A New Procedure to Calibrate the Weibull Stress Modulus (*m*)

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ABSTRACT: *This paper describes a new procedure to calibrate the Weibull stress modulus, m, which employs SSY values for cleavage fracture toughness measured in the DBT region. The calibration scheme utilizes only one set of fracture toughness data while, at the same time, avoiding the non-uniqueness of calibrated Weibull stress parameters. An application follows to predict cleavage fracture behavior in different fracture specimens for an A515-70 structural steel tested in the transition region. The methodology predicts the measured statistical distribution of cleavage fracture toughness for the cracked specimens which provides a compelling support to the new calibration procedure.*

INTRODUCTION

Current transferability models for elastic-plastic fracture toughness values based upon the Weibull stress (σ_w) rely on the notion of σ_w as a *probabilistic* crack-tip driving force [1-3]. Under increased remote loading (as measured by J), differences in evolution of the Weibull stress reflect the potentially strong variations of near-tip stress fields due to the effects of constraint loss while, at the same time, incorporating statistical effects of the material microstructure on toughness. In this context, the Weibull modulus, m, plays a major role in the process to correlate fracture toughness for varying crack configurations and loading modes (tension *vs.* bending). Consequently, robust schemes to calibrate parameter m become a key element in fracture assessment procedures based upon σ_w.

Previously developed procedures to calibrate the Weibull modulus, m, (see [1,9] for additional details) employ toughness data for cleavage fracture (such as J_c-values) measured from only *one* set of specimens. However, Gao et al. [2] clearly show that such a widely adopted methodology based on a single set of specimens provides nonunique parameters, *i.e.*, many pairs of (m, σ_u) yield equally good correlation of critical Weibull stress values with the measured distribution of toughness data. Advanced calibration procedures now under development [2,3,7,13] employ a toughness scaling model based upon the Weibull stress to determine parameter m based upon fracture toughness data measured from *two* sets of specimens. By using the Weibull stress trajectories, σ_w *vs.* J, for two crack configurations exhibiting different constraint levels (*e.g.*, a deep notch and a shallow notch SE(B) specimen), the process seeks the *m*-value which *corrects* the corresponding measured toughness distributions. This approach eliminates the non-uniqueness of calibrated Weibull stress parameters that arises when using only one set of fracture toughness data but at an extra cost of requiring fracture testing of different crack configurations. While this calibration procedure has proven effective to predict cleavage fracture in different specimen geometries and materials [2,3,7,13], more improved and yet simpler schemes to calibrate the Weibull modulus, *m*, become necessary in routine engineering analysis.

This paper describes a new procedure to calibrate the Weibull stress modulus, *m*, which employs SSY values for cleavage fracture toughness measured in the DBT region. The calibration scheme utilizes only one set of fracture toughness data while, at the same time, avoiding the non-uniqueness of calibrated Weibull stress parameters. An application follows to predict cleavage fracture behavior in different fracture specimens for an A515-70 structural steel tested in the transition region. The proposed methodology predicts the measured statistical distribution of cleavage fracture toughness for these cracked specimens which provides a compelling support to the new calibration procedure.

THE WEIBULL STRESS MODEL

Experimental studies consistently reveal large scatter in the measured values of cleavage fracture toughness for ferritic steels tested in the DBT region. A continuous probability function derived from weakest link statistics conveniently characterizes the distribution of toughness values in the form [8]

$$F(J_c) = 1 - \exp\left[- \left(\frac{J_c - J_{\min}}{J_0 - J_{\min}} \right)^{\alpha} \right], \tag{1}$$

which is a three-parameter Weibull distribution with parameters $(\alpha, J_0, J_{\min})$. Here, α denotes the Weibull *shape* parameter, J_0 defines the characteristic toughness (*scale* parameter) and $J_{\min}$ is the *threshold* fracture toughness. Often, the threshold fracture toughness is set equal to zero so that the Weibull function given by Eq. (1) assumes its more familiar two-parameter form. The above limiting distribution remains applicable for other measures of fracture toughness, such as K_{Jc} or CTOD. A central feature emerging from this model is that, under SSY conditions, the scatter in cleavage fracture toughness data is characterized by $\alpha = 2$ for J_c-values or $\alpha = 4$ for K_{Jc}-values [5,9].

