DAMAGE MECHANICS APPLICATION ON DESTROYED STEEL BRIDGE

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ABSTRACT: In the present paper the way of determination of elastic parameters of the body weakened by elliptical voids is presented. Starting from arbitrary placed single elliptical void in plane sheet, decrease of Young's modulus and Poisson's ratio is obtained. After that, using statistical distribution of voids, the decrease of overall elastic constants is found. In this procedure there are two ways, Taylor and Self-consistent model. In the first method interaction of defects is ignored, while in the second, so-called weak interaction is incorporated. Both methods are applied for small concentration of defects. In the paper it is shown that cracks and circles are special cases of the model derived in this study. Such obtained model is applied for determination of stiffness of steel members of the truss of bridge destroyed during lust war by shrapnel of bombshells. Such calculated stiffness is input for static and dynamic analysis of bridges using FEM. This approach is applied for analysis of The Pivnica Bridge, across The Ibar River, on the railway track Belgrade-Thessaloniki, destroyed during lust war. It is shown that with increasing a damage of members of the bridge the time period of free vibrations is increasing to, while natural frequency is decreasing.

INTRODUCTION

Damage Mechanics found its place not only in scientific papers and books in the past, but rather in the codes for engine parts design. The best example is aircraft industry. In recent times, the design specifications for fatigue of civil engineering structures, such are bridges, have been introduced. For example American Association of State Highway Officials (AASHO) proposed Fatigue

Design Curves for Fabricated Bridge Components. In the present paper a way of Damage Mechanics application on steel bridge destroyed by projectile is investigated.

PLANE SHEET WEAKEND BY ELIPTICAL VOID

Consider the problem of the elliptic cylinder ($a_3 \to \infty$) (see Figure 1) embedded in the elastic isotropic material with the same elastic parameters E (Young's modulus) and ν (Poisson's ratio). Eshelby in his 1957 [1] paper referred to "eigenstrains" as stress-free transformation strains. He proved that the uniform "eigenstrain" $\varepsilon_{ij}^{*'}$ within the elliptical inclusion, cause the uniform "eigenstresses" $\sigma_{ij}^{*'}$ in the same region (see also Mura, [4]):

$$\begin{aligned}
\sigma_{11}^{*'} &= \frac{\mu}{1-\upsilon}\left\{-2+\frac{a_2^2+2a_1a_2}{(a_1+a_2)^2}+\frac{a_2}{a_1+a_2}\right\}\varepsilon_{11}^{*'} \\
&\quad+\frac{\mu}{1-\upsilon}\left\{\frac{a_2^2}{(a_1+a_2)^2}-\frac{a_2}{a_1+a_2}\right\}\varepsilon_{22}^{*'}-\frac{2\mu\upsilon}{1-\upsilon}\frac{a_1}{a_1+a_2}\varepsilon_{33}^{*'}. \\
\sigma_{22}^{*'} &= \frac{\mu}{1-\upsilon}\left\{-2+\frac{a_1^2+2a_1a_2}{(a_1+a_2)^2}+\frac{a_1}{a_1+a_2}\right\}\varepsilon_{22}^{*'} \\
&\quad+\frac{\mu}{1-\upsilon}\left\{\frac{a_1^2}{(a_1+a_2)^2}-\frac{a_1}{a_1+a_2}\right\}\varepsilon_{11}^{*'}-\frac{2\mu\upsilon}{1-\upsilon}\frac{a_2}{a_1+a_2}\varepsilon_{33}^{*'}. \\
\sigma_{33}^{*'} &= -\frac{2\mu\upsilon}{1-\upsilon}\frac{a_1}{a_1+a_2}\varepsilon_{11}^{*'}-\frac{2\mu\upsilon}{1-\upsilon}\frac{a_2}{a_1+a_2}\varepsilon_{22}^{*'}-\frac{2\mu}{1-\upsilon}\varepsilon_{33}^{*'}, \\
\sigma_{12}^{*'} &= -\frac{2\mu}{1-\upsilon}\frac{a_1a_2}{(a_1+a_2)^2}\varepsilon_{12}^{*'}, \\
\sigma_{23}^{*'} &= -2\mu\frac{a_2}{a_1+a_2}\varepsilon_{23}^{*'}, \\
\sigma_{31}^{*'} &= -2\mu\frac{a_2}{a_1+a_2}\varepsilon_{31}^{*'},
\end{aligned} \tag{1}$$

