# FATIGUE CRACK INITIATION IN STRESS CONCENTRATION AREAS

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**ABSTRACT**

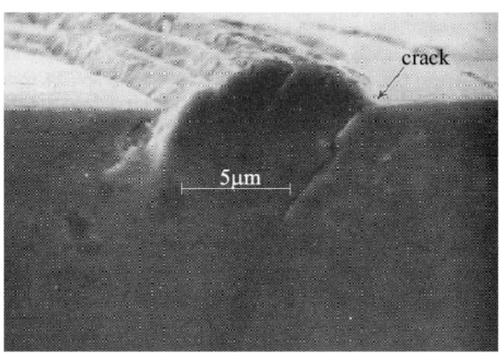
The objective of the present work is to account for stress gradient effect on fatigue initiation. A non local fatigue criterion is proposed, with a single additional parameter compared to traditional criteria, by averaging a classical criterion over a damaged area. The damaged area is primarily determined by the geometry, the loading and the additional parameter. The coefficients of the criterion have been identified to two distinct types of material (steel and light alloy), based on literature data on the one hand and tests carried out at EADS CCR and CROMEP on the other hand. The effect of the stress concentration on initiation life seems correct based on first experimental results.

## Introduction

Materials used in aircraft structures have to sustain repetitive loading conditions; they are therefore candidates to fatigue failure. Consequently the durability of aeronautical structures has to be assessed, and this process requires a large amount of testing. There is therefore a great interest in optimizing the test database in order to shorten the development cycle. Using computer simulation instead of the time consuming actual test should help. However, to do so, one needs a good understanding of the fatigue process.

The fatigue life of structural components is determined by the sum of the elapsed cycles required to initiate a fatigue crack and to propagate this crack up to the critical dimension. The relative importance of these two phases depends on the level of loading. In the case of aeronautical structure, the loads are supposed to be far enough below the elasticity limit to consider that the greater contribution is the one due to fatigue crack initiation. As a consequence, as a first approximation, fatigue crack initiation is the primary phenomenon to be taken into account in a fatigue model for (moderately loaded) aeronautical structures.

Fatigue crack initiation has been extensively observed by many researchers, and the micromechanics of fatigue initiation is nowadays well established (notably for FCC crystals) as a sequence of events which begins with dislocation motion in the slip systems of a grain (causing extrusion and intrusion on the surface, see figure 1, from Suresh [1]) and that ends up by the propagation of a macrocrack many times larger than the average grain size.



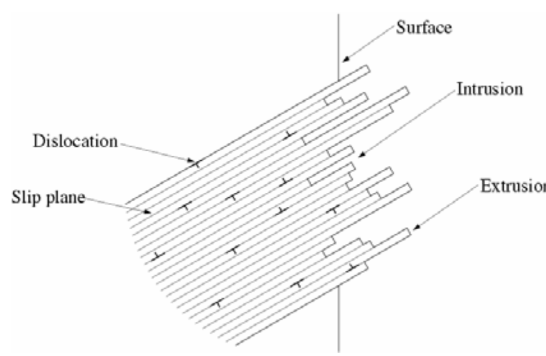




Figure 1 : crack initiation

Microplasticity seems to be at the origin of fatigue crack initiation. This phenomenon occurs for some grains of the tested sample, although during a fatigue testing, the macroscopical behaviour is globally elastic. However the loading may induce some local plastic strains in stress concentration areas, the concentrators being structural details as well as microstructural defaults. In these areas the macroscopical behaviour becomes elastic after a limited number of cycles (compared to the number of cycles to failure) as elastic shakedown is reached, but this does not reflect the behavior of each grain in the area. For some of these grains, the cyclic response may never reach the elastic shakedown. Instead they enter a state of plastic shakedown eventually leading to their failure: these grains are candidates for fatigue crack initiation sites as illustrated on figure 1.

The numerical modelisation of this phenomenon from "first principle" (that is to say dislocation dynamic) has been developed by Brinkmann and Needleman [2]. However the computation time needed for these simulations is prohibitive, even for a non industrial purpose. Macroscopical phenomenological models have also been developed in order to qualitatively represent the irreversible process of microscopical plasticity. These models follow the initial work of Cailletaud [3] who has described a constitutive rule which derives from the slip theory for a single crystal. Their application to polycrystalline aggregate has been done by Barbe [4]. However the number of finite element dof needed for such a model is directly linked to the number of grains contained in the structure; therefore the application of this model may require considerable computation resources.

