Modelling of Fatigue Crack Propagation and Brittle Fracture Problems

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ABSTRACT: *A non-local critical plane model was proposed by Seweryn and Mróz [1,2] and applied to prediction of crack initiation and propagation in brittle materials for monotonic loading and also for cyclic loading, involving high cycle fatigue life prediction for both in-phase and out-of-phase loading cases. In this paper, the crack propagation model is considered assuming a constant length damage zone translating at the crack front. The paper contains results of parametric analysis of model, particular overloading effect, unstable crack growth condition, crack growth curves, shape of damage zone.*

INTRODUCTION

Modelling of fatigue crack propagation has a great significance in fracture mechanics. Accurate prediction of the rate of crack growth allows for determination of service life of machine elements containing different type of defects. Crack initiation in structural elements subjected to a cyclically varying loading does not necessitate immediate renewing or replacement of the element. In most engineering cases, it is sufficient to monitor crack growth and replace the cracked element before the crack reaches its critical length. This critical length can be estimated by using fatigue crack propagation models.

In present paper we shall develop a non-local critical plane model proposed by Seweryn and Mróz [1,2] for prediction of fatigue crack propagation in uniaxial and multiaxial loading conditions.

CRACK GROWTH UNDER MODE I CONDITION

Let's consider a plate of uniform thickness, see Figure 1a, with the edge crack of length l, loaded by a cyclically varying stress σ of range $\Delta\sigma$ and mean value $\sigma_m = \Delta\sigma/2$. The material is assumed to be linear elastic, but exhibiting a process or damage zone Ω of length d_0 ahead of the crack tip, see Figure 1b. Damage growth occurs only in the damage zone and is specified by the mean value of damage parameter $\overline{\omega}_n$ affecting the critical stress σ_c.

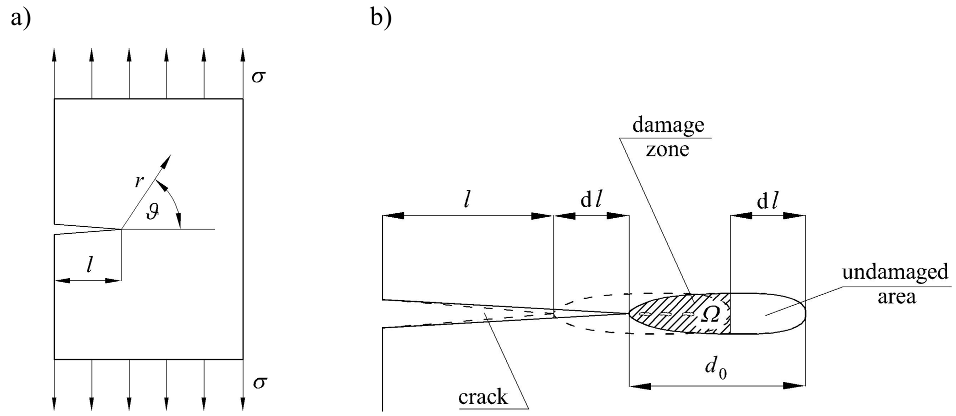

Figure 1: a) Plate with the edge crack, b) damage zone propagation.

The normal stress averaged in the zone Ω is

$$\overline{\sigma}_n = \frac{1}{d_0}\int_0^{d_0} \sigma_n \mathrm{d}r = \frac{2K_\mathrm{I}}{\sqrt{2\pi d_0}} \tag{1}$$

where K_I is the stress intensity factor for mode I.

The cycle of fatigue loading we divide into four stages, see Figure 2.

When stress in a cycle increases from zero, in the stage I there is no damage accumulation as $\overline{\sigma}_n < \sigma_o$ (so $K_\mathrm{I} < K_\mathrm{Ith}$) where σ_o denotes the treshold stress and K_Ith is the treshold value of K_I. Let us note that both σ_o and K_Ith depend on the damage state, thus

$$\sigma_o = \sigma_o^*\left(1-\overline{\omega}_n\right)^p, \qquad K_\mathrm{Ith} = K_\mathrm{Ith}^*\left(1-\overline{\omega}_n\right)^p \tag{2}$$

where σ_o^* and K_Ith^* are the respective values for the undamaged material and p is the material parameter.