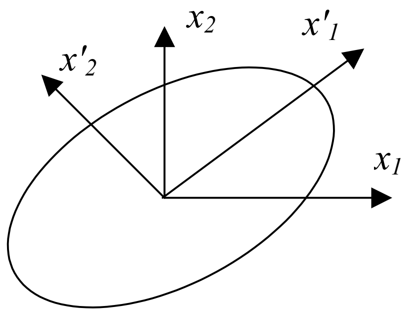

Figure 1: Elliptical void in the global and local (primed) coordinate system

In the above expressions $a_1=a$ and $a_2=\alpha a$ are half axes of the elliptical region, while μ and ν are the shear modulus and Poison's ratio respectively. According to equivalent inclussion method, Mura 1987 [4], the total stress within the elliptical region under far field stresses σ_{ij}', and one that is caused by the "eigenstrain" given by the expressions (2.1) should be zero everywhere in the eliptical region if the region should represent the void:

$$\sigma_{11}^{'} + \sigma_{11}^{*'} = 0, \quad \sigma_{22}^{'} + \sigma_{22}^{*'} = 0, \quad \sigma_{12}^{'} + \sigma_{12}^{*'} = 0. \tag{2}$$

Equation (2) is written for plane stress condition. Substituting governing values from the Eq. (1) into Eq. (2) leads to the system of equations with respect to unknown "eigenstrains" $\varepsilon_{11}^{*'}$, $\varepsilon_{22}^{*'}$ and $\varepsilon_{12}^{*'}$. The solution of the system of equations is:

$$\left.\begin{aligned} \varepsilon_{11}^{*'} &= \frac{1-\upsilon}{2\mu}\left[(1+2\alpha)\sigma_{11}^{'} - \sigma_{22}^{'}\right], \\ \varepsilon_{22}^{*'} &= \frac{1-\upsilon}{2\mu}\left[\frac{2+\alpha}{a}\sigma_{22}^{'} - \sigma_{11}^{'}\right], \\ \varepsilon_{12}^{*'} &= \frac{1-\upsilon}{2\mu}\frac{(1+\alpha)^2}{\alpha}\sigma_{12}^{'} \end{aligned}\right\} \tag{3}$$

Once the $\varepsilon_{ij}^{*'}$ are known, the increase of the strain energy of the body due to presence of elliptical void is obtained as:

$$\Delta W = -\frac{1}{2} V \sigma_{ij}^{'} \varepsilon_{ij}^{*'} = -\frac{1}{2} \pi a_1 a_2 \sigma_{ij}^{'} \varepsilon_{ij}^{*'}, \tag{4}$$

Substituting (3) into (4) yields to:

$$\Delta W = \frac{\pi \alpha a^2}{2E} \left[(1+2\alpha)(\sigma_{11}')^2 - 2\sigma_{11}^{'}\sigma_{22}^{'} + \frac{(2+\alpha)}{\alpha}(\sigma_{22}^{'})^2 + \frac{2(1+\alpha)^2}{\alpha}(\sigma_{12}^{'})^2 \right] \tag{5}$$

Differentiating expressions (5) twice with respect to stresses yields to the compliances:

$$S_{ij}^{'(k)*} = \frac{\partial^2 W}{\partial \sigma_i^{'} \partial \sigma_j^{'}} \tag{6}$$

where Voigt notation, $\sigma'_1 = \sigma'_{11}$, $\sigma'_2 = \sigma'_{22}$ and $\sigma'_6 = \sigma'_{12}$ is used. Also in the expression (6) (k) refers to a single eliptical void and (*) stands for the increase of the governing value of the compliance due to presence of the void. Once the compliances $S_{ij}^{'(k)*}$, in the local coordinate system are determined, using the transformation rule, Horii and Nemat-Nasser [2] the compliances in the global coordinate system $S_{ij}^{(k)*}$ can be determined.

Mean field theory (uniform distribution of voids)

In the case of many voids the total compliance would be, Horii and Nemat-Nasser [2], Sumarac and Krajcinovic [5]:

$$\overline{S}_{ij} = S_{ij} + \widehat{S}_{ij}^{*}. \tag{7}$$

In the above expression ( )* refers to the increase of the value due to presence of voids, and S_{ij} is the compliance matrix of the undamaged (virgin) material.
In the case of of Taylor model system of Eq. (2.7) leads to:

$$\frac{\overline{E}^{tm}}{E} = \frac{1}{1+\omega(\alpha^2+\alpha+1)}, \tag{8}$$

$$\frac{\bar{\nu}^{tm}}{\nu}=\frac{1+\dfrac{\omega\alpha}{\nu}}{1+\omega(\alpha^2+\alpha+1)}. \qquad (9)$$

In the case of Self-consistent model Eq. (7) are:

$$\frac{1}{\bar{E}}=\frac{1}{E}+\frac{\omega}{\bar{E}}(\alpha^2+\alpha+1), \qquad -\frac{\bar{\nu}}{\bar{\mathrm{E}}}=-\frac{\nu}{E}-\frac{\omega}{\bar{E}}\alpha. \qquad (10)$$

Solution of them is:

$$\frac{\bar{E}^{sc}}{E}=1-\omega(\alpha^2+\alpha+1), \qquad (11)$$

$$\frac{\bar{\nu}^{sc}}{\nu}=1-\omega(\alpha^2+\alpha+1)+\frac{\omega\alpha}{\nu}. \qquad (12)$$