Since precise, physical modelisation seems too costly, a practical point of view is considered: prediction of fatigue crack initiation is commonly achieved through the use of criteria. As the analysis of the micromechanics of fatigue tends to prove that the physical phenomenon at the basis of the fatigue failure is the accumulation of plastic deformation in some grains, the criteria should be grounded on an analysis of this plastic accumulation. The influence of other phenomenon (such as micro crack coalescence and growth) is then accounted for by the use of macro variables which are known to have impact on these phenomenons (typically the hydrostatic pressure). These considerations lead us to the usual formulation of a fatigue criterion:

$$J_2 + \alpha P_H \leq \beta \tag{1}$$

where $J_2$ is a shear stress related term (accounting for the accumulated plastic deformation in the grains) and $P_H$ is a pressure related term. This formulation has to be interpreted as follow: if at any point of the structure and at any time during the load this relation is satisfied, fatigue failure will never occur.

Although a wealth of these criteria is available in the literature, there is not yet a universally accepted approach, each criterion having significant limitations. One of the main drawbacks, shared by many criteria, is that the formulation (1) is a local formulation which is therefore unable to predict the well known effect of the stress gradient. Such a formulation is thus not directly applicable to a structure where stress concentrations exists. To alleviate this limitation some proposals have been made, which rely on the two usual methods to introduce spatial interaction terms either by introducing gradients of some constitutive variables in the model (Panoskaltsis [5]) or by using integral relations (Palin-Luc [6], Lanning [7]). The model described in this article can be related to the latter kind of approaches. We propose to average over a damaged volume the value of one of the most efficient fatigue criterion available in the literature: the Papadopoulos criterion. The definition of the damaged volume relies heavily on the Papadopoulos criterion formulation which is therefore detailed in the first section. The definition of the damaged volume itself is given in the second section. The description of the integral model and the identification of its parameters for an aeronautical steel (30NCD16) and an aluminium alloy (2524T3) are then discussed. Some applications are given in the final section.

## Papadopoulos criterion Highlights

In this part we present briefly the Papadopoulos fatigue criterion. For more details please refer to Papadopoulos [8].

In keeping with the fact that the plastic strain accumulation in some crystals eventually leads to their failure and creates potential crack initiation sites, Papadopoulos defined a first criterion, which we will call the Papadopoulos mesoscopic criterion. This criterion prevents any grain from entering a plastic shakedown state by limiting the accumulated plastic strain in each slip system of each grain. This criterion could be written as follow:

$$\forall\ (grain, slip\ \ system)\ \ in\ \ the\ \ aggregate,\ \varepsilon^{plastic}_{accumulated}(grain, slip\ \ system) \leq g' \tag{2}$$

where g' is a crystal property (called the ductility of the crystal by Papadopoulos). In order to transform this equation into a useful statement two steps are required:

1. Estimate the mesoscopic quantities from the macroscopic ones thanks to a homogenization model.
2. Estimate the accumulated plastic strain in a grain after a great number of cycles.

The first part (estimation of the mesoscopic quantities) is done by using the Lin Taylor model (which states that the deformation of the grains is imposed by the surrounding grains) as proposed by Dang Van in his PhD thesis, Dang Van [9].

Two approaches are readily available for the second step: we can either use a complex hardening model and determine the accumulated plastic strain by numerical calculation or use simplified hardening rules and determine analytically the accumulated plastic strain after an infinite number of cycles. The latter has been chosen by Papadopoulos, who showed that the accumulated plastic strain along a slip direction on a slip plane happens to be nearly proportional to the macroscopic resolved shear stress amplitude (denoted as $T_a$) when the number of cycles tends to infinity. For details on the assumption made on the behavior of the crystal, refer to [8].

