hand of the Equation (9) is assumed to be a fracture criterion, it only requires the determination of the two constants α and β which can be evaluated by achieving two tests at least but exhibiting different loading paths (for example an uniaxial tension and an equibiaxial tension).

In practice, it is useful for the engineers to express this criterion in terms of ultimate elongations. Thus, for a given material, assuming that Equation (9) is true and by combining Equations (8) and (9), the rupture stretch λ_{1b} can be written as a function of the ultimate tensile stretch $\lambda_{1b_{(UT)}}$, the rate of biaxialty n and the material constants α, β

$$\lambda_{1b} = \lambda_{1b_{(UT)}}^{\frac{\alpha n+\beta}{2}\cdot\sqrt{\frac{3}{1+n+n^2}}} \quad (10)$$

The principal elongations in the two other perpendicular directions are deduced from the general form of the tensor $\overline{\overline{F}}$ (Eq. 7) and lead to

$$\lambda_{2b} = \lambda_{1b}^{n} \text{ and } \lambda_{3b} = \lambda_{1b}^{-(1+n)} \quad (11)$$

The evaluation of λ_{1b} from the Equation (10) needs to previously determine the experimental coefficients α and β. Let us briefly remind the subsequent steps allowing the determination of these coefficients.

The Equation (8) gives the parameter $I^{\ln}$ in the case of simple tension (n = -0.5) and equal biaxial tension (n = 1) under the following forms, respectively

$$\begin{cases} I^{\ln}_{(UT)} = \frac{3}{\sqrt{2}}\ln\left(\lambda_{1b}^{UT}\right) \\ I^{\ln}_{(ET)} = 3\sqrt{2}\ln\left(\lambda_{1b}^{ET}\right) \end{cases} \quad (12)$$

By introducing $I^{\ln}_{(UT)}$ and $I^{\ln}_{(ET)}$ given by (12) in Equation (9), we obtain an algebric equation system, solution of which is

$$\begin{cases} \alpha = \frac{2}{3}\left(2\frac{\ln\left(\lambda_{1b}^{ET}\right)}{\ln\left(\lambda_{1b}^{UT}\right)} - 1\right) \\ \beta = \frac{2}{3}\left(\frac{\ln\left(\lambda_{1b}^{ET}\right)}{\ln\left(\lambda_{1b}^{UT}\right)} + 1\right) \end{cases} \quad (13)$$

So, the determination of α and β requires the experimental failure elongations of both a tensile test (λ_{1b}^{UT}) and of an equal biaxial test (λ_{1b}^{ET}).

The table 3 gives the average values of α and β. These parameters are calculated using linear regression including the whole data. The obtained values are slightly different from those obtained with the uniaxial and equal biaxial tests.

Material	α (whole data)	α (UT&ET data)	β (whole data)	β (UT&ET data)
NR	0.40	0.38	1.20	1.19
PU	0.47	0.43	1.22	1.21
SBR	0.34	0.35	1.17	1.17
TPE	0.55	0.51	1.25	1.26

Table 3. Parameters α and β for the 4 elastomers

Finally, the principal elongations λ_{1b}, λ_{2b} and λ_{3b} are estimated using Equations (10) and (11). As an example, the evolutions of the reduced elongations λ_{1b} and λ_{2b} are plotted against the biaxiality n in the Figures 7a and b in the case of the vulcanizable (SBR) and the theromoplastic (TPE), (the third breaking elongation λ_{3b} is deduced, using the incompressibility constraint. A nice agreement is highlited between the experimental data and the analytical expression (Eq. 10). The analytical model predicts a higher value of the ultimate elongation under pure shear loading (n=0) than under uniaxial tension (n=-0.5). Although this result is not expected, it is in agreement with some conclusion reported in the literature Smith, Rinde [14] and Kakavas, Blatz [25]. Nevertheless, further fracture data under pure shear loading are required to confirm the predicting capability of this criterion.

