Analysis of Tubular Composite Cylindrical Shells

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ABSTRACT

In this paper, tubular composite cylindrical shells with different geometries are analyzed. The aim of this research is to investigate the behavior of the previously mentioned cylindrical shells, under different load ratio and different winding angles. The problem was solved using the classical theory of composite laminated shells and finite element methods. A fiber glass filament wounded thin composite cylindrical shell is used for study. Vessel is investigated under various meridians to axial load ratio. Also in each load condition filament winding angle is being varied. Based on classic theory of composite laminated shells, on-axis and off-axis stresses and strains of each ply is determined. A Tsai-Wu failure criterion is used as design limit. It is assumed that layers are completely bounded to each other and there is no sliding between two conjunction layers. Comparing the results of analytical method and finite element method shows the reliability of procedure. Based on the results, by increasing the ratio of (k), minimum of Tsai-Wu failure criterion leads to the small winding angles. So the fibers align according to maximum stress and make structures more efficient.

Introduction

Composite materials due to their small weight to strength ratio, have found wide application particularly in aerospace industries. Case of Space Shuttle and Trident II rocket and most of new rocket systems are carbon fiber filament wounded structures with wonderful strength [1].

Mechanical property of composite materials significantly depends on measurement direction. One major advantage of composites is that they can change to satisfy design requirement. This is possible by alignment of fibers in principal applied force directions. Hereby we let fiber's most large strength directly carry loads and force be minimized in unsuitable directions [1].

Composite pressure vessels commonly are made by filament winding. Accurate and fast placement of continues reinforcements in predefined pattern is the base of filament winding method, (Figure 1). The most important advantage of filament winding method is cost reduction. This reduced cost in filament winding is because of the fact that we can combine an expensive fiber with an inexpensive resin to obtain an inexpensive composite material. Studying the behavior of filament

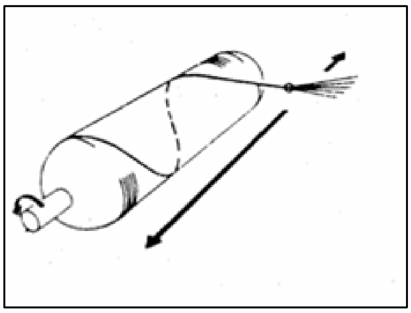

Figure 1. Schematic of filament winding process

wounded composite cylindrical shells are very important in optimum design of composite pressure vessels that are used in aerospace, gas engine and chemical industry. According to unique capability of composite pressure vessels, in the last decade various studies have been done about them.

G.C ECKOLD (1985) has employed anisotropic elastic theory to analyze filament wound laminates as used in the manufacturing of pipework, tanks and vessels for chemical and allied industries. Both the effect of uniaxial and combined loads have examined and discussed. A design method based on the results of the elastic analysis has been presented [2].

J. F. Wilson et al. (1986) within the framework of classical elasticity, the nonbuckled deformation have calculated for orthotropic, right circular, thin-walled cylinders under uniform load condition. Through parametric studies deformation patterns have calculated that are unique to orthotropy. They have employed classical elasticity to predict the nonbuckled deformation for uniform, orthotropic right circular cylinders with thin walls, subjected to three types of uniform loads [3].

B. W. TEW (1995) has presented a design approach based on netting theory which enables engineers to develop preliminary structural design for these structures using composite materials. The integration of creep, cyclic loading, and environmental degradation factors into initial design calculation has also discussed and illustrated. The design approach presented can be used for preliminary design and analysis of composite tubular structures subjected to internal pressure, axial loads, and bending loads, however, laminate analysis and/or finite element analyses combined with a physically based, interactive failure criteria should be performed to verify preliminary design [4].