Indexing the slip systems by their Euler angles (the slip plane is given by two angles *(φ,θ)*, the slip direction in this plane is then given by *ψ*), the equation (2) can now be written:

$$\forall M \in V, \forall(\theta,\varphi,\psi) \in [0,\pi]\times[0,2\pi]\times[0,2\pi], T_a^M(\theta,\varphi,\psi) \leq g \quad (3)$$

where *V* denotes the volume occupied by the structure, $[0,\pi]\times[0,2\pi]\times[0,2\pi]$ defines all the slip systems available at a material point M (by default it is supposed there are enough grains to cover all the orientations) and $T_a^M$ stands for the resolved shear stress at the material point *M* in the direction defined by a slip system, *g* is a microscopic constant proportional to *g'*.

Satisfying the criterion precludes the breaking of a crystal; however it does not adequately represent the fatigue limit as observed in experiments since cracks of the size of a grain are not likely to threaten the integrity of the structure. Instead we have to search for cracks of the size of a representative volume (containing many grains). Accordingly, Papadopoulos proposed to average this mesoscopic criterion over all the slip system orientations available at a material point using the volumetric root mean square:

$$\forall M \in V, \sqrt{\frac{5}{8\pi^2}\iiint_{[0,\pi]\times[0,2\pi]\times[0,2\pi]}\left(T_a^M(\theta,\varphi,\psi)d\varphi\sin\theta d\theta d\psi\right)^2} \leq g \quad (4)$$

In addition, it must be noted that the formulation (4) does not prevent the nucleation of cracks as (3) did. Papadopoulos proposed to take into account the possible propagation of these micro cracks by taking into account the influence of the normal stress over these cracks. As the volumetric average of the normal stress is equal to the hydrostatic pressure, Papadopoulos proposed the following final expression for his criterion:

$$\forall M \in V, \sqrt{\left\langle T_a^M \right\rangle} + \alpha P_{\max}^M \leq \beta \quad (5)$$

where $P_{max}$ is the maximum hydrostatic stress during the cycle, and α and β are two material parameters to be identified. We will not detail precisely the numerous advantages of this criterion (which in many usual cases is identical to the Crossland criterion). Let us just notice that it predicts adequately the independence of the fatigue limit over the mean shear stress, it does predict a difference between compression and tension loading (contrary to the energetic criteria), and its predictions in the case of out of phase loading are more reliable than the ones of other criteria. However, it cannot predict the experimental difference observed between the bending and tensile fatigue limits, which is not surprising since all developments are carried out at the scale of the material point, whereas gradient effects involve volume quantities.

**Damaged Volume definition**

In order to take into account the stress gradient effect, we propose a non local approach by averaging the Papadopoulos criterion over a damaged area. The existence of this area has been investigated at the LAMEFIP (ENSAM, Bordeaux, France). Fatigue tests have been carried out using blocks on different type of materials (from cast iron to titanium alloy), alternating blocks with a high level of load with low level blocks (this kind of loading is represented in figure 2).

