Estimating 3-D SE(B) Fracture Parameters using 2-D Equations

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ABSTRACT: *The present study was conducted to assess the accuracy of plane-stress or plane-strain approximations when used to assess the state of three-dimensional samples. SE(B) samples were modelled by 2-D and 3-D finite elements and load/CMOD compliance and elastic J-integrals were calculated for elastic deformations; no attempt was made to study the effects of plasticity. The calculated 2-D and 3-D results were compared with those estimated using the equations given in ASTM E 1820. For a standard B×2B SE(B) sample with a/W = 0.5, the crack front is in a near-plane-strain state over most of the specimen thickness, although the specimen as a whole is in a mixed plane-strain/plane-stress state. The stress state changes continuously from plane strain to plane stress as the distance from crack tip (r) increases, and is close to plane stress when r/b = 0.6 to 0.8 where b is the width of the ligament. Near the point of load application (r/b = 1), the stress state is close to plane strain. In general, the out-of-plane stress state is closest to plane strain in the regions of highest in-plane stress gradient. The crack length calculated from CMOD compliance is approximated better by a 2-D plane-stress equation than by a plane-strain one. Experimental results are reported which substantiate this conclusion. Also, the relation between the elastic J-integral (calculated by the virtual crack extension method) and the stress intensity factor is in closer agreement to a 2-D plane-stress equation than a plane-strain one.*

INTRODUCTION

The first widely-accepted standard to measure fracture toughness was ASTM E 399, "Standard Test Method for Plane-Strain Fracture Toughness of Metallic Materials". This Standard specified a B×2B test-piece geometry (thickness B, width W=2B) with a/W.0.5, and stated that this specimen geometry assured that the crack tip would be in a condition of plane-strain constraint. The crack length is calculated from specimen compliance. Early experiments were done using load-line-displacement compliance, but currently crack-mouth-opening compliance is preferred, as outlined in the

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current standard ASTM E 1820, Standard Test Method for Measurement of Fracture Toughness.

Since the fracture toughness test ensures plane-strain constraint, it is natural to assume that the specimen compliance would be a plane-strain compliance. However, in recent revisions of the Standard, the compliance expressions are those of plane-stress constraint, raising the question of whether the overall deformation is in reality best described as plane strain or plane stress. In addition, the situation has been further confused by the introduction in the first edition of ASTM E 1921, "Standard Test Method for Determination of Reference Temperature, T_0 , for Ferritic Steels in the Transition Range", of an equation to calculate the elastic component of the J-integral from the plane-stress relation between J and stress intensity factor K rather than the plane-strain one. It has been recently explained that this was the result of an editorial slip-up, but nevertheless it focused attention on the question of whether the constraint is plane-stress or plane-strain. It was the intent of the current work to contribute to the resolution of this question.

CALCULATIONS

Single-edge bend SE(B) samples were modelled by 2-D and 3-D finite elements and load/CMOD compliance and elastic J-integrals were calculated for elastic deformations; no attempt was made to study the effects of plasticity. The finite element code used in the calculation is ADINA [1]. The calculated 2-D and 3-D results were compared with those estimated using the equations given in ASTM E 1820.

Two-Dimensional Modelling

To confirm the accuracy of the calculations, 2-D plane strain finite element analyses (FEM) were conducted for two SE(B) samples (a/W = 0.5 and 0.6) with plane strain conditions. Crack lengths calculated from the equations in E 1820 using the FEM compliances agreed within 0.2% with the assumed geometries. Also, J-integrals were calculated using the virtual crack extension (VCE) method and used to deduce values of stress intensity factor K from $J = K^2/EN$ where $EN = E/(1-v^2)$. Values of K were within 0.1% of those calculated using the ASTM equation for K (which was fitted to earlier FEM calculations for plane-strain constraint).

Three-Dimensional Modelling

A B×2B SE(B) sample (a/W = 0.5) was modelled by 3-D finite elements. Because of symmetry, only a quarter of the sample was modelled. A fine mesh was used with 12 layers of elements of non-uniform spacing in the thickness direction. There are, in all, 83905 nodes and 19200 eight-node parametric elements. Singular elements were used around the crack tip to model the crack tip singularity. Load was applied by means of a uniform displacement along the centre line. The material was assumed to be elastic with modulus of elasticity E = 207 GPa and Poisson's ratio $\nu = 0.3$.