Xia et al. (2001) have studied composite pipes composed of multi-layered filament-wound (FW) structures. Each layer of the pipes has been assumed to be anisotropy. Based on three-dimensional (3-D) anisotropic elasticity, an exact elastic solution for stress and deformations of pipes under internal pressure has been presented. Moreover, detailed stress and strain distributions for three given angle-ply pipe designs have been investigated, using the presented theory [5].

A. Be'akou et al. 2001, by using reliability analysis of a $[\pm\alpha]_n$ filament wound composite, have showed that the fiber winding angle can vary hugely with the scattering of some design variables; however, cylindrical laminated composites are generally made with a fiber winding angle equal to 55º. Studied structures were pressure vessels and axially loaded pressure pipes. The stresses have computed using the classical laminated membrane theory and the Tsai-Wu failure criterion has adopted as the limit state function of ply [6]. At the presented study two different approaches is used to analysis of the orthotropic cylindrical shell, classical theory of laminated membrane and finite element method.

Classical theory of laminated shells

The analysis for composite tanks, pressure vessels and piping subjected to internal pressure, has been done using different methods. The British Standard assumes thin wall theory and calculates the average stress and calculates the average stresses and strains in the wall. These in turn are compared to laminate strength value for failure determination [7]. A rigorous analysis for composite pressure vessels was given by Sherrer 1967, in which equilibrium equations are written for each individual layer and equilibrium and compatibility equations are written for the layer interfaces. This analysis is applicable to locations away from the ends because average boundary conditions are imposed [8].

Another study based on the laminate theory for the analysis of laminates has don by Puck (1969). Puck gave micromechanics equations to calculate layer properties from constituent properties, and the summation of layer elastic coefficients to make laminate elastic. In failure criterion, Puck pays more attention to the stresses transverse to fiber direction and in-plan shear stresses [9].

An approximation of the exact elasticity equations for the analysis of cylindrical shells was given by Whitney (1971). For thin shells as in fiber reinforced vessels, tanks and piping considered here, shell theory represents a good balance between rigor and simplicity [10].

A portion of the cylinder can be represented as a shell shown in Figure 2. In this figure:
z,θ,x : Coordinates along the axial, circumferential and radial directions respectively

w,v,u : Displacements along the axial, circumferential and radial directions respectively

t : Thickness of the laminate

R : Radius to the mid-plane

R_i , $R_\circ$: Radius to the inside and outside surface of the cylinder respectively

l : Length of the portion of the cylinder under consideration

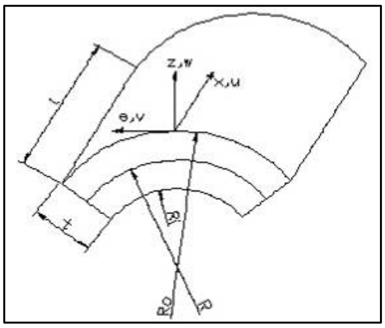

Figure 2. Typical element of cylindrical filament wound shell

According to Vlasov-Ambartsumyan (1964) shell theory for anisotropic laminates, we have the kinematic relations as:

$$\varepsilon_x = \varepsilon_x^\circ + zk_x$$

$$\varepsilon_\theta = \varepsilon_\theta^\circ + zk_\theta \qquad (1)$$

$$\gamma_{x\theta} = \gamma_{x\theta}^\circ + zk_{x\theta}$$

Where

$\gamma_{x\theta}^\circ, \varepsilon_\theta^\circ, \varepsilon_x^\circ$ = The mid-plane strains

$k_{x\theta}, k_\theta, k_x$ = The shell curvatures

In terms displacements

$$\varepsilon_x^\circ = u,_x^\circ$$

$$\varepsilon_\theta^\circ = \frac{1}{R}\left(v,_\theta^\circ + w\right)$$

$$\gamma_{x\theta}^\circ = \frac{1}{R}\left(u,_\theta^\circ + v,_x^\circ\right)$$

$$k_x = -w,_{xx} \qquad (2)$$

$$k_\theta = -\frac{1}{R^2}\left(w,_{\theta\theta} + w\right)$$

$$k_{x\theta} = -\frac{1}{R}\left(2w,_{x\theta} + \frac{1}{R}u^\circ,_\theta - v,_x\right)$$

