CRACK PROPAGATION IN GEAR TOOTH ROOT

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ABSTRACT

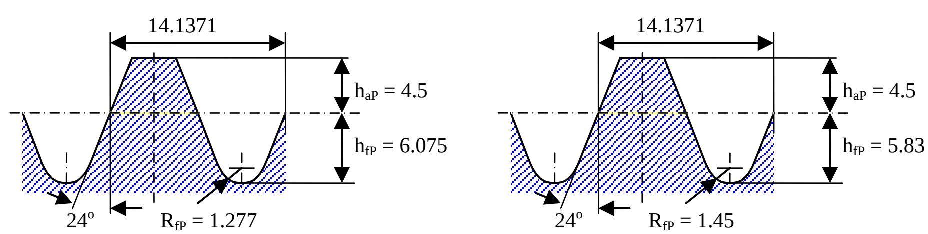
The paper deals with the problem of the crack beginning location and its influence on the crack propagation. The object of the consideration is the common truck gearbox; actually a pair of 1st step gears. Theoretically the crack beginning can occur in very narrow area on the tooth root surface where the principal stresses are the highest. From the experiments, which are executed on a special device, it is derived that the crack can appear at very different locations in tooth root and then propagates in their own way. Possible reasons are discussed that can influence the start position of the crack initiation. The primary attention is focused into the loading distribution over the tooth flank, the undercutting of the tooth and to the tracks that remain from the cutting tool. It is confirmed that all counted phenomena cause a wide scatter of crack propagation in tooth root.

Introduction

The service life of the gearbox already exceeds one and half million kilometers, which means that all components should be designed very carefully in order not to become a critical component. One of the most exposed elements in the gearbox are obvious the gears, because they carry high specific loads all the time. For that reasons the material and heat treatment of gear are also chosen very carefully. Due to the good thermal treatment of the gear the tiny surface layers are very hard and have high resistance to wear but consequently such gears are sensitive to crack appearance in the tooth root. This paper discusses the reasons for starting a crack at the appointed location.

The Problem Definition

Many theoretical models about the crack initiation and the crack propagation are developed. The most effective theories about the crack propagation are mostly based on the linear fracture mechanic. For example, the theory predicts that the initial crack should appear on the surface where the tensions or principal stresses are the highest. Theoretically, this exact location in tooth root is easy to find. But the problem is that the experimental results deviate from the theory quite a lot. The experts explain this phenomenon by not understanding all the influences about the crack propagation procedure. In order to improve the theoretical models and to make it more efficient, a lot of scientists try to modify the theoretical equations by simply adding numerous additional parameters. In presented paper it will be shown that theoretical accessories may work correctly if appropriate boundary conditions are taken into account. To illustrate the problem of the crack propagation a real case, taken from the automotive industry, will be considered.



a) Reference Profile of Gear, $z_2 = 39$ b) Reference Profile of Pinion, $z_1 = 11$

Figure 1. The reference profile of 1st step gearbox gear pair

The pair of 1st step gears of 11-tons truck that is used for delivery purposes is observed. The reference profile is given on Figure 1 from which it can be seen that the engagement angle, namely 24°, is larger than the most common one of 20°. The position of the observed gear pair is shown on Figure 2.This larger engagement angle should ensure sufficient strength of the tooth root and it should assure enough resistance against fracture. In the continuation only the bigger gear, called gear wheel, is to be considered because the stresses in its tooth root are higher as in the engagement pinion.