Figure 1 shows the three normal stress components (normalized by E) as a function of distance r (normalized by ligament b) from the crack front on the centre plane (distance normal to the crack plane y = 0, distance along the crack front from the centre plane z = 0).

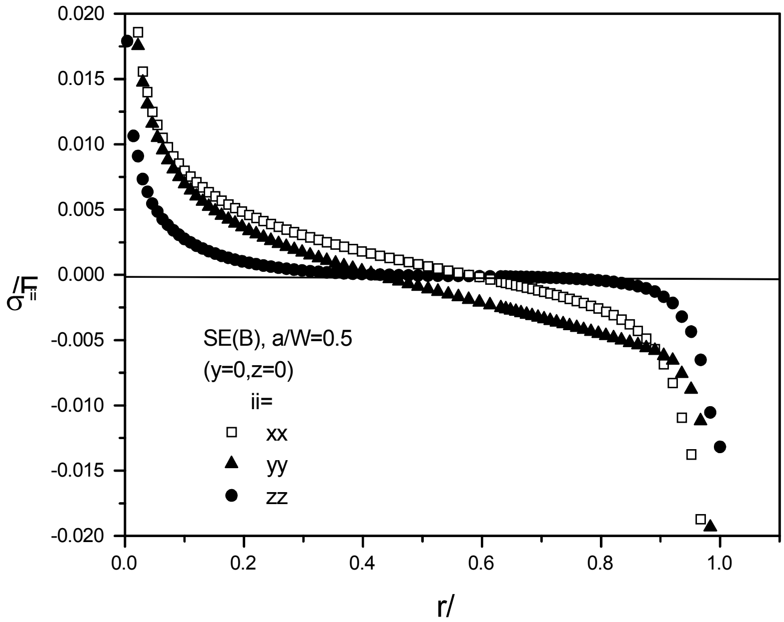

Figure 1: Normal stress components along centre layer as a function of distance r from crack tip (y = 0, z = 0, b = ligament length).

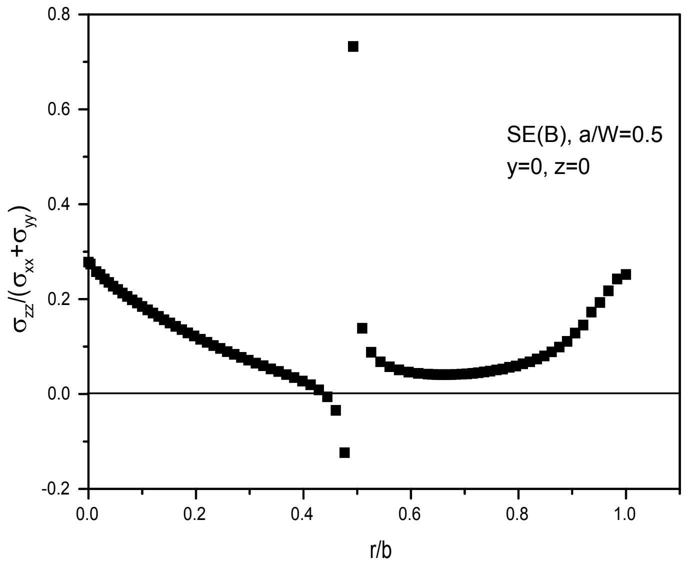

Figure 2: Ratio of Φ_{zz} to $(\Phi_{xx} + \Phi_{yy})$ at ligament (y = 0, z = 0).