Where

w, v°, u° Are mid-plane displacements in the z, θ, x directions respectively and, a comma denotes partial differentiation.
Each layer of shell is assumed to be orthotropic with respect to a plane perpendicular to the z axis in an approximate state of plane stress. Employing contracted notation, the constitutive relations of each ply as below:

$$\sigma_i = Q_{ij}\varepsilon_j \qquad (i, j = 1,2,6) \qquad (3)$$

Where

$1 = x$, $2 = \theta$ and $6 = x\theta$

σ_i = off axis normal and shear stresses respectively

Q_{ij} = laminate modulus components

According to Loves's first approximation (1944) the force resultants N_i and moment resultants M_i written in abbreviate form as in equations followed [7]:

$$(N_i, M_i) = \int_{-t/2}^{t/2} \sigma_i (1, z) dz \qquad (i, j = 1,2,6) \tag{4}$$

Equations (1) and (2) in conjunction with Equation (4) yield the following constitutive relations for the shell

$$N_i = A_{ij} \varepsilon_j^\circ + B_{ij} k_j \qquad (i, j = 1,2,6) \tag{5}$$

$$M_i = B_{ij} \varepsilon_j^\circ + D_{ij} k_j$$

Where

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-t/2}^{t/2} Q_{ij} (1, z, z^2) dz \qquad (i, j = 1,2,6) \tag{6}$$

The shell equation of equilibrium can be written as:

$$N_{x,x} + \frac{1}{R} N_{x\theta,\theta} = 0$$

$$N_{x\theta,x} + \frac{1}{R} N_{\theta,\theta} + \frac{1}{R} M_{x\theta,x} + \frac{1}{R^2} M_{\theta,\theta} = 0 \qquad (i, j = 1,2,6) \tag{7}$$

$$M_{x,xx} + \frac{2}{R} M_{x\theta,x\theta} + \frac{1}{R^2} M_{\theta,\theta\theta} - \frac{N_\theta}{R} + p = 0$$

Where

p = Internal pressure.

For most composite pressure vessels, there is always a head at the end. This head stiffens up the end of the cylinder. As a first approximation, the cylindrical portion of the vessel can be assumed to be clamped at both ends, but in such a manner that it is free to rotate and extend (or contract) at $x = l$. Based on this assumption, the following boundary conditions are appropriate.

$$u^\circ(0,\theta) = v^\circ(0,\theta) = w(0,\theta) = w,_x(0,\theta) = w,_x(l,\theta) = 0$$

$$u^\circ(l,\theta) = Constant \tag{8}$$

$$v^\circ(l,\theta) = Constant$$

Assuming that the mid-plane displacements u°, v° only vary along the x direction, using Equation (2), the displacements can be rewritten as:

$$u^\circ = \varepsilon_x^\circ x$$

$$v^\circ = \gamma_{x\theta}^\circ x \tag{9}$$

$$w = \varepsilon_\theta^\circ R$$

Where $\varepsilon_\theta^\circ, \gamma_{x\theta}^\circ, \varepsilon_x^\circ$ are assumed to be constant.

Substituting Equation (9) into Equation (2) yields the following curvatures (k).

$$k_x = 0$$

$$k_\theta = -\frac{\varepsilon^\circ \theta}{R} \tag{10}$$

$$k_{x\theta} = \frac{\gamma_{x\theta}^\circ}{R}$$

Substituting the curvature expression in Equation (10) into Equation (5) yields to:

$$\begin{bmatrix} N_1 \\ N_2 \\ N_6 \\ M_1 \\ M_2 \\ M_6 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} \\ A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{61} \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} \\ B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^\circ \\ \varepsilon_\theta^\circ \\ \gamma_{x\theta}^\circ \\ 0 \\ -\dfrac{\varepsilon_\theta^\circ}{R} \\ \dfrac{\gamma_{x\theta}^\circ}{R} \end{bmatrix} \tag{11}$$