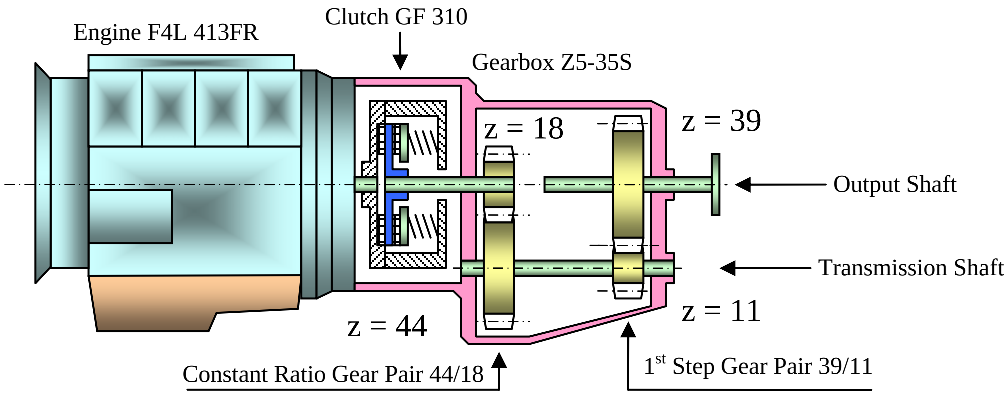


Figure 2. The position of observed 1st step gears

The loading spectrum that is characteristic for a typical delivery truck is calculated on the basis of torques measured on the truck propeller shaft. The loading spectrum is actually a list that consists from numerous amounts of stresses and its densities. Both are measured and then calculated to meet the conditions, which are characteristics for the observed gear pair. To get the nominal stress in tooth root from the measured torque on the propeller shaft, first the tangential load in the outer point of single engagement A is calculated. Taking into account this load and the geometrical properties, the nominal bending stress is calculated. Then the nominal stress is multiplied by dynamic coefficient and by other coefficients that consider many geometrical deviations. The results of such procedure are so called real stresses in the tooth root. The density, or the frequency, of each stress in tooth root depends on the number of loading cycles only. The density is calculated from measured torques and their durations considering the geometrical ratio between the propeller shaft and observed gear pair. To get the representative loading spectrum that is characteristics for a delivery truck, three driving condition have been measured: city drive, intercity drive and highway drive. All three driving conditions are combined into one loading spectrum by considering following shares: 50% city drive, 25% intercity drive and 25% highway drive. The final loading spectrum, Fig. 3, is representative one for a 1000 kilometers route. It is supposed that this representative loading spectrum is repeated continually. This unit route in amount of 1000 km is chosen because a 1000-times longer service life of the gear pair is expected and the tolerance of one cycle in the whole spectrum would mean a discrepancy of one thousandth only.

Stress in tooth root [MPa]	Density [times]
1172	1
1086	13
1004	202
920	487
836	160
760	65
681	82
595	62
515	86
433	108
354	114
270	105
187	94
106	69

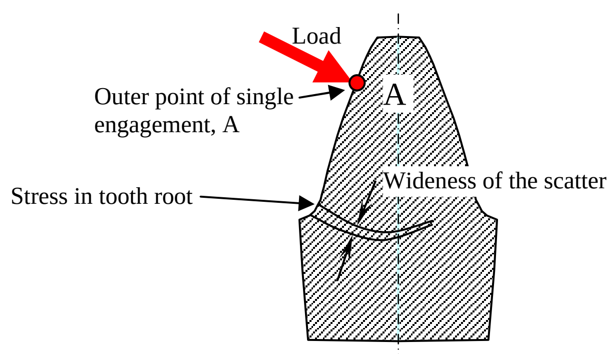


Figure 3. The loading spectrum of 1st step gear characteristics for 1000 km route done

On the basis of long-term monitoring of many truck transmissions, it was concluded that the real cause for final failure of 1st step gear pair is newer the wear but always the breakdown of few teeth. Many crack path measurements have been investigated [1] where a big scatter or dissemination of the crack paths was obviously shown. For the module of 4.5 mm the wideness of the scatter is greater than one millimeter, which is a cause for great be concern. This wide scatter of fractures means namely that also the service life of such a gear is shorter as expected, because of uncontrolled influences.

Due to presented facts the basic problem clearly appears in the front: To clarify the circumstances about the crack initiation and crack propagation. The analysis of crack propagation due to the different possible tooth flank loading distribution is presented. Special care is dedicated to the crack path.

Objectives

The most loaded part of the each gear pair is always the tooth flank and consequently the tooth root also. The tooth flank is loaded by pressure due to the contact force, which consists from normal one and tangential one. Both components of the contact forces produce friction that results in heat. Consequently, on the tooth flank wear appears. Normally, all these phenomena are more or less controlled by the hard surface layer on the tooth flank. It should be noted that during the gear engagement the contact point is traveling. It starts at the top of the tooth, in point A, and travels along the tooth flank over the whole working depth to the other end, to the point E. In other words, that means that the loading, which acts on the appointed surface, lasts for a very short time period.

The kind of steel used for the gear wheel plays a decisional role. In the automotive industry and also in the common mechanical engineering, the gears are made from surface heat treated steels, so called surface hardened steels. The surface is hard in order to stand the sliding contact loading. The core material of surface hardened steels should remain tough in order to carry the bending loadings. Firstly, the gears are cut out to get the teeth over the circumference. After the finished mechanical handling the gears are thermally treated in order to give the material the final strength. This treating is sometimes followed by grinding in order to raise the quality or accuracy of the gear. The result of such treatment is a hard surface and a tough core. Hard surface extends over the whole gear body. Hard surface layer behave excellently when they are loaded by pressure, but due to its brittleness this surface layer become very problematic. Such problematic fields where the high hardness can cause the problems are at the bottom lands of the gears. Usually, the geometry and the kinematics of the gear pair leads to relative overloading of one gear only. That means that the tooth root of the pinion is relatively more loaded than the tooth root from the gear wheel, or opposite.