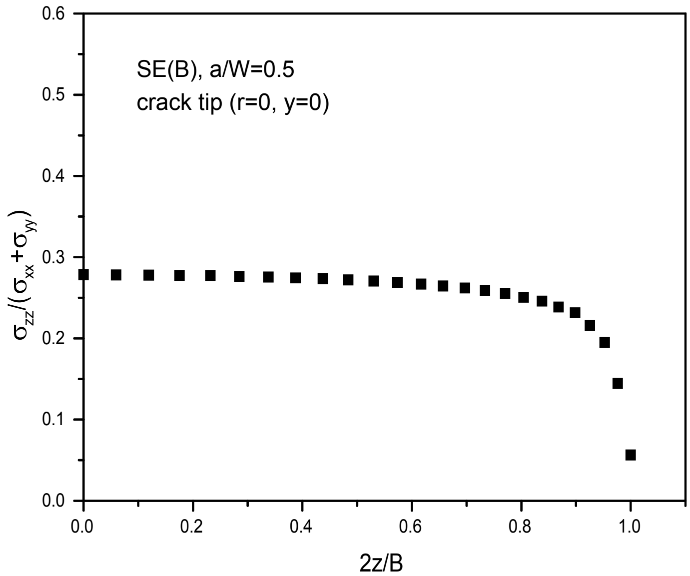

Figure 3: Ratio of of Φ_{zz} to $(\Phi_{xx} + \Phi_{yy})$ along thickness direction z at the crack tip (r = 0).

Figure 1 shows that the neutral axis (for which $\Phi_{yy} = 0$) is near the centre but closer to the crack tip, as expected. Figure 2 shows that the stress state changes continuously from plane strain to plane stress as the distance from the crack tip (r) increases, and is close to plane stress when r/b = 0.6 to 0.8. The degree of plane strain constraint drops to half at a distance from the crack tip of the order of 20% of the ligament. Close to the point of load application (r/b = 1), the stress state is close to plane strain. Figure 3 shows that the stress state is one of near-plane-strain along most of the crack front.

Estimation of crack length from compliance

The load/CMOD compliance was calculated by FEM, and used to evaluate crack length from the ASTM compliance equation. Values of the non-dimensional compliance for both 3-D and 2-D geometries are reported in Table 1. The crack length was evaluated from the 3-D compliance assuming both plane stress and plane strain constraint (in the latter case replacing E in the ASTM equation with $E/(1-\nu^2)$). Similar calculations were conducted with a/W = 0.6 for B×2B and B×B SE(B) samples. All the calculated crack lengths are listed in Table 1. In the table, C_i is the CMOD compliance and S is the loading span.

TABLE 1: Comparison of crack lengths

Geometry	a/W (actual mesh)	$BWEC_i/(S/4)$			a/W evaluated from compliance			
		2-D		3-D	plane stress	difference from actual a/W (%)	plane strain	difference from actual a/W (%)
		plane stress	plane strain					
Bx2B	0.5	35.58	32.37	34.85	0.496	-0.8	0.513	+2.6
Bx2B	0.6	63.82	58.07	62.40	0.595	-0.8	0.610	+1.7
BxB	0.6	63.82	58.07	60.85	0.591	-1.5	0.606	+1.0

The 2-D compliance is larger for plane stress than for plane strain, the ratio between the two being $1/(1-\nu^2)$ as expected. The 3-D compliance lies between the two limits, again as expected. For the standard sample geometry (B×2B) the value of a/W calculated from the compliance using the plane stress equation is much closer to the actual value for both a/W = 0.5 and a/W = 0.6 than that using the plane strain equation.

Evaluation of Stress Intensity Factors from Near-Crack-Tip Stress

The local stress intensity factor K was also evaluated from the local opening stress Φ_{yy} by extrapolation of $\Phi_{yy}(2Br)^{1/2}$ to r = 0 ("stress method"). K was then normalized to obtain $F = K/K_0$ where $K_0 = \Phi(Ba)^{1/2}$; $\Phi = 3PS/(2BW^2)$ for the SE(B) sample (a/W = 0.5) in three-point bending and P is the applied load. Results are shown in Fig. 4. The stress intensity factor calculated from the applied load using the ASTM formula is also shown in the figure for comparison. The normalized stress intensity factor at the centre of the 3-D sample is about 7% higher than that calculated from the applied load. In terms of the normalized stress intensity factor, F = 1.416 from the ASTM formula, and F = 1.500 (centre) and 0.775 (surface) from the 3-D calculation. As a consistency check, the plane strain FEM calculations gave F = 1.407 for both the 2-D and 3-D geometries, in essential agreement with the ASTM result.