Equation (11) can be rearranged to have the form

$$\begin{bmatrix} N_1 \\ N_2 \\ N_6 \\ M_1 \\ M_2 \\ M_6 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} - \dfrac{B_{12}}{R} & A_{16} + \dfrac{B_{16}}{R} \\ A_{21} & A_{22} - \dfrac{B_{22}}{R} & A_{26} + \dfrac{B_{26}}{R} \\ A_{61} & A_{62} - \dfrac{B_{62}}{R} & A_{66} + \dfrac{B_{66}}{R} \\ B_{11} & B_{12} - \dfrac{D_{12}}{R} & B_{16} + \dfrac{D_{16}}{R} \\ B_{21} & B_{22} - \dfrac{D_{22}}{R} & B_{26} + \dfrac{D_{26}}{R} \\ B_{61} & B_{62} - \dfrac{D_{62}}{R} & B_{66} + \dfrac{D_{66}}{R} \end{bmatrix} \begin{bmatrix} \varepsilon_x^\circ \\ \varepsilon_\theta^\circ \\ \gamma_{x\theta}^\circ \end{bmatrix} \tag{12}$$

Equation (12) can be used to write the relation between stress resultants and mid-plane strains to be

$$\begin{bmatrix} N_1 \\ N_2 \\ N_6 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} - \dfrac{B_{12}}{R} & A_{16} + \dfrac{B_{16}}{R} \\ A_{21} & A_{22} - \dfrac{B_{22}}{R} & A_{26} + \dfrac{B_{26}}{R} \\ A_{61} & A_{62} - \dfrac{B_{62}}{R} & A_{66} + \dfrac{B_{66}}{R} \end{bmatrix} \begin{bmatrix} \varepsilon_x^\circ \\ \varepsilon_\theta^\circ \\ \gamma_{x\theta}^\circ \end{bmatrix} \tag{13}$$

Or in terms of the modified components of laminate modulus we have

$$\begin{bmatrix} N_1/t \\ N_2/t \\ N_6/t \end{bmatrix} = \begin{bmatrix} A_{11}^* & A_{12}^* - \dfrac{B_{12}^* t}{R} & A_{16}^* + \dfrac{B_{16}^* t}{R} \\ A_{21}^* & A_{22}^* - \dfrac{B_{22}^* t}{R} & A_{26}^* + \dfrac{B_{26}^* t}{R} \\ A_{61}^* & A_{62}^* - \dfrac{B_{62}^* t}{R} & A_{66}^* + \dfrac{B_{66}^* t}{R} \end{bmatrix} \begin{bmatrix} \varepsilon_x^\circ \\ \varepsilon_\theta^\circ \\ \gamma_{x\theta}^\circ \end{bmatrix} \tag{14}$$

From equilibrium consideration it can be shown that

$$\begin{aligned} \frac{N_1}{t} &= \frac{N_x}{t} = \frac{pR}{2t} \\ \frac{N_2}{t} &= \frac{N_\theta}{t} = \frac{pR}{t} \\ \frac{N_6}{t} &= \frac{N_{x\theta}}{t} = 0 \end{aligned} \tag{15}$$

Defining the modified modulus matrix in Equation (14) as $[C']$, Equation (14) can be written as

$$\{N_i/t\} = [C']\{\varepsilon_i^\circ\} \tag{16}$$

$$\{\varepsilon_i^\circ\} = [C']^{-1}\{N_i/t\} \tag{17}$$

Equation (17) can be used to obtain the mid-plan strain. The strain at any point can then be obtained using Equations (9) and (10).