Gear Wheel Testing on the Crack Propagation in Tooth Root

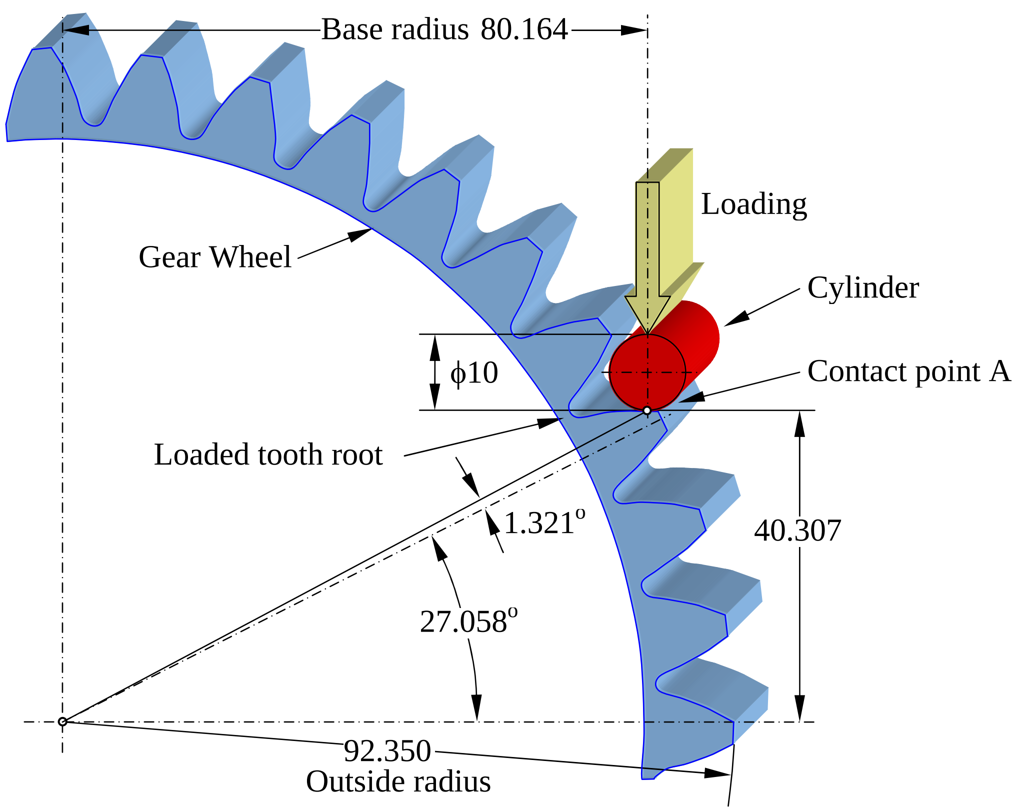


Figure 4. The position of gear wheel to get the vertical position of load

In order to get a clear figure about the crack propagation, a special-purpose device was designed. It enables to put the testing device on the common testing machine, which is intended for running the standard tension and compression tests and is available in many mechanical laboratories. To avoid mistakes, it was required to test each tooth separately. The common testing machine is designed to act in vertical direction and its fixing plate is positioned perpendicularly to the acting direction. For this reason the gear wheel should be rotated for such angle that the loading force, which is pressing on the tooth in outer point of single engagement A, is positioned perfectly vertical, Figure 4. The load is simulated by using the horizontally positioned cylinder.

The length of the acting cylinder normally doesn't fit the gear wheel width, which is 28 mm. The acting cylinder should represent the working pressure distribution that can generally take three different forms, as shown on Figure 5. The contact load can be positioned in the middle of the tooth flank, it can be located on one side only or it can be extended equally over the whole width. In the first two cases the length of acting cylinder is 2/3 of tooth width only.

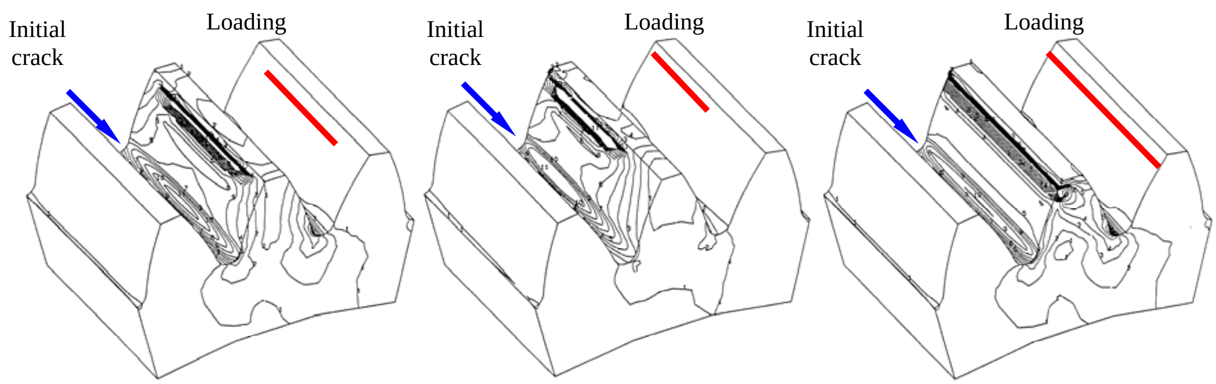


Figure 5. The basic forms of contact load distribution over the tooth flank

The final concept of the testing device is shown on Figure 6. At the beginning, the tested gear wheel is freely placed on the thick central spindle. By activating the position device the proper angle of gear wheel is ensured. The fastening of the gear wheel body on the basic skeleton is performed by tightening the spindle nut. In that way the gear wheel position is locked and its body fully blocked but the position device is not loaded at all. The fixed skeleton of testing device is fasted on the bottom base plate of the testing machine. The upper base plate of the testing machine is acting on the loading device that is placed at the top of our testing device. This loading device consists from a moving shaft which is leaded in bush. The only movements of the shaft are vertical movements.