In the second stage $\sigma_o < \overline{\sigma}_n < \sigma_c$, the damage accumulation occurs in the zone Ω, according to the rule [1,2]

$$\mathrm{d}\overline{\omega}_n = A\left(\frac{\overline{\sigma}_n - \sigma_o}{\sigma_c - \sigma_o}\right)^n \frac{\mathrm{d}\overline{\sigma}_n}{\sigma_c - \sigma_o}, \qquad \mathrm{d}\overline{\sigma}_n > 0 \tag{3}$$

and

$$\sigma_c = \sigma_c^*\left(1 - \overline{\omega}_n\right)^p, \qquad K_{Ic} = K_{Ic}^*\left(1 - \overline{\omega}_n\right)^p \tag{4}$$

where σ_c^* and K_{Ic}^* are the critical values for the undamaged material and A and n are the material parameter.

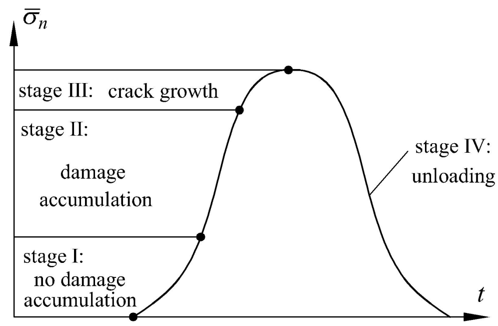

Figure 2: Consecutive stages of loading in one cycle.

When $\overline{\sigma}_n$ reaches the critical value $\overline{\sigma}_n = \sigma_c$ and $K_I = K_{Ic}$, the crack growth process occurs, so that the condition

$$\overline{\sigma}_n = \sigma_c, \qquad K_I = K_{Ic}, \qquad \mathrm{d}l > 0 \tag{5}$$

is satisfied. The stable crack growth condition is

$$\mathrm{d}\overline{\sigma}_n = \mathrm{d}\sigma_c, \qquad \mathrm{d}K_I = \mathrm{d}K_{Ic}\left(\overline{\omega}_n\right) \tag{6}$$

Let us note that $K_I = K_I(\sigma, l)$, so we have

$$\mathrm{d}K_I = \frac{\partial K_I}{\partial \sigma}\mathrm{d}\sigma + \frac{\partial K_I}{\partial l}\mathrm{d}l \tag{7}$$

In most cases the first term dominates as the crack growth value $\mathrm{d}l/\mathrm{d}N$ is small and then $\mathrm{d}K_I \cong M_k\sqrt{\pi l}\,\mathrm{d}\sigma$ where M_k depends on the geometry of the plate.

The damage accumulation during the stage III is decomposed into two terms

$$\mathrm{d}\overline{\omega}_n = \mathrm{d}\overline{\omega}_{n1} + \mathrm{d}\overline{\omega}_{n2} = A\frac{\mathrm{d}\overline{\sigma}_n}{\sigma_\mathrm{c} - \sigma_\mathrm{o}} - \overline{\omega}_n \frac{\mathrm{d}l}{d_0} \tag{8}$$

where the first term is associated with loading increment and the second is associated with damage zone propagation. Introducing ratio $f = \sigma_\mathrm{o}^* / \sigma_\mathrm{c}^*$ and integrating Eq. 8 we obtain

$$\frac{\Delta l}{d_0} = \left[1 + \frac{pA}{1-f}\right] \ln\left(\frac{\overline{\omega}_{n\mathrm{k}}}{\overline{\omega}_{n\mathrm{p}}}\right) - \frac{pA}{1-f} \ln\left(\frac{1-\overline{\omega}_{n\mathrm{k}}}{1-\overline{\omega}_{n\mathrm{p}}}\right) \tag{9}$$