Finite element method

A most common way for analysis of complex structures is finite element method (FEM). Using composite laminates stiffness matrix and finite element theory we can provide programs for analyzing of this structures. These programs can analyze orthotropic laminates and using a secondary program, transform multi layer laminates to an equal orthotropic layer and analyze using previous program. Input parameters for analysis of orthotropic laminates in finite element environments are members of stiffness matrix or engineering coefficients ($\upsilon_{xy}, E_z, E_y, E_x$) which using secondary program and these values stiffness matrix is determined [11].

Design example

A filament wound thin walled fiberglass reinforced epoxy cylinder is considered for this study. Dimensions are as Table 1. And structural material properties are as Table 2. Using classical theory of laminated membrane, under various hoop to axial load ratios, mentioned thin walled cylinder is analyzed. Various winding angles are applied in each load condition. Schematic of cylindrical shell is presented in Figure 3.

Table 1. Characteristic of filament wound cylinder

Diameter (mm)	Ply thickness (mm)	Number of layers	Winding angle (degree)	Internal pressure (Mpa)
300	0.6	3	0 - 22.5 – 45 - 67.5 - 90	2.2

Table 2. Material properties of used fiber glass epoxy composite

Mechanical Propertie	Value	Meaning
E_1	60673.88[Mpa]	Normal elastic modulus
E_2	24821.136[Mpa]	Transverse elastic modulus
G_{12}	11996.882[Mpa]	Shear modulus
υ_{12}	0.23	Normal poison ratio
υ_{21}	0.09	Transverse poison ratio
X_t	1289.32[Mpa]	Maximum tensile strength
X_c	820.476[Mpa]	Maximum compressive strength
Y_t	45.988[Mpa]	Transverse tensile strength
Y_c	162.027[Mpa]	Transverse compressive strength
S	44.316[Mpa]	Maximum shear strength

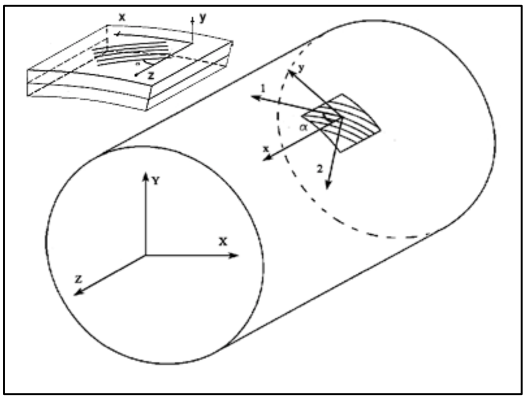

Figure 3. Schematic of filament wound cylindrical shell

Acting forces due to internal pressure over the cylindrical shell on each arbitrary element can be defined as axial unit load (N_ϕ) and hoop unit load (N_θ). Analyses are done under various axial to hoop load ratios (k) where:

$$N_\phi = PR \tag{18}$$

$$k = \frac{N_\phi}{N_\theta} \tag{19}$$

Table 3 shows the ratios (k) and used winding angle for cylinder model. Totally 25 various condition are obtained from combination of mentioned values for analyses. Based on classical theory of orthotropic shells stresses and strains of considered cylinders in off axis and off axis directions and for each ply in upper and lower surface, is determined. According to material ultimate strength, safety factors for failure criterion which here is maximum stress and Tsai-Wu failure criteria, for first layer fracture risk are obtained. It is assumed that plies are completely coupled to each others and no sliding happens between them.

Table 3. Various load ratio (k) and winding angle

Axial to hoop load, ratio	0	0.5	1	1.5	2
Winding angle (α) [Degree]	0	22.5	45	87.5	90

When k=0, cylinder acts like a pipe with two open ends. For k=0.5 it is equal to a pressure vessel with two close ends, subjected to uniform internal pressure. Other load conditions simulate special situations like a cylinder subjected to internal pressure and an axial load, similar to what occurs in a rocket casing.