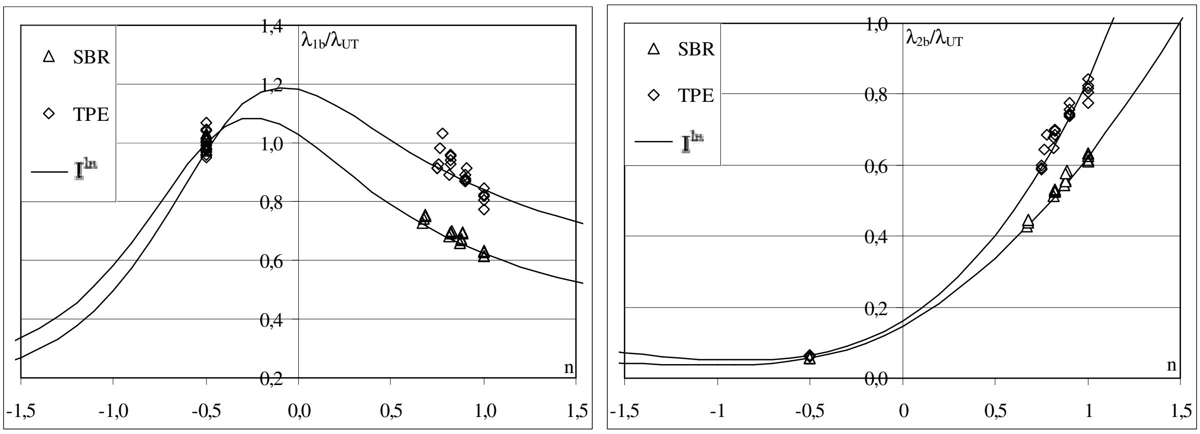

Figure 7. Reduced breaking elongation versus the biaxiality n : (a) λ_{1b}, (b) λ_{2b}

In the Figures 8a and b, the breaking points are again reported in the reduced plan of the principal elongations. In this plan, two examples of loading paths corresponding to an equal biaxial and a biaxial tests, n=1 and n=0.68 respectively, are also shown (Figure 8a).
It can be clearly seen that the fracture experimental data are well fitted by the theoretical curves obtained from the criterion of the equivalent elongation I^{ln} . Note that the same trends are obtained for the two other materials (NR) and (PU).

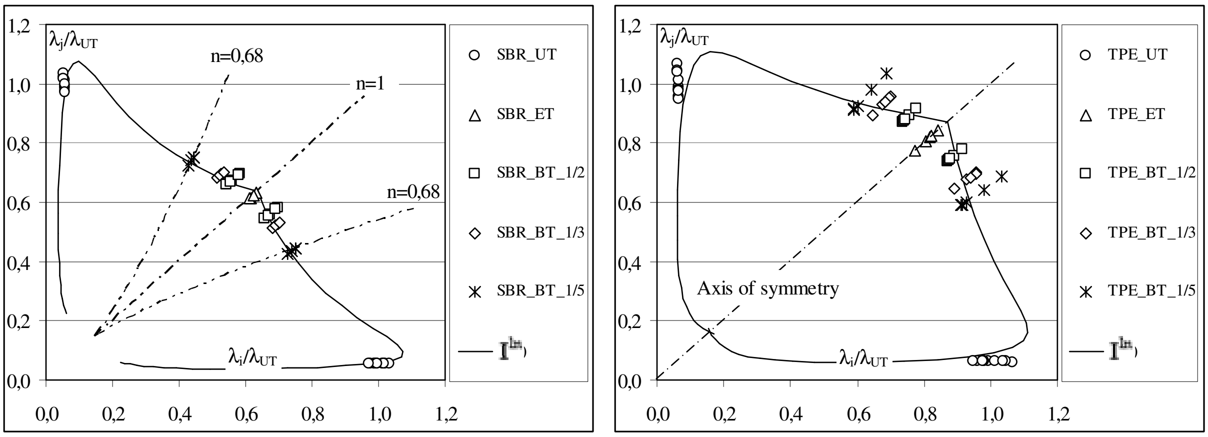

Figure 8. I^{ln} fracture criterion in the reduced plan (λ_i/λ_{UT}, λ_j/λ_{UT}) : (a) SBR, (b) TPE

Conclusion

No unified criterion exists for predicting the rupture of elastomers under mutiaxial loading conditions. In this study, failure criteria extracted from the literature have been applied to experimental data we have performed on filled and unfilled, rubber-like materials. The experimental tests have combined both tensile and biaxial loading paths.

We have pointed out that the criterion proposed by Kawabata, initially established using unfilled NR and SBR elastomers can not be extended to our materials.
When considering the fracture stresses no evident criterion based upon these quantities can be proposed yet. Nevertheless, the investigation in this direction must be pursued. As must also be continued the work concerning the establishment of a criterion based upon strain enregy considerations.

In this work, a new original failure criterion, named I^{ln} based upon an equivalent logarithmic elongation concept is introduced. By studying the failure experimal data, this parameter has been found to be directly proportional to the biaxiality ratio. Furthermore, the principal elongations at break have been analytically written as function of both the biaxiality ratio n and two material constants. A quite good agreement is highlighted when comparing the failure experimental data with the analytical results which suggests that I^{ln} is a reasonable criterion of the fracture of elastomers. Nevertheless, the experimental data basis must be enriched principally for some loading paths and more particularly for the pure shear loading.

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