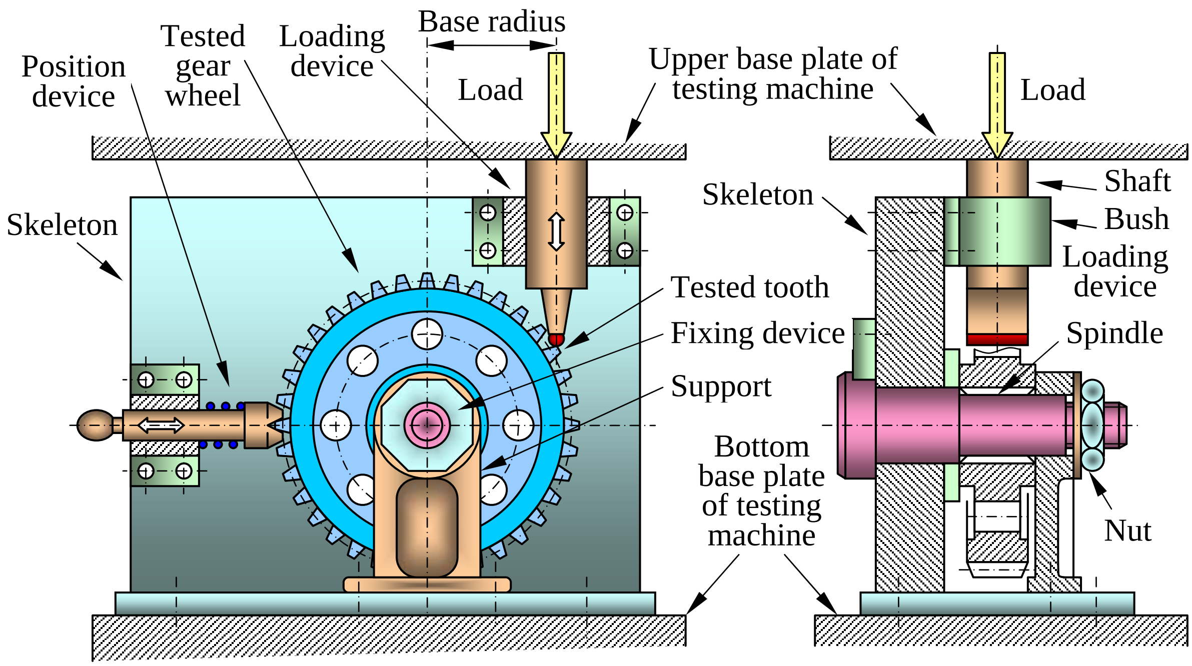


Figure 6. The basic principles of the testing device

The most common case of loading is shown on Figure 7 where the load is concentrated in the middle of the tooth. The decisional pressure is acting at the outer point of single engagement because in this case the largest bending moment occurs. The tooth length is extending along x-axis; the tooth height is extending along y-axis and tooth thickness is extending along z-axis. The highest stress lies in the tooth root on the 30^{o} line, exactly as it is predicted by theory. According to the presumption that the crack would initiate exactly in the point where the tension stress on the surface is the highest the crack should start in the doted line. The scatter of real crack paths is evident. Already about the starting point of the crack a great discrepancy between the theoretical forecast and practical measurements is obvious.