Cylindrical shell with characteristic mentioned in Tables (1), (2) and load condition shown in Table (3) is analyzed using Ansys Finite element software. According to example data, Shell-99 element is used to analyzing this model. Finite element software provides translation of boundary condition, material properties and load condition for processing operation. Figure 4 shows the cylinder finite element model.

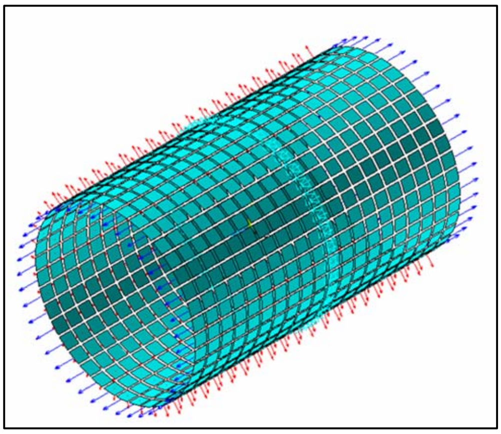

Figure 4. The finite element model of cylindrical shell

Results and discussion

For thin walled orthotropic cylinder, the results of analyses, obtained using classical theory of laminated shells and finite element method, for an arbitrary element is investigated. It is seen that results obtained using classical theory of laminated shells and finite element method are coincide in acceptable condition.

According to Figure (5), hoop stresses (σ_θ) shows very small changes due to the variations in winding angle.

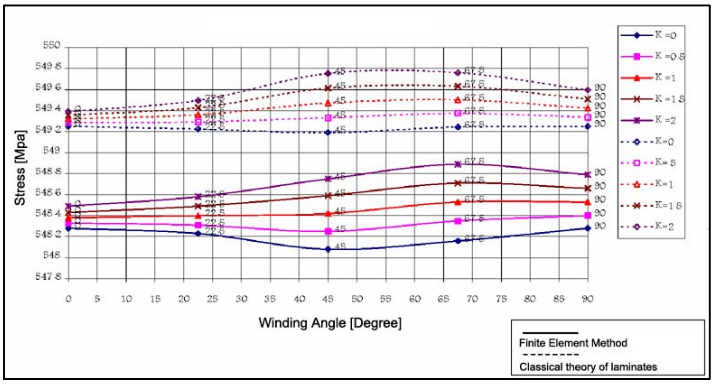

Figure 5. The hoop stress (σ_θ) against winding angle

According to Figure (6), axial stresses (σ_z) have direct relation with axial loads. Axial stress for k=0 is zero and for k=2 shows maximum value.

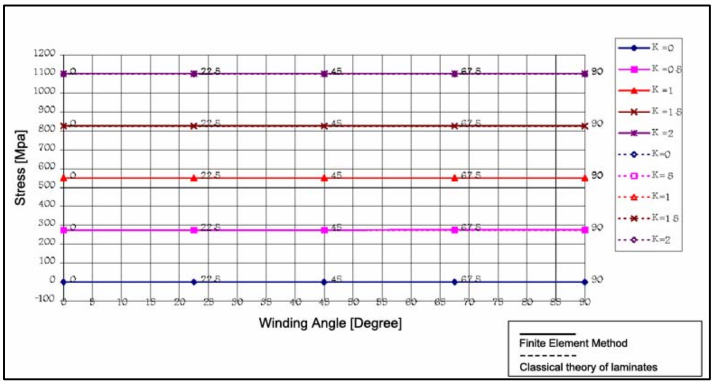

Figure 6. The axial stress (σ_z) against winding angle

For shear stresses ($\tau_{\theta z}$), as shown in Figure (7), along with direct relation to axial load, we will have significant change due to winding angle variations. For k=0 situation, pipe with two open ends under uniform internal pressure, a minimum shear stress can be seen in 55°. By change in k value, minimum varies from 47.5° for k=0.5 to 45° for k=1 and 42.5° for k=2.