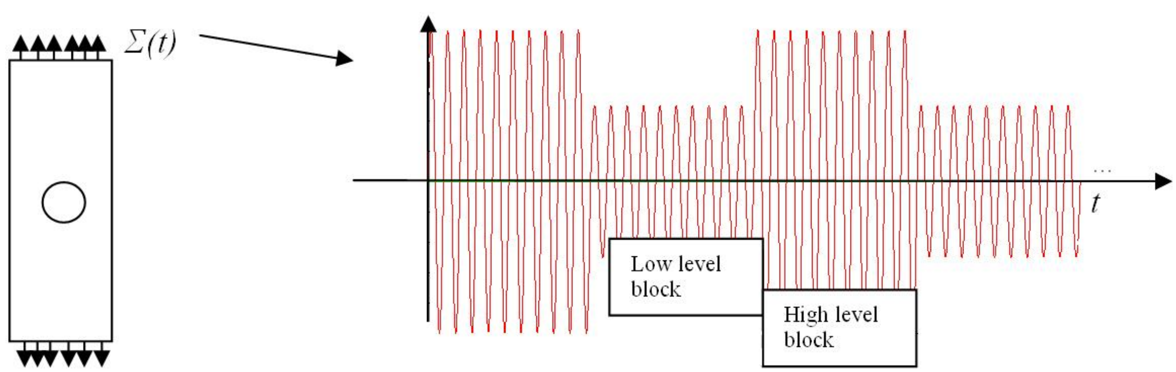



Figure 2 : traction fatigue load by blocks on a holed specimen

The low-level blocks are theoretically unable to provoke fatigue failure because their magnitude is far below the endurance limit. However they found out that the lower level blocks do have an influence over the fatigue life of the sample. More precisely, when the load applied during the low blocks exceeds a given threshold, which happens to be lower than the usual endurance limit, the life of the structure is reduced. This means these lower blocks participate in the fatigue damage. According to Palin-Luc this is due to the fact that these low blocks are allowing nucleation of micro cracks which are afterwards propagated by the high level blocks.

In the presence of a stress concentrator, the fatigue failure usually happens at the point where the stress reaches its peak value (the critical point). However, an area exists around this point where the stress value is theoretically insufficient to cause fatigue failure, but high enough to cause initiation of micro cracks (see figure 3) as did the low level blocks in Palin-Luc's experiment. These micro cracks contribute to a redistribution of the stress in the whole area, in particular the stress seen by the macro crack at the critical point is lower. As a consequence, the existence of the damaged area raises the fatigue limit of the structure. This could be an explanation of the well known positive effect of stress gradient on fatigue life.

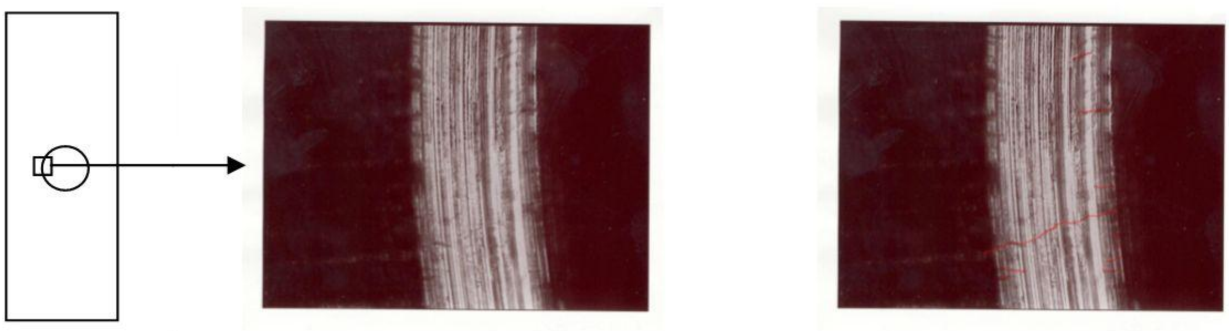


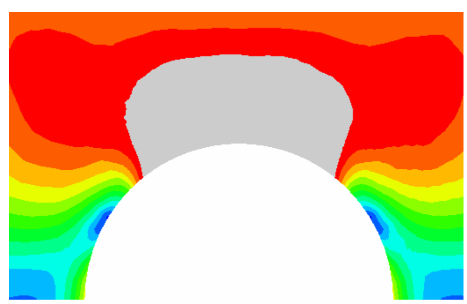
Figure 3 : Damaged area on the edge of the central hole of the specimen
(Cracks are highlighted in red on the right)

According to these considerations, we can define the damaged area as the area where micro cracks have developed. To determine such an area, it seems logical to use the Papadopoulos mesoscopic criterion which has been precisely designed to determine whether or not micro cracks have initiated at a material point. As a consequence the damaged area – denoted VES in the following– is then defined by the following equation:

$$VES = \{M, \exists(\theta,\psi,\varphi), T_a(\theta,\psi,\varphi) > G\} \tag{6}$$

An example of a VES is given in the following figure:

Geometry of the specimen and loading



VES (depicted in grey)

Figure 4 example of VES determined numerically

**Model presentation**

In this section, we present the model designed to take into account stress gradient effect. The basic idea is to average the criterion over the damaged area. By doing so, it is hoped that the effect of possible competition between several micro cracks will be taken into account.
The averaging process is given by the following equation:

$$\frac{1}{vol(VES)} \iiint_{VES} \left( \sqrt{T_a^2} + \alpha P_{\max} \right) dV \le \beta \tag{7}$$

where $\alpha$ and $\beta$ are material parameters to be identified on test results. Although it is believed $G$ could be related to some intrinsic monocrystal properties, we propose to identify its value from macro testing results as well. This approach has the main advantage of including in the *G*-value the effect of grain boundary on the resistance of the grains. Therefore, it is no longer a crystallographic property, but a material property (in particular it depends on the fabrication process). The identification procedure requires three different tests.