where $\overline{\omega}_{n\mathrm{k}}$ and $\overline{\omega}_{n\mathrm{p}}$ denote the damage values at the beginning and the end of the propagation stage III.
The relation 9 specify the crack growth during one cycle, so that $\Delta l = \mathrm{d}l/\mathrm{d}N$. The consecutive stage IV corresponds to elastic unloading:

$$\mathrm{d}\overline{\sigma}_n < 0, \qquad \mathrm{d}\overline{\omega}_n = 0, \qquad \mathrm{d}l = 0 \tag{10}$$

Using the double logarithmic scale, the crack propagation curves are shown in Figure 3 for varying values of damage growth parameter $A = 0.1, 0.5, 1$.

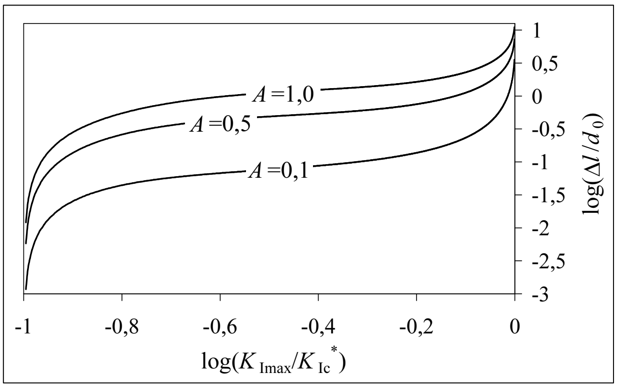

Figure 3: Influence of parameter *A* on crack propagation curves.

The curves can be compared with the usual diagrams $\mathrm{d}l/\mathrm{d}N = \mathrm{f}(\Delta K_\mathrm{I})$ available in literature. It is seen that the crack propagation curves correspond qualitatively well to experimental curves. When K_I tends to K_Ic^*, the crack propagation rate tends to infinity, when K_I tends to K^*_{Ith}, the propagation rate tends to zero.

Figure 4 illustrates the effect of overloading on crack propagation rate for different values of *f*.

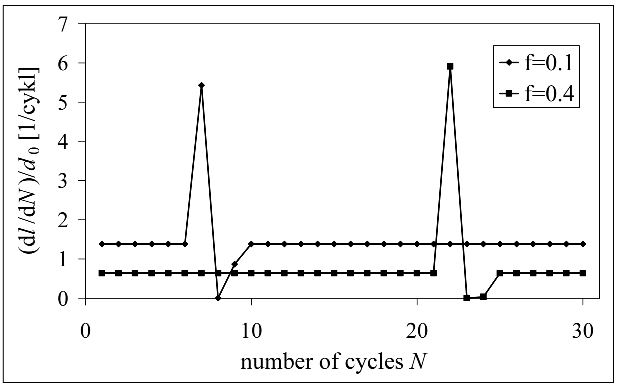

Figure 4: The effect of overloading on crack propagation rate: a) single overloading cycle $K_{Imax}/K^*_{Ic} = 0.9$ and subsequent cycles $K_{Imax}/K^*_{Ic} = 0.5$.

Let us note that when K_I tends to K^*_{Ic} (or $\overline{\sigma}_n$ tends to σ^*_c), then the case of brittle fracture occurs. Let us remind that the first term of Eq. 7 dominates for stable crack growth, and the second term – for the unstable growth. To formulate brittle fracture condition then, we can disregard the first term of Eq. 7 because

$$\frac{\partial K_I}{\partial \sigma}\mathrm{d}\sigma << \frac{\partial K_I}{\partial l}\mathrm{d}l, \qquad \text{so} \qquad \mathrm{d}K_I \approx \frac{\partial K_I}{\partial l}\mathrm{d}l \tag{11}$$

It is justified in the case of load control (then $\mathrm{d}\sigma/\mathrm{d}l \geq 0$). When kinematic control occurs, we have $\mathrm{d}\sigma/\mathrm{d}l < 0$ and it is necessary to consider the complete form of Eq. 7.