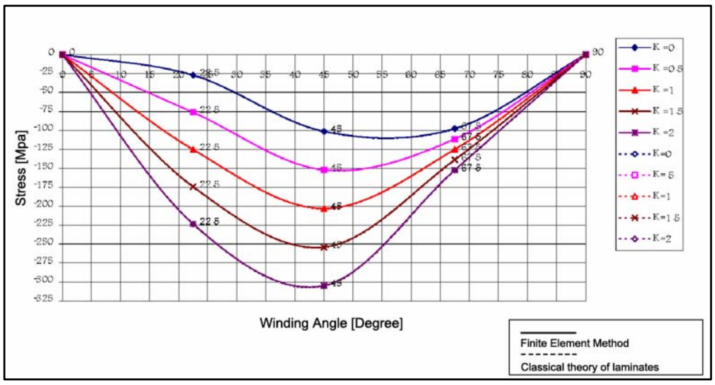

Figure 7. The shear stress ($\tau_{\theta z}$) against winding angle

Ratio of axial to hoop stresses $\left(\frac{\sigma_z}{\sigma_\theta}\right)$, shows a value equal to k which is not depended to cylinder winding angle, (Figure 8).

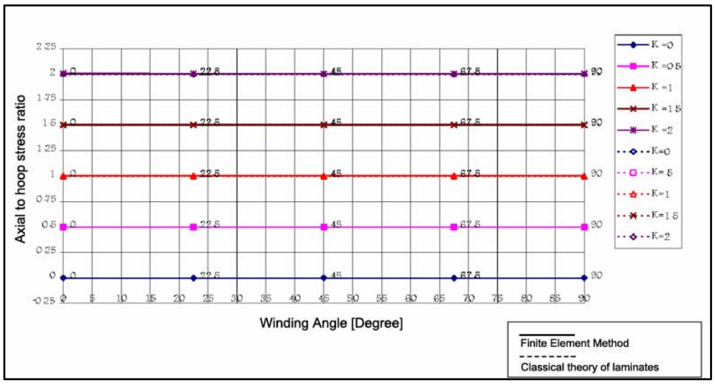

Figure 8. The ratio of axial to hoop stress $\left(\frac{\sigma_z}{\sigma_\theta}\right)$ against winding angle

On axis stresses determined using classical theory of laminated shells and finite element method, also, are similar. On axis normal stresses (σ_1) directly change with k but, by rising the winding angle all of the obtained stress values converge to the common amount at 90° as shown in Figure(9).

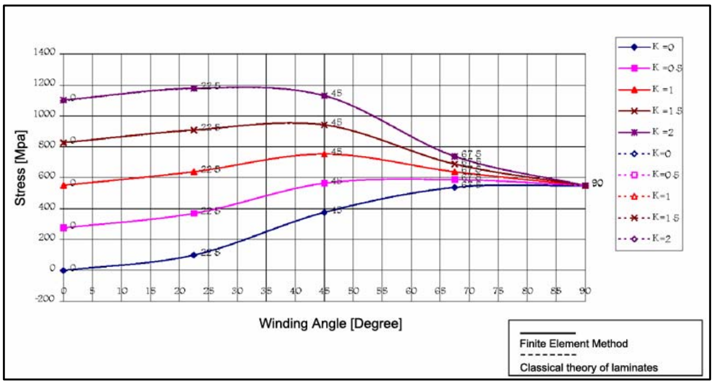

Figure 9. The on axis normal stress (σ_1) against winding angle

As shown in Figure (10) on axis transverse stresses (σ_2), also, are directly related to k like σ_1 but, convergence point is 0°.