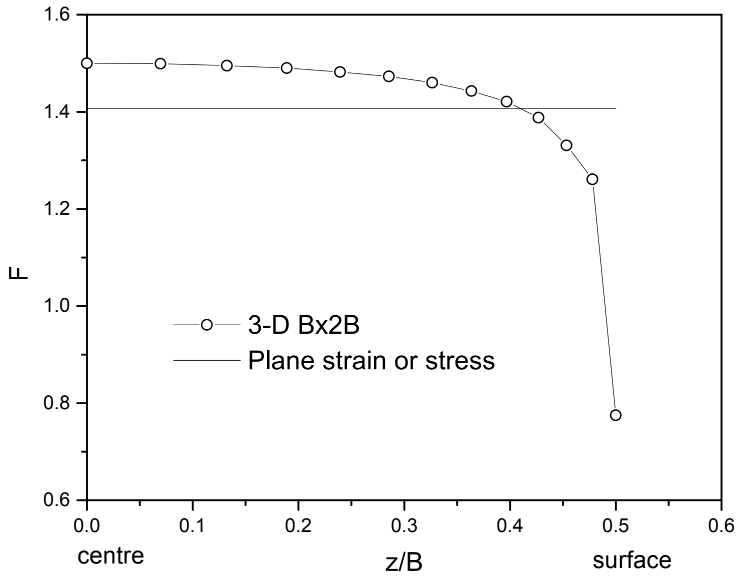

Figure 4: Normalized local stress intensity factors of a 3-D B×2B SE(B) sample along the crack front as a function of distance z from the centre plane.

Evaluation of J-integral

The J-integral at the centre of the crack front (z = 0) was evaluated by the virtual crack extension (VCE) method. ADINA provides for calculation of local J-integral values, i.e. point-by-point values rather than average ones. The J-integral at the same load was also calculated using the equations given in ASTM E 1820 by calculating the stress intensity factor from the applied load and then the J-integral from $J = K^2/EN$. The latter calculation was performed assuming both plane strain and plane stress constraint, i.e. by using the relevant effective modulus EN with the 2-D relation. The ratios of the 3-D J-integral to the values calculated from the stress intensity factors using the effective modulus for plane stress and plane strain are 0.967 and 1.062 respectively.

Experimental Verification

Twenty-six SE(B) samples (B×2B, a/W = 0.5, W = 38 mm) were tested in three-point bending. The material was CSA 350A steel (E = 207 GPa, ν= 0.3, Φ_y = 390 MPa, Φ_u = 535 MPa). All the samples were tested at -110°C. Standard tensile tests were conducted to measure the mechanical properties, including the modulus of elasticity, from -160° to 60°C. The temperature dependence of the modulus over this temperature range was well approximated by a linear equation:

$$E = F_1 - F_2T \tag{1}$$

where T is the temperature in °C, F_1 = 208.5 GPa and F_2 = 0.0435 GPa/°C. The modulus calculated from eqn (6) is 213 GPa at T = -110°C.

The load and crack mouth opening displacement were recorded for each sample. Crack lengths were then evaluated from the CMOD compliance for each sample using the equations given in ASTM E 1820. For comparison, the crack lengths were also evaluated using plane strain equations. After testing, the crack length of each sample was measured on the fracture surface by the 9-point-average technique described in ASTM E 1820. Figure 5 shows the ratio of crack length measured from compliance (both plane stress and plane strain) to that measured using the 9-point average. The mean of the ratios is also shown in the figures. The mean using the plane-stress assumption (1.002) is much closer to unity than that assuming plane strain (1.035).