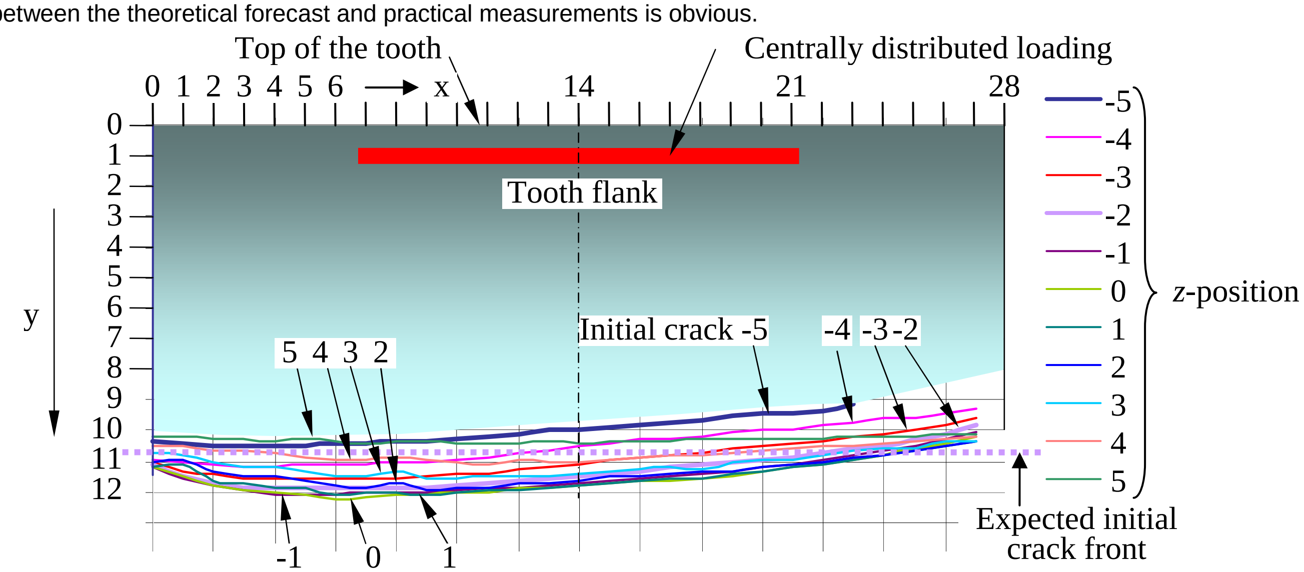


Figure 7. The broken tooth loaded with centrally distributed loading

On Figure 8 the broken tooth that was loaded with one sided loading is presented. This is the case for which one should be worried about. According to the theoretical stress distribution in tooth root, the initial crack should be positioned in the tooth root parallel to the tooth length, along the gear wheel width, respectively. But this is surely not the case. The explanation of this phenomenon should probably be searched for in load distribution. According to the testing device design, the theoretical one sided load distribution can be applied at the beginning only. After the initial crack appears, the stiffness of the tooth changes significantly what consequently leads to load distribution changes. On Figure 9 the broken tooth, loaded with perfectly equal distributed loading is presented. The crack initiation is more or les at the expected location.

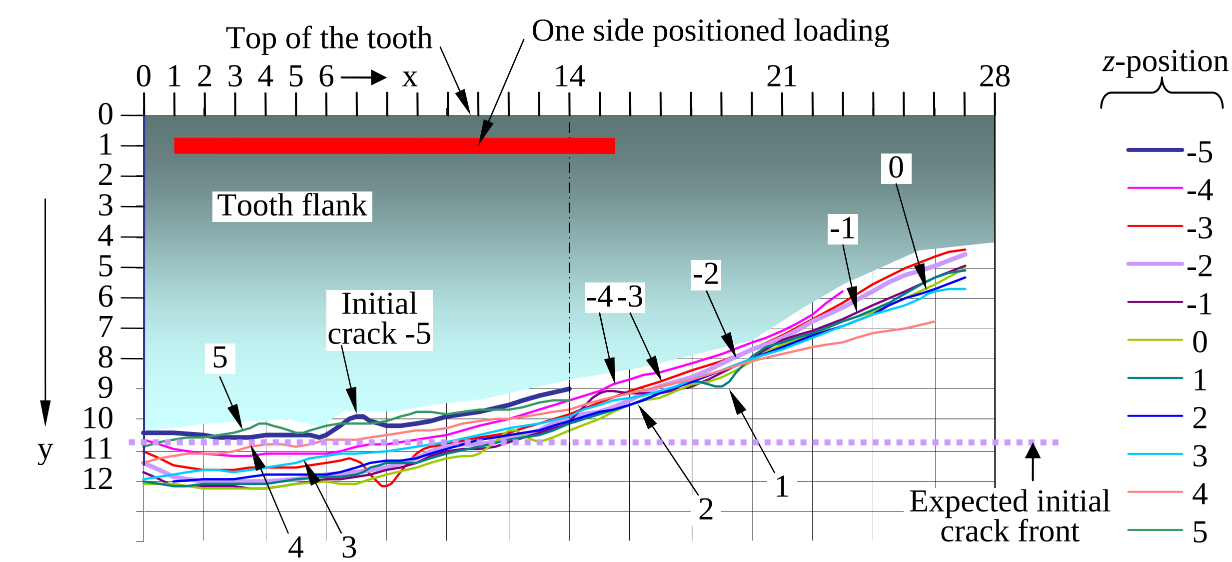


Figure 8. The broken tooth loaded with one side positioned loading

A closer look to the quite wide scatter of crack paths should make one worried because the discrepancy between the theoretical forecasts and practical measurements are still abnormally and unexpectedly great. Many broken teeth were measured and all are indicating approximately the same situation. The most outstanding cases were not presented. Some of analyzed teeth indicate even greater discrepancies as it is presented in the appointed cases.