Current probabilistic models to extend the previous methodology to multiaxially stressed, 3-D crack configurations employ weakest link arguments to couple the micromechanical features of the fracture process (such as the inherent random nature of cleavage fracture) with the inhomogeneous character of the near-tip stress fields. The *Weibull stress* (σ_w), a term coined by the Beremin group [4], provides the basis for generalizing the concept of a probabilistic fracture parameter and supports the development of procedures (often termed local approaches) that unify toughness measures across different crack configurations/loading modes. These models adopt a two-parameter Weibull distribution for the fracture stress of a cracked solid in the form [1,4,9]

$$F(\sigma_w) = 1 - \exp\left[-\frac{1}{V_0}\int_V \left(\frac{\sigma_1}{\sigma_u}\right)^m dV \right] = 1 - \exp\left[-\left(\frac{\sigma_w}{\sigma_u}\right)^m \right], \quad (2)$$

where V denotes the volume of the (near-tip) fracture process zone, V_0 is a reference volume and $\sigma_1 = f(J)$ is the maximum principal stress acting on material points inside the fracture process zone defined by the loci $\sigma_1 \geq \lambda\sigma_0$, with $\lambda \approx 2$. Parameters m and σ_u appearing in Eq. (2) denote the Weibull modulus and the scale parameter of the Weibull distribution. Following Beremin [4], the Weibull stress is then defined as the stress integral

$$\sigma_w = \left[\frac{1}{V_0}\int_V {\sigma_1}^m dV \right]^{1/m} . \quad (3)$$

which describes local conditions leading to unstable (cleavage) failure.

CALIBRATION PROCEDURE FOR THE WEIBULL MODULUS

The proposed scheme to calibrate the Weibull modulus, m, uses a toughness scaling methodology (TSM) based upon σ_w [1] to correct measured toughness distributions for fracture specimens. Cleavage fracture toughness values (such as J_c-values) measured from one set of high constraint standard specimens (configuration **A**) define parameter J_0 of the statistical distribution given by Eq. (1) as the basis for calibration; this parameter is denoted J_0^A. Consider now a different high constraint crack configuration with different thickness (configuration **B**) at the same temperature and loading rate (here taken as quasi-static). Because parameter m is assumed independent of specimen geometry, the calibrated Weibull modulus is the m-value that *corrects* the characteristic toughness for configuration **A** to its equivalent characteristic toughness for configuration **B**, denoted J_{0-TSM}^B ($J_0^A \rightarrow J_0^B$ correction).

However, a key feature of the procedure adopted here is that no additional fracture testing is needed (other than testing of fracture specimens for configuration **A**). For cleavage fracture toughness under SSY conditions measured in the DBT region, the weakest link model (WLM) correctly describes the effects of thickness on toughness. By using a thickness correction based on weakest link statistics, parameter J_0^A is simply scaled to the characteristic toughness value for configuration **B** (with different thickness), denoted J_{0-WLM}^B, which is given by

$$J_{0-WLM}^{B} = J_{\min} + (B_A/B_B)^{(1/\alpha)}(J_0^A - J_{\min}) \qquad (4)$$

where B_A and B_B denote the specimen thickness for **A** and **B**. The thickness correction expressed by Eq. (4) thus provides the *second* toughness parameter required in applications of the TSM. Consequently, the calibrated *m*-value for the material is defined as the value at which $J_{0-TSM}^{B} \equiv J_{0-WLM}^{B}$. This condition enforces a rigorous correspondence between predictions based on the Weibull stress approach and the weakest link model for measured toughness values under SSY conditions.