For the 30NCD16 steel, traction, rotative bending and torsion fatigue limits are available in the literature. This data was found in Palin-Luc and Lasserre [10].

| Loading | Fatigue limit (MPa) |
|---|---|
| Traction R-1 | 560 |
| Rotative bending R-1 | 658 |
| Torsion R-1 | 428 |

Table 1: 30NCD16 fatigue limits from [10]

The application of the formulae (7) to the loading cases described in table 1 (the fatigue limit for the fully reversed tensile test, the fully reversed rotative bending test and the fully reversed torsion test are denoted respectively $\sigma_{tens}$, $\sigma_{rotBend}$, $\sigma_{tors}$) yields the following system which has to be solved in *α, β, G*.

$$\left\{ \begin{array}{c} \sigma_{tens}\left(\frac{1}{\sqrt{3}}+\frac{\alpha}{3}\right)=\beta \\ \frac{2}{3}\tau_{\max}\frac{1+G/\sigma_{tors}+\left(G/\sigma_{tors}\right)^2}{1+G/\sigma_{tors}}=\beta \\ \frac{2}{3}\left(\frac{1}{\sqrt{3}}+\frac{\alpha}{3}\right)\sigma_{RotBend}\frac{1+G/\sigma_{RotBend}+\left(G/\sigma_{RotBend}\right)^2}{1+G/\sigma_{RotBend}}=\beta \end{array} \right. \quad (8)$$

In addition, the application of the meso criterion in these case yields the following constraints on *G*: $G< \sigma_{tens}/2$, $G< \sigma_{tors}$, $G< \sigma_{rotBend}/2$. The equation given in (8) can be solved analytically. In particular, it is easy to see that these constraints are fulfilled by the solution of (8).

For the 2524T3 aluminum alloy, a test campaign dedicated to this identification has been carried out. We have used two tension tests with different R ratios together with a bending test. The influence of the choice of the identification tests over the prediction of the criterion has been found to be reasonable (see the part application for details on this subject).
The identification of the model has been done on two different type of data: the first set of data is the usual fatigue properties determined from the number of cycles to failure, the second one is fatigue properties determined from the number of cycles to initiation of a macro crack (the macro crack size has been considered to be close to 0.1mm). This distinction has been made in order to determine whether identifying the model (which is an initiation model) on initiation data could result in better predictions. At the moment, we do not have enough data to assess this hypothesis. The resulting fatigue properties (these properties have been determined at $10^5$ cycles) are given in table 2, together with their attached 95% confidence intervals. The initiation of a fatigue crack was estimated visually by observation of the specimen through a Questar microscope at different times of the fatigue life.

| Loading | Usual Fatigue limit (MPa) | Initiation fatigue limit (MPa) |
|---|---|---|
| Tension $R_{0.1}$ on holed specimen | 200 [185; 210] | 185 |
| Tension $R_{0.5}$ on holed specimen | 275 [255; 300] | 260 |
| Plane bending $R_{-1}$ on plane bending specimen | 285 [240; 310] | 260 |

Table 2: 2524T3 fatigue limits

The same identification procedure has then to be done. Namely, we have to solve a system similar to (8). However, simple equations are not available since the tests have been carried out on complex specimens (an open hole specimen) for which a simple analytical description of the state of stress is not available. As a consequence, the system we want to solve is described by the following equations:

$$
\begin{cases}
f_1\,(\alpha,\beta,G) = criterion(\alpha,\beta,G)\big(FE(Traction, R_{0.1}, holed\ sample\big) = 0 \\
f_2\,(\alpha,\beta,G) = criterion(\alpha,\beta,G)\big(FE(Traction, R_{0.5}, holed\ sample\big) = 0 \\
f_3\,(\alpha,\beta,G) = criterion(\alpha,\beta,G)\big(FE(Bending, R_{-1}, plane\ bending\ sample\big) = 0
\end{cases}
\qquad (9)
$$

where FE stands for finite element analysis. This numerical system is solved with a classical penalization algorithm by minimizing on *(α, β, G)* the quantity $f_1^2(\alpha,\beta,G) + f_2^2(\alpha,\beta,G) + f_3^2(\alpha,\beta,G)$.