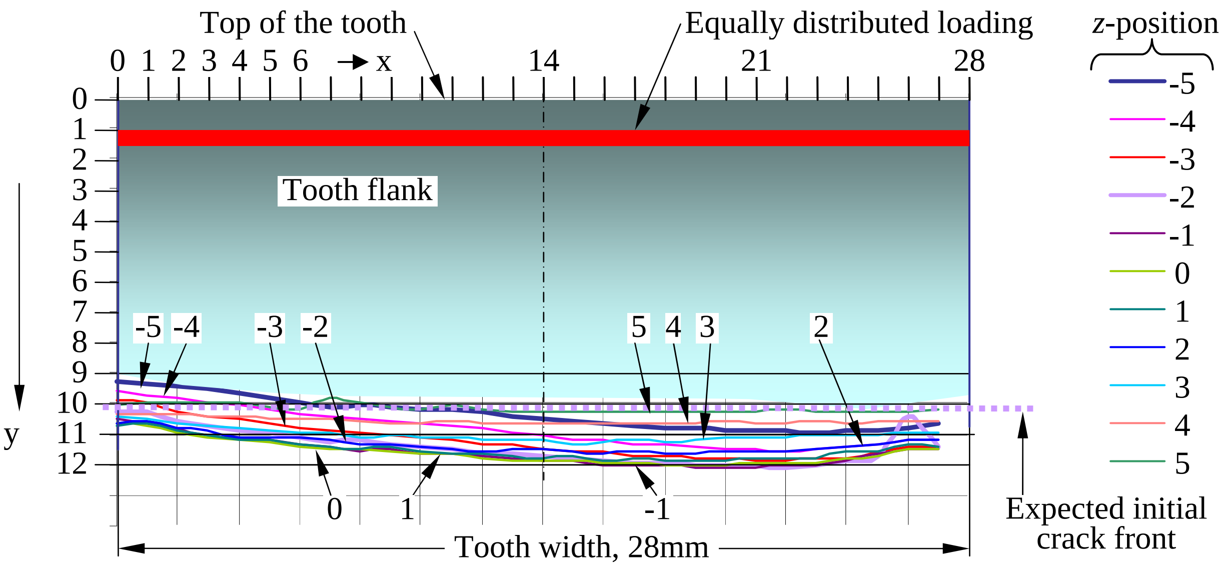


Figure 9. The broken tooth loaded with equally distributed loading

Calculation of Crack Velocity and Crack Path in Tooth Root

According to the valid laws of linear elastic fracture mechanics the stress field at the tip of a crack is usually fully described by the stress intensity factor. Due to the complex load distribution and complex gear tooth geometry the stress intensity factor can only be determined by numerical methods [2]. The gear thickness of 28 mm allows plain strain conditions to be assumed, but the validity of linear fracture mechanics should be permanently monitored by checking the size of the plastic zone at the crack tip. It should be small enough (less than 40 % of crack length).

The description of the stress field around the crack tip using only the singular term of asymptotic expansion is justified for a distance much smaller than the length of the last rectilinear crack segment. The elastic stress field can be described with a series, where the number of terms depends on the required accuracy of the solution. For the first and second order terms of asymptotic expansion of elastic stress fields near the crack tip, the stress components are given by [3]:

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix} = \frac{K_{\mathrm{I}}\cos\frac{\theta}{2}}{\sqrt{2\pi r}}\begin{bmatrix}1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\\ 1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\\ \sin\frac{\theta}{2}\cos\frac{3\theta}{2}\end{bmatrix} + \frac{K_{\mathrm{II}}\sin\frac{\theta}{2}}{\sqrt{2\pi r}}\begin{bmatrix}-\left(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)\\ \cos\frac{\theta}{2}\cos\frac{3\theta}{2}\\ \cot\frac{\theta}{2}-\cos\frac{\theta}{2}\sin\frac{3\theta}{2}\end{bmatrix} + \begin{bmatrix}T\\ 0\\ 0\end{bmatrix} \tag{1}$$

where $K_{I,II}$ are the stress intensity factors and represent the geometry and a loading dependent quantity. T is referred to as T-stress and it is a non-singular stress. It acts parallel to the crack plane, it is independent of r and proportional to the applied stresses.

For each crack length the stress intensity factors: K_I , K_{II} and K_{III} (in 3D model) and T-stress are calculated and plotted in graph. In order to get confidence in obtained results 2-dimensional and 3-dimensional analysis were done. The biggest importance on crack propagation has the stress intensity factor K_I. The observation of T-stress was primarily introduced in order to better explain the crack propagation in tooth root. T-stress is small in case of crack in tooth root. Therefore its influence can be neglected. In case of the exactly known position of the initial crack the crack path and the distribution of so called effective stress intensity factor along the crack can be determined easily, Fig. 10. This figure shows that some influences can really be neglected. When the theoretic value of K_{eff} reaches the critical material value called K_{Ic} (core material) then the crack gets the sound velocity and the tooth root brakes instantly. That means that the critical length of the crack in tooth root won't exceed approximately 6.5 mm.