The total overall compliance for matrix in the case of Self-consistent approximation for uniform distribution of elliptical voids is:

$$\frac{\bar{S}_{ij}^{sc}}{S_{ij}}=\begin{bmatrix} \dfrac{1}{1-\omega A} & -1-\dfrac{\omega\alpha}{\nu(1-\omega \mathrm{A})} & 0 \\ -1-\dfrac{\omega\alpha}{\nu(1-\omega A)} & \dfrac{1}{1-\omega \mathrm{A}} & 0 \\ 0 & 0 & \dfrac{1+\nu-\omega\nu\mathrm{A}+\omega\alpha}{(1+\nu)(1-\omega\mathrm{A})} \end{bmatrix} \qquad (13)$$

DAMAGE AND REPAIR OF BRIDGE PIVNICA

In the present paper the damage of the bridge Pivnica across The Ibar River on the railway Belgrade-Thessaloniki is presented. The bridge was destroyed during the bombardment of our country. Rebuilding of the bridge was performed using one temporary support at the place of most severe damage. The two new spans were built in factory, but other damage was repaired on site. The static and dynamic characteristics of rebuild structure are analyzed in the present paper according to damage mechanics and theory of structures. It is shown that for more amount of damage structure of the bridge becomes more

compliant or in another words period of free vibration is slightly increased. In the paper the problem of fatigue of material, especially of parts which undergone the low cycle fatigue is shortly outlined.
Bridge Pivnica was hit during bombardment two times. First bombshell hited the middle part of bridge, but it didn't fell down. Second projectile hit diagonal above support, and then bridge felt into river, see Figure 2. Due to impact, the bottom members suffered plastic deformation. Besides that there was a lot of damages due to bomb shrapnel. Holes in the members can be approximated as ellipses. In the first section it is explained in details how the decrease of Young's modulus can be expressed in terms of damage, see Eq.(8) and (11).
It should be noted that for reconstruction of bridge it is spent 75t of steel. Total weight of the bridge is 440t, see Figure 4. In Figure 3 statical scheme is shown for finite element method.

Figure 2: Destroyed bridge Pivnica

Dynamical characteristics of reconstructed bridge

All static and dynamic characteristics are analyzed using FEM procedure. First step was to find free vibrations for first three modes. They are: T_1=0.731s,

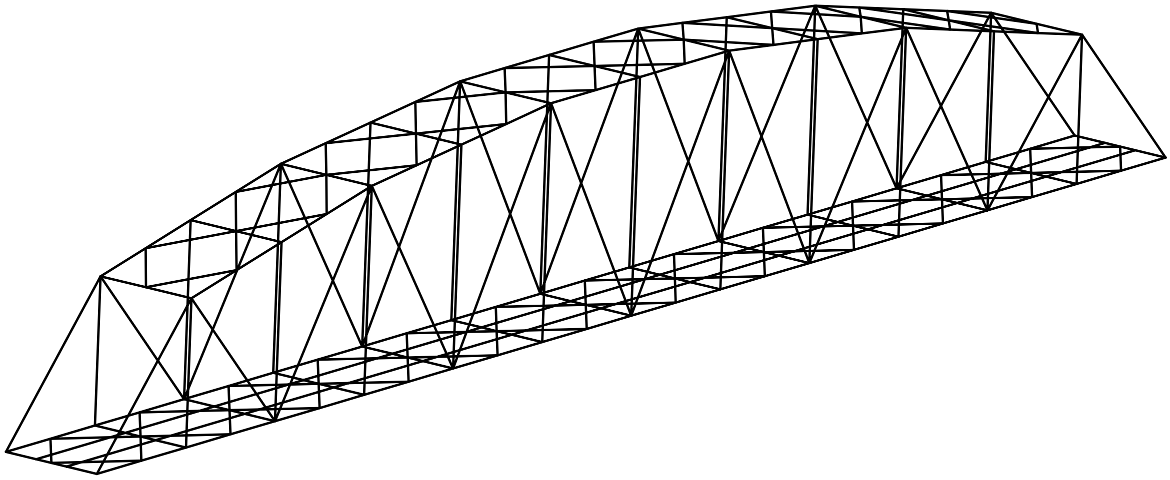

Figure 3: Reconstructed bridge Pivnica-statical scheme

T_2=0.2491s, T_3=0.1212s. For the damaged bridge it is calculated ω=0.1, ω=0.2 and ω=0.3 for the two spans of bottom members and four spans of top members. For instance in case of ω=0.3 $T_2(\omega=30\%)$=0.2551. This result was expected. If structure is more damaged, period of vibration is larger.

Figure 4: Reconstructed bridge Pivnica

Problem of Fatigue

It is well known that railway bridges are designed against high cycle fatigue. However during the bombardment some elements are destroyed. Neighboring parts suffered low cycle fatigue. It was impossible to change all elements. It is important to check behavior of elements, which suffered low cycle fatigue, and they are still in construction. Especially this is important during the winter when temperatures are well below zero.

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