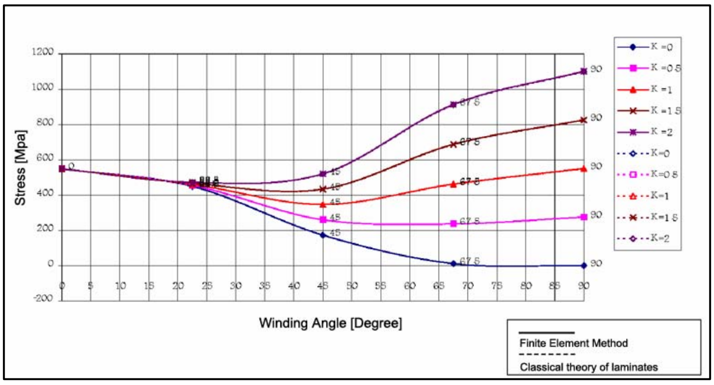

Figure 10. The on axis transverse stress (σ_2) against winding angle

On axis shear stresses (τ_{12}) show a different behavior. It is increases by rising of k but shows a sinusoidal change by varying of winding angle, as shown in Figure (11).

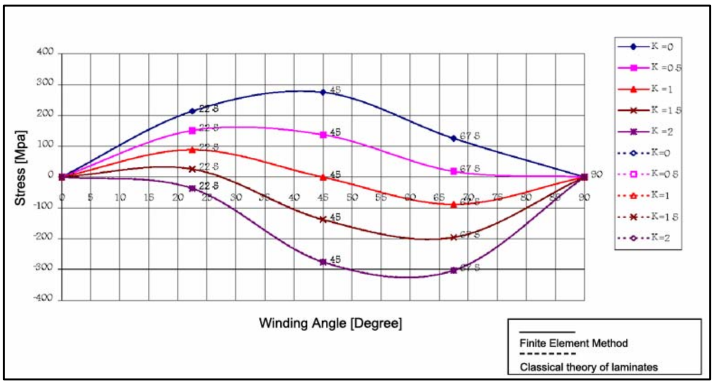

Figure 11. The on axis shear stress (τ_{12}) against winding angle

Figure (12) shows variation of Tsai-Wu failure criterion due to the winding angle. It can be seen that by increasing k minimum of graph moves toward the small winding angles. It means that elements try to aliening to achieve maximum available efficiency.

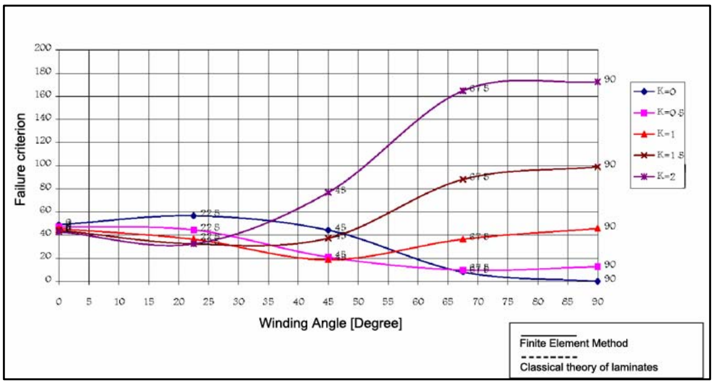

Figure 12. The Tsai-Wu failure criterion against winding angle

Conclusion

An orthotropic thin walled cylindrical shell under 5 different ratios of axial to hoop load (k) and in each step with 5 different winding angles (α) is analyzed. Analyses results are studied using classical theory of laminated shells and finite element method. The results can be summarized as follow:

- It is seen that the results obtained using classical theory and finite element method are the same satisfactorily.
- Generally increasing of k raises the hoop and axial stresses. Axial stresses directly depend on k and ratio of axial to hoop stresses is equal to k.
- Changes in winding angle don’t have any clear effect on hoop and axial stresses.
- Increasing the ratio of (k) minimum of Tsai-Wu failure criterion moves to the smaller winding angles. Hence the fibers aliens according to maximum stress and it makes structures more effective.
- Results show that optimum winding angle depends on axial to pressure loads and is not a constant value.

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