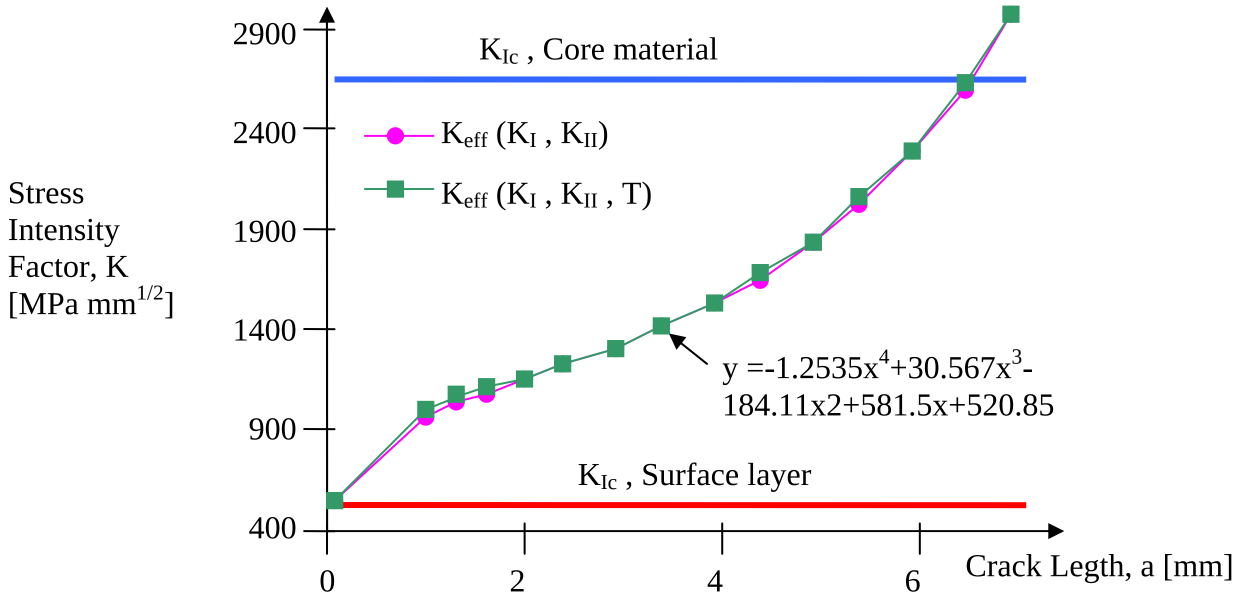


Figure 10. The distribution of effective stress intensity factors along the crack length

Due to the preliminary treatment that easily causes the surface defects, the initial crack can appear in many places, Figure 11. The path of crack propagation strongly depends on initial crack. Presumably the direction of crack propagation is determined by the position of highest virtual stress intensity factor in each crack tip. Consequently also the distributions of the stress intensity factor along the length depend on the crack propagation path, as it is shown in Figure 12.