Rearranging Eq. 6 we can finally obtain brittle fracture criterion in the following form:

$$K_I = K^*_{Ic}\left(1-\overline{\omega}_n\right)^p \qquad \text{and} \qquad \frac{\partial K_I}{\partial l} \geq \frac{pK^*_{Ic}\left(1-\overline{\omega}_n\right)^p\overline{\omega}_n}{d_0\left[1-\overline{\omega}_n+\dfrac{pA}{(1-f)}\right]} \tag{12}$$

Figure 5 shows the graphic illustration of this equations.

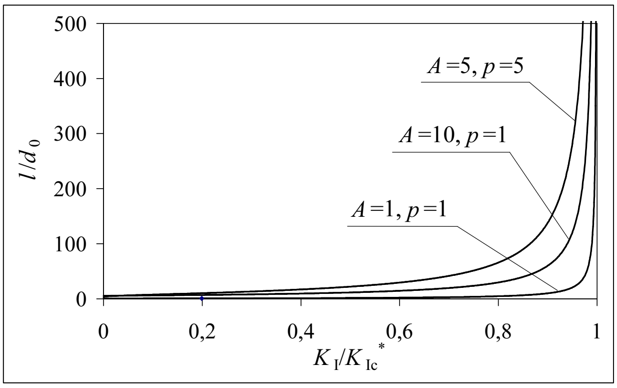

Figure 5: Dependence of the critical crack length on K_I value for unstable crack growth with varying values of A and p.

CRACK PROPAGATION UNDER COMBINED MODE I AND MODE II CONDITIONS

The approach presented in the previous section can now be extended to the case of biaxial loading with occurance of modes I and II. Then the mean values $\overline{\omega}_n(\vartheta)$, normal stress $\overline{\sigma}_n(\vartheta)$, shear stress $\overline{\tau}_n(\vartheta)$, critical stresses $\sigma_c(\vartheta)$ and $\tau_c(\vartheta)$ depend on angle ϑ in a polar coordinate system connected with the crack tip, see Figure 6.

The stress distribution on critical planes at the crack tip according to asymptotic solution for the linear elastic material is expressed as follows

$$\sigma_n = \frac{1}{4\sqrt{2\pi r}}\left[K_I\left(3\cos\frac{\vartheta}{2}+\cos\frac{3}{2}\vartheta\right)-3K_{II}\left(\sin\frac{\vartheta}{2}+\sin\frac{3}{2}\vartheta\right)\right],$$
$$\tau_n = \frac{1}{4\sqrt{2\pi r}}\left[K_I\left(\sin\frac{\vartheta}{2}+\sin\frac{3}{2}\vartheta\right)-K_{II}\left(\cos\frac{\vartheta}{2}+3\cos\frac{3}{2}\vartheta\right)\right] \tag{13}$$

where K_I and K_{II} are the stress intensity factors for modes I and II respectively. Let us note that singular terms are only considered in Eq. 13.

The cycle of biaxial loading is divided into four stages similarly as in the uniaxial loading case. There is no damage growth in the first stage and stresses in the zone Ω are lower than the treshold values. Let us describe this condition by the damage initiation function $R_{\sigma o}$ [3,4]:

$$\overline{R}_{\sigma o} = \frac{1}{d_0} \int_0^{d_0} R_{\sigma o}\left(\frac{\sigma_n}{\sigma_o}, \frac{\tau_n}{\tau_o}\right) \mathrm{d}r < 1 \tag{14}$$

where $\overline{R}_{\sigma o}$ is the non-local stress damage initiation function and τ_o is the treshold value of shear stress depending on value of $\overline{\omega}_n$, namely

$$\tau_o = \tau_o^* \left(1 - \overline{\omega}_n\right)^p \tag{15}$$

where τ_o^* denotes the treshold value for the undamaged material.