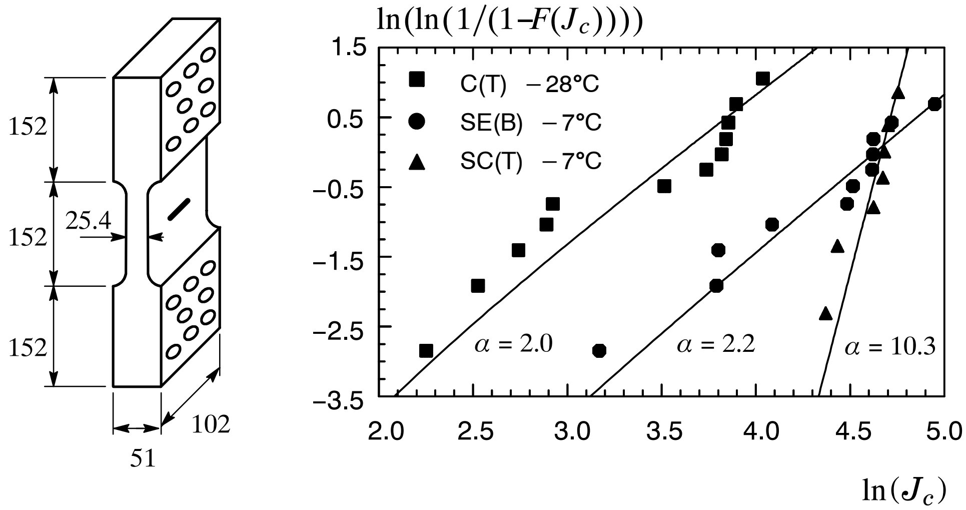
This approach also retains contact with the same scaling procedure applied on measured toughness distributions developed by Gao et al [2] and Ruggieri et al. [3] while, at the same time, using the WLM to provide the additional toughness data needed for calibration of *m*. Moreover, the calibration process maintains consistency in both the statistical procedures and finite element procedures for subsequent fracture assessments of structural components using the TSM based on the calibrated Weibull stress model. While there exist no strict requirements for a specific choice of the thickness ratio B_A/B_B as the basis for the scaling $J_0^A \rightarrow J_{0-WLM}^{B}$, the two crack configurations (**A** and **B**) must have substantially different σ_w *vs.* J histories computed for the same *m*-values to insure appropriate levels of J_0^B/J_0^A ratios.

CLEAVAGE FRACTURE PREDICTIONS

Specimen Material and Fracture Testing

Gao et al [7] recently reported a series of fracture toughness tests conducted by Joyce and Link [11] and Tregoning [12] on a C-Mn alloy pressure vessel steel. The fracture mechanics tests include: (1) a conventional, plane sided 1T C(T) specimen with $a/W=0.6$, $B=25$ mm and $W=50$ mm; (2) a conventional, plane sided SE(B) specimen with $a/W=0.2$, $B=25$ mm, $W=50$ mm and $S=4W$ and (3) a bolt-loaded SC(T) specimen with $a/t=0.25$, $c/a=3$ and $t=25$ mm. For the C(T) and SE(B) specimens, a is the crack length, W is the specimen width, B is the specimen thickness and S is the bend specimen span. For the SC(T) specimen, a is the maximum depth of the surface crack, $2c$ is the length of the semi-elliptical crack and t is the thickness of the cracked section. The material is an A515-70 pressure vessel steel (280 MPa yield stress at -7°C) with relatively high hardening properties ($\sigma_t/\sigma_{ys} \approx 2$). Testing of these configurations was performed at $T=-28$°C for the C(T) specimens and $T=-7$°C for the SC(T) specimens (DBT transition behavior for the material). Figure 1(a) shows the geometry for the bolt-loaded SC(T) specimen.

Figure 1(b) provides a Weibull diagram of the measured toughness values for both test temperatures. The solid symbols in the plots indicate the experimental fracture toughness data for the specimens. The straight lines indicate the three-parameter Weibull distribution, Eq. (1), obtained by a maximum likelihood analysis of the data set with $J_{\min} = 1.8$ KJ/m^2 ($K_{J-\min} = 20$ MPa$\sqrt{\text{m}}$).