## Application

Some fatigue data for various loading conditions are available for the 30NCD16 in [10]. This data allows us to test the predictive capability of the criterion. We have used the criterion, identified as explained above, to predict some Haigh's diagrams for different load conditions. The criterion used is the one exposed in the previous section slightly modified (the dependence to the pressure term has been chosen to be quadratic). These diagrams are given in the following figures, the blue squares represent the experimental data found in [9], the green lines are the prediction of our criterion, the red lines - when displayed - are the prediction of the classical Papadopoulos criterion or of the Papadopoulos - Panoskaltsis criterion exposed in [4]. A fair agreement between predictions and experimental results is observed in the range of moderate loading.

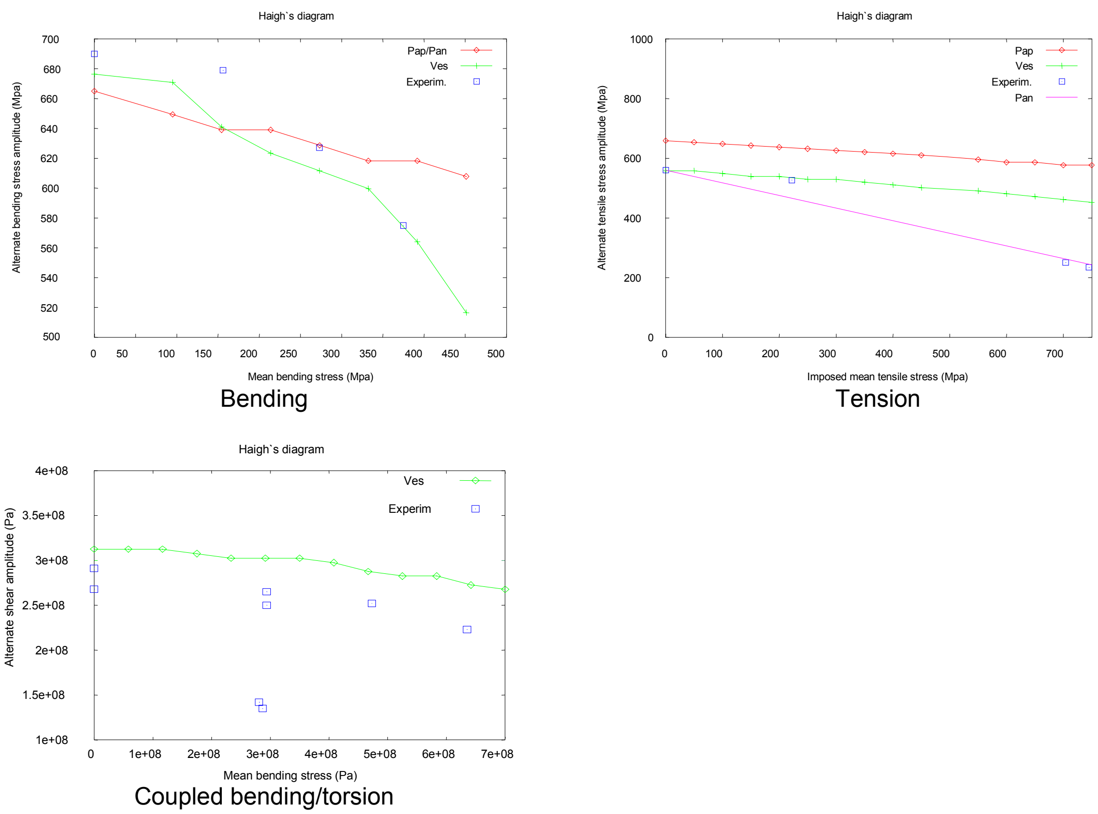
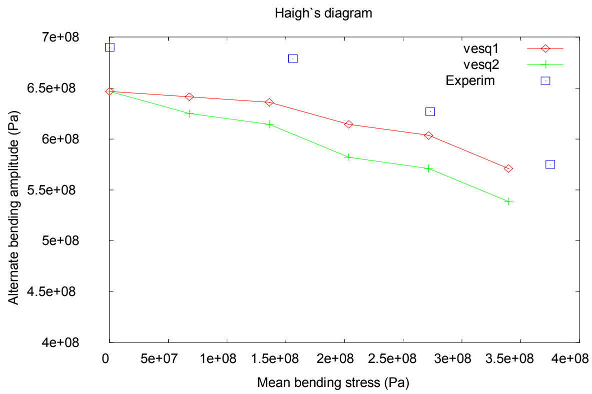
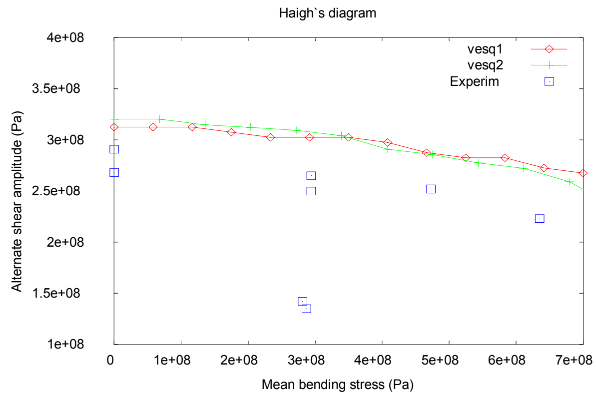