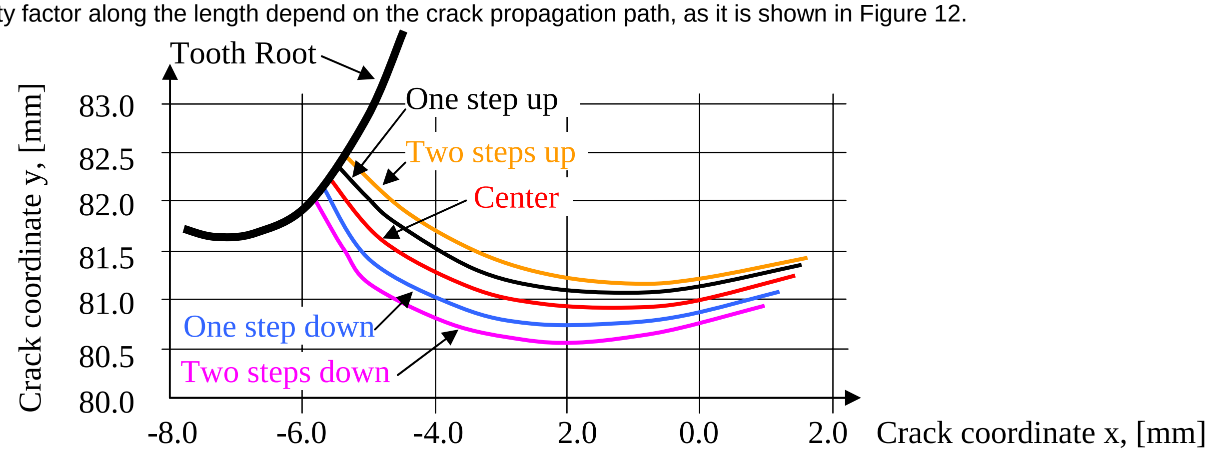


Figure 11. The crack propagation paths depending on initial cracks

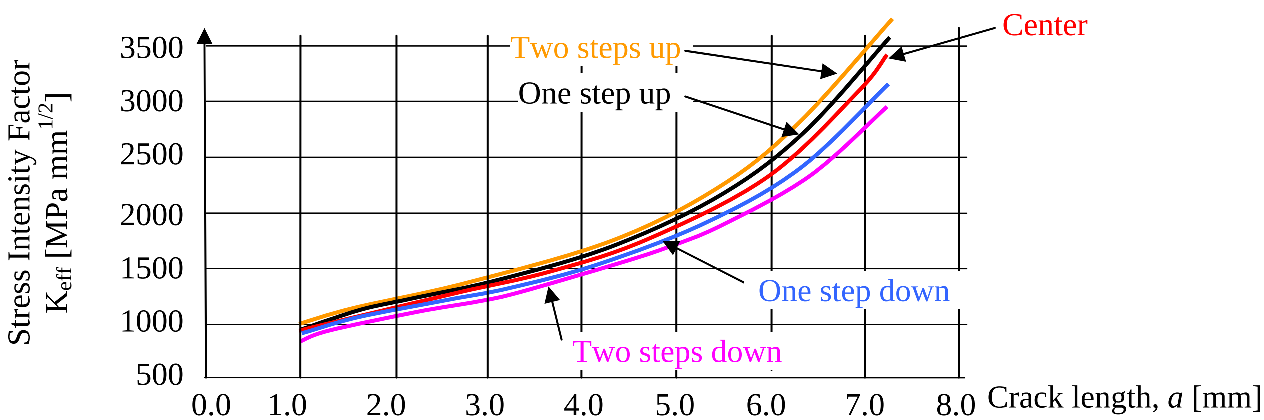


Figure 12. The distribution of stress intensity factors along the crack length

Discussion about the Causes for Wide Scatter of Crack Path and the Conclusion

A few possible causes that might lead to the wide scatter of crack path in the tooth root are discussed. From the gearbox design it is known that the loaded shafts are not perfectly parallel. Due to the radial forces the shaft is deformed and the gear mounted on it is positioned in such a way that the load over the tooth flank is not distributed equally.

As it is follows from stress analyses the position of the von Mises pick stress in tooth root is not always at the exactly the same position. It is discussable where actually the point of the highest stress is. One can easily fix different positions of the crack initiation by considering different influences. However, the theoretical position of the initial crack is discussable. It seems that the influence of FEM mesh density also has to be considered. For that reason the tooth model was meshed in different ways and it becomes clear that the mesh indeed has quite a big influence on the appearance of initial crack. Especially important becomes the FEM mesh in case of accounting different effects that are the results of preliminary treatments. The final treatments for vehicle gears are normally grinding, preliminary treatment are usually milling, which lefts evident tracks on the tooth root as it is shown on Figure 13.

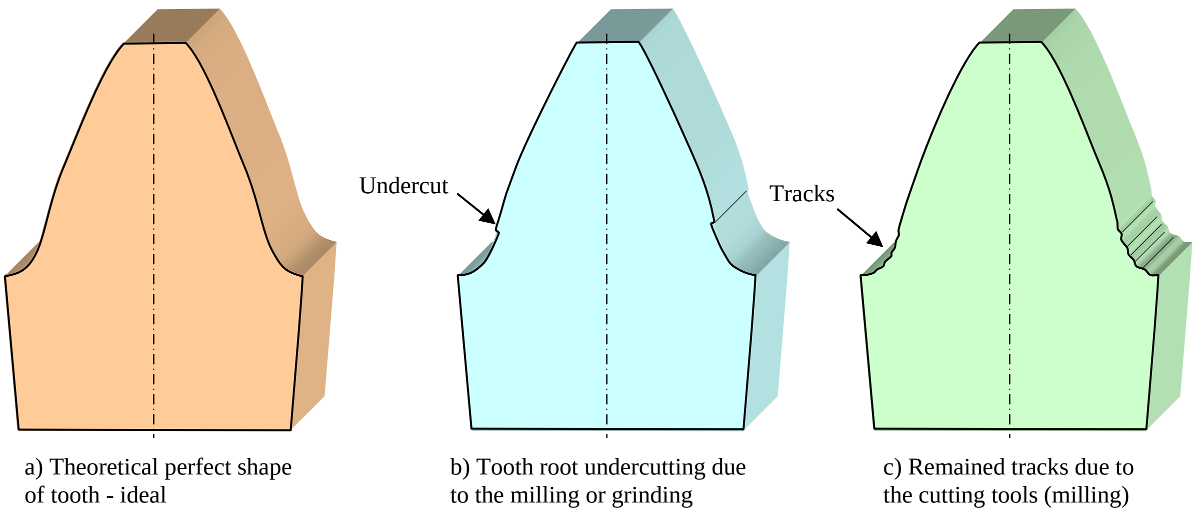


Figure 13. In the tooth root are frequently presented defects due to the mechanical treatment

The existence of few obvious influences that might have important effect on appearance of initial crack was clearly shown. Before the designers calculate the service life of gears they must carefully investigate the actual state of tooth root in order to be sure that the initial crack is positioned on the proper place. More of the research is in progress in order to evaluate the effects of different influences.

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