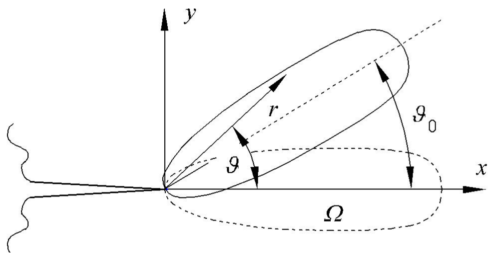

Figure 6: Damage zone at the crack tip in a polar coordinate system.

The function $R_{\sigma o}$ can be expressed in the form of the elliptic condition for $\sigma_n \geq 0$ and the Coulomb condition for $\sigma_n < 0$, thus

$$R_{\sigma o} = \begin{cases} \left[\left(\frac{\sigma_n}{\sigma_o}\right)^2 + \left(\frac{\tau_n}{\tau_o}\right)^2\right]^{0.5}, & \sigma_n \geq 0 \\ \frac{1}{\tau_o}\left(\left|\tau_n\right| + \sigma_n \tan\varphi\right), & \sigma_n < 0 \end{cases} \tag{16}$$

where φ is the friction angle.

In stage II of loading we have the damage growth on the selected physical plane. This condition is described by the stress failure function R_σ which can be expressed in form similar to Eq. 16. Because of singular stress field we have to use the non-local stress failure function $\overline{R}_\sigma$ namely [5,6]:

$$\overline{R}_\sigma = \frac{1}{d_0}\int_0^{d_0} R_\sigma\left(\frac{\sigma_n}{\sigma_c},\frac{\tau_n}{\tau_c}\right)\mathrm{d}r \tag{17}$$

If the damage accumulation occurs on the physical plane with no crack propagation we have

$$f \le \overline{R}_\sigma < 1, \qquad f = \frac{\overline{R}_\sigma}{\overline{R}_{\sigma 0}}, \qquad \mathrm{d}\overline{R}_\sigma > 0 \tag{18}$$

The damage growth $\overline{\omega}_n(\vartheta)$ is now governed by the relation

$$\mathrm{d}\overline{\omega}_n = A\left(\frac{\overline{R}_\sigma - f}{1-f}\right)^n \frac{\mathrm{d}\hat{\overline{R}}_\sigma}{1-f} \tag{19}$$

where
$$\mathrm{d}\hat{\overline{R}}_\sigma = \frac{\partial \overline{R}_\sigma}{\partial \sigma_n}\mathrm{d}\sigma_n + \frac{\partial \overline{R}_\sigma}{\partial \tau_n}\mathrm{d}\tau_n, \qquad \mathrm{d}\hat{\overline{R}}_\sigma \ge 0 \tag{20}$$

When stress failure function on the physical plane reaches the critical value, the growth of the crack occurs, thus

$$\overline{R}_\sigma\left(\sigma_n/\sigma_c, \tau_n/\tau_c\right) = 1, \qquad \mathrm{d}l > 0 \tag{21}$$

The stable growth takes place when

$$\mathrm{d}\overline{R}_\sigma = \mathrm{d}\hat{\overline{R}}_\sigma + \frac{\partial \overline{R}_\sigma}{\partial \overline{\omega}_n}\mathrm{d}\overline{\omega}_n = 0 \tag{22}$$

The damage evolution is induced by the load increment and crack increment:

$$\mathrm{d}\overline{\omega}_n = \mathrm{d}\overline{\omega}_{n1} + \mathrm{d}\overline{\omega}_{n2} = A\frac{\mathrm{d}\hat{\overline{R}}_\sigma}{1-f} - \overline{\omega}_n \frac{\mathrm{d}l}{d_0} \tag{23}$$

Introducing effective length of the damage zone:

$$d_{\mathrm{eff}}(\vartheta) = \int_0^{d_0} R_\sigma\left(\sigma_n, \tau_n, \overline{\omega}_n\right) \mathrm{d}r \tag{24}$$

crack growth condition takes the form of $d_{\mathrm{eff}} = d_0$.