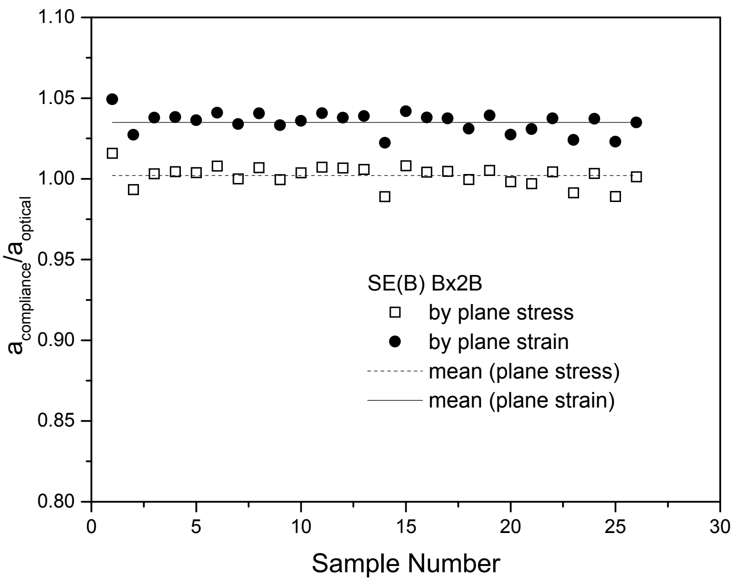

Figure 5: Ratio of crack length calculated from CMOD compliance to that measured by nine-point average technique; nominal a/W = 0.5.

DISCUSSION

The most striking feature of these results is the observation that the local stress intensity factor derived from the crack-tip stresses is actually larger than the stress intensity factor calculated from the applied load using the ASTM formula. This is surprising, since the value of the geometry factor F is identical in two dimensions for either plane strain or plane stress; the natural assumption is that the factor seems to be independent of constraint and should therefore be the same for mixed plane strain/plane stress conditions. It may be that the average value of the geometry factor F averaged along the crack front in the 3-D case is close to the 2-D result, but this was not checked. It is also surprising that the J integral at the mid-plane $z = 0$ ($J_{3\text{-}D}$) is related to the stress intensity factor $K_{2\text{-}D}$ (calculated from the applied load using the ASTM equation) through the plane stress relation $J_{3\text{-}D} = K_{2\text{-}D}^2/E$ even though the constraint at this location is one of plane strain.

These results are in accord with those of Nakamura and Parks [2] who studied the three-dimensional stress field near the crack front of a thin elastic plate. They are also in general agreement with the results of two independent studies [3,4] that appeared while this work was in progress. The former [3] includes FEM calculations for the SE(B) geometry. The latter [4] addresses only the centre-cracked plate tension geometry, but many of the results can be applied to the SE(B) case.

CONCLUSIONS

1. The 2-D finite element calculations in this work have generated values of compliance and stress intensity factors in close agreement (within 0.2%) of values calculated using equations in ASTM standard E 1820 for the limiting cases of plane stress and plane strain for standard single-edge bend SE(B) samples with a/W = 0.5 and 0.6.
2. Elastic 3-D finite element calculations for a standard B×2B SE(B) sample with a/W = 0.5 show that the crack front is in a near-plane-strain state over most of the specimen thickness, although the specimen as a whole is in a mixed plane-strain/plane-stress state. In general, the out-of-plane stress state is closest to plane strain in the regions of highest in-plane stress gradient.
3. Crack lengths calculated from CMOD compliance are approximated better by a plane-stress equation than a plane-strain one for a B×2B sample with a/W = 0.5 and 0.6, in support of recommendations in ASTM E 1820. However, for a B×B geometry with a/W=0.6, crack lengths are approximated better by a plane-strain equation.
4. The stress intensity factor $K_{3\text{-}D}$ on the mid-plane of a B×2B SE(B) sample with a/W = 0.5 estimated from the local stress field is higher than that calculated from the equation for K given in ASTM E 1820.
5. The relation between the local 3-D elastic J-integral at the mid-plane (calculated by the virtual crack extension method) and the stress intensity factor $K_{2\text{-}D}$ calculated using a 2-D equation ($J = K^2/EN$) is approximated better using the plane stress relation (EN = E) than that for plane strain ($EN = E/(1-\nu^2)$).
6. Physical crack lengths measured experimentally for B×2B samples with a/W.0.5 by the nine-point-average technique agree well with crack length estimates calculated from crack-mouth opening compliance assuming a state of plane stress.

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