Figure 5 : Haigh's diagram

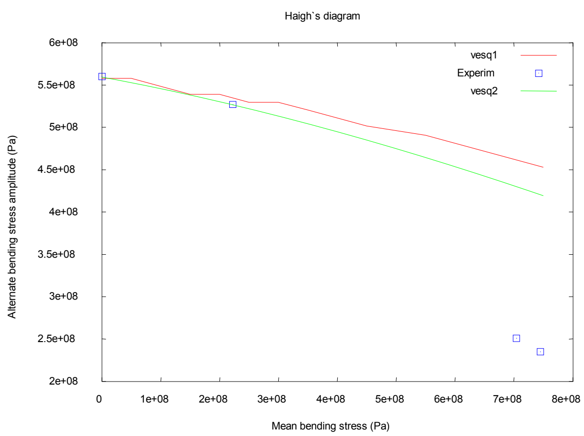
We have also used the available data to investigate the effect of the choice of the tests used for the identification of the criterion. Identifying the criterion on two sets of fatigue data (set 1 : fully reversed rotative bending, traction and torsion; set 2 : fully reversed rotative bending, traction with R ratios of 0.1 and 0.4), we obtain two different criteria named vesq1 and vesq2 in the following figure. Determining again Haigh's diagrams with these two criteria yield the following figure (figure 6). The influence over the predictions of the set of fatigue data used to identify the criterion seems to be reasonable.



Plane bending



Coupled bending-torsion



Tension-compression

Figure 6 : Haigh’s diagram for different type of loads

The predictive capabilities of the criterion have been tested for a 2524T3 aluminum alloy. Several test campaign on this alloy have been conducted at the EADS Corporate Research Center France in Suresnes, France. In particular, the effect of the mean stress over the tensile fatigue limit of a holed sample has been studied, as well as the effect of the ratio *Φ/W* where *Φ* is the hole diameter and *W* the width of the specimen. These experimental results are plotted below together with the prediction of our criterion. Again a good agreement between experiments and predictions is observed. The 95% confidence intervals plotted around the experimental values are determined according to the French standard [11] (section 7).