In the stage IV we have:

$$\mathrm{d}\overline{R}_\sigma < 0, \qquad \mathrm{d}\overline{\omega}_n = 0, \qquad \mathrm{d}l = 0 \tag{25}$$

Assume that the stress failure function takes the form of the elliptic condition for $\sigma_n \geq 0$ and the Coulomb condition for $\sigma_n < 0$ we shall present results of calculation of the damage zone shape for varying parameters of model. Figure 7a illustrates influence of parameter $K_{\mathrm{II}}/K_{\mathrm{I}}$ on orientation and shape of the damage zone. Let us note that N_{f} denotes number of cycles required to crack increment. In the Figure 7b we can see dependence of zone shape on amplitude of loading for mode I ($K_{\mathrm{II}}/K_{\mathrm{I}} = 0$).

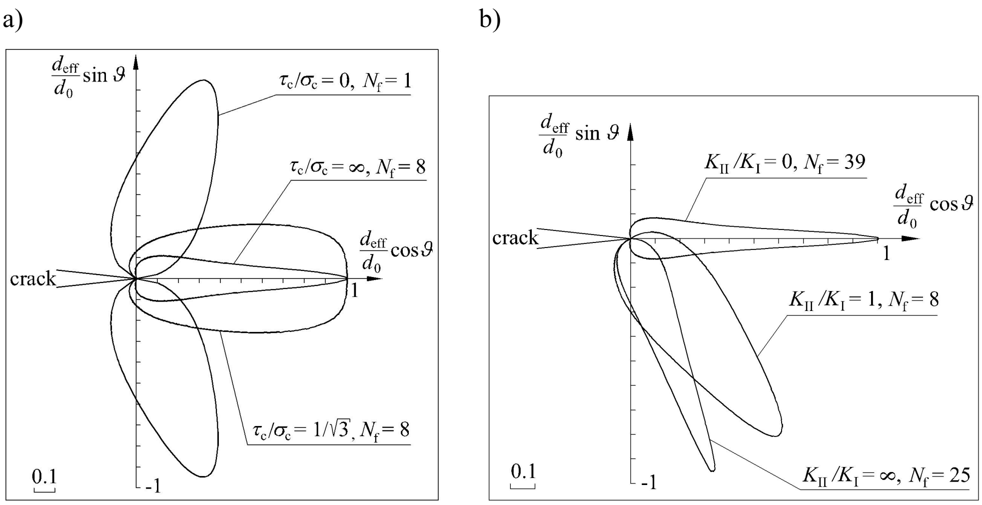

Figure 7: Damage zone shapes a) influence of parameter τ_c/σ_c for mode I, b) influence of parameter $K_{\mathrm{II}}/K_{\mathrm{I}}$.

CONCLUDING REMARKS

The present paper provides the model of analysis of crack initiation and propagation for monotonic and variable loading. The damage zone of constant length was introduced with the averaged measures of stress and damage within the zone. The zone is assumed to translate with the crack tip when the critical propagation condition is reached.

The model proposed enables calculation of crack growth in the linear elastic material, analysis of the effect of overloads on crack propagation rate, specification of crack trajectory for arbitrary biaxial non-proportional loading and specification of unstable crack growth regimes. The analysis was referred to asymptotic stress fields near the crack tip. However, it can be extended to more complex descriptions containing more terms of asymptotic expansions or generated by the approximate methods. The analysis can also be extended to three dimensional stress states and the associated damage zones.

Let us remind that presented model considers only the translation of the damage growth zone at the propagating crack tip. However it is possible to propose an approach considering both: motion of the damage zone and growth of this zone. This approach has been discussed in details in the article [7].

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