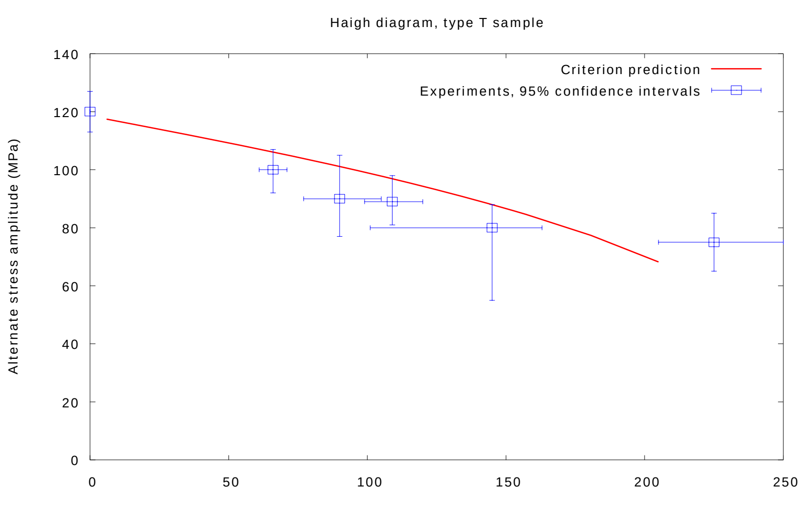


Figure 7: Mean stress effect

The experimental points plotted above have been obtained on a different batch of material than the one we used to identify the criterion. We have therefore rescaled this data in order to match the overlapping values between this new database and the database we used to identify our criterion. It is also noticeable on the previous graph that the last experimental point (with the highest mean stress) seems to lie outside of the general trend given by the other experimental points. This is most probably due to the fact that, at that level of loading, significant plasticity (in terms of both size and amount of deformation) is to be observed around the hole, perturbing in this area the shakedown phenomenon. This data point has nevertheless been used to identify the parameters of the criterion, in order to obtain an adequate formulation for a wide range of loading conditions. However, a refined formulation, taking into account cyclic properties, is certainly a more commendable way to deal with significant plasticity, and it should be addressed in future work.

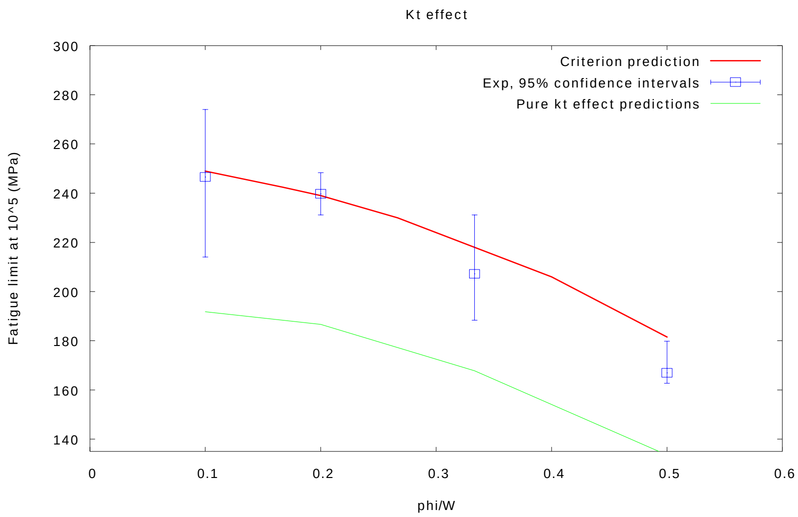



Figure 8: Φ/W effect

Again, the database plotted above has been rescaled for similar reasons. The green curve represents the prediction of a local fatigue criterion which would simply account for gradient by dividing the tensile fatigue limit by the $K_T$ value, whereas the proposed criterion seems to be able to represent the beneficial effect of the stress gradient.

## Conclusions

The criterion proposed here seems to be a suitable tool in order to predict fatigue crack initiation in the presence of stress concentrations, as its prediction are good compared to our first experimental results. In particular, the effect of the hole size on the fatigue resistance (for finite life) of open hole coupons is very well captured by the non local averaging process. This can alleviate or eliminate the need for additional tests required for the tuning of local criteria. R ratio effects are fairly well predicted but the influence of cyclic properties needs further investigation. It is of interest to notice that the approach described here is different from the usual non-local approach as it does not introduce an internal length. Instead a damaged volume has been identified as a consequence of the geometry and the loading (obviously, materials parameters do have an influence over this volume). It can be fruitfully compared to a somewhat similar approach developed since 2000 by Palin-Luc at the LAMEFIP.
The use of a macro fatigue criterion guarantees that this approach can be introduced more efficiently in an engineering design process.

### Acknowledgments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