(a) **(b)**

Figure 1 *(a) Geometry of bolt-loaded surface crack specimen; (b) Weibull plots of experimental toughness values at* $T = -28°C$ *and* $-7°C$ *[7].*

Calibration of Weibull Modulus for the A515 Steel

The procedure previously outlined is applied to calibrate the Weibull modulus for the tested steel using the measured toughness values for the C(T) specimens, taken here as configuration **A**. Because the fracture specimens were not tested at the same temperature, the J_0-value for the C(T) specimens at $-28°C$ is adjusted to the corresponding J_0-value at $-7°C$ using the Master Curve fitting given by ASTM E-1921 [14]. The characteristic toughness values for the C(T) specimens at $-7°C$ is then given as $J_0^{1T-C(T)} \approx 54$ KJ/m^2. This value is corrected to the characteristic toughness for a different thickness (denoted as configuration **B**) to define the required J_0^B / J_0^A -ratio. To examine the potential effect of this toughness ratio on cleavage fracture predictions for the tested material, the procedure considers both 4-T and 8-T deep notch SE(B) specimens with $a/W = 0.5$ to generate the required toughness for configuration **B**. The weakest link model (WLM) given by Eq. (4) with $a = 2$ and $J_{min} = 1.8$ KJ/m^2 yields $J_0^{4T-SE(B)} \approx 28$ KJ/m^2 and $J_0^{8T-SE(B)} \approx 19$ KJ/m^2.

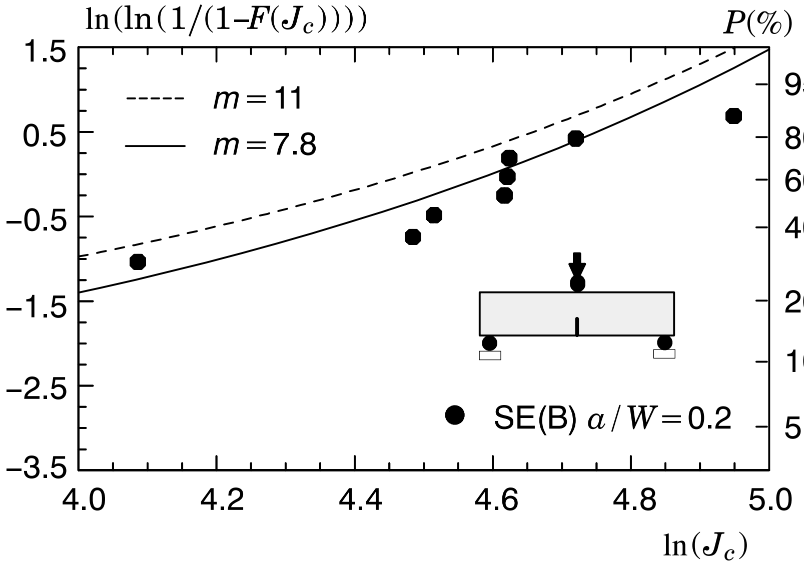
The process now proceeds by determining the m-value which *corrects* the characteristic toughness values for configurations **A** and **B** using the toughness scaling model (TSM) based upon σ_w. Nonlinear finite element analyses using the finite element code WARP3D [10] are performed on very detailed models for the 1T C(T), and 4T and 8(T) SE(B) fracture specimens to generate the Weibull stress trajectories needed to scale the J_0-values for the expected range of

m-values. The research code WSTRESS [6] is employed to calibrate parameter m using a function minimization based upon a golden section search algorithm. Ruggieri [15] provides further details on the computational models employed in the numerical analyses of the fracture specimens. For the tested material, the calibrated Weibull stress modulus is $m = 11$ for $J_0^{4T-SE(B)} / J_0^{1T-C(T)}$ and $m = 7.8$ for $J_0^{8T-SE(B)} / J_0^{1T-C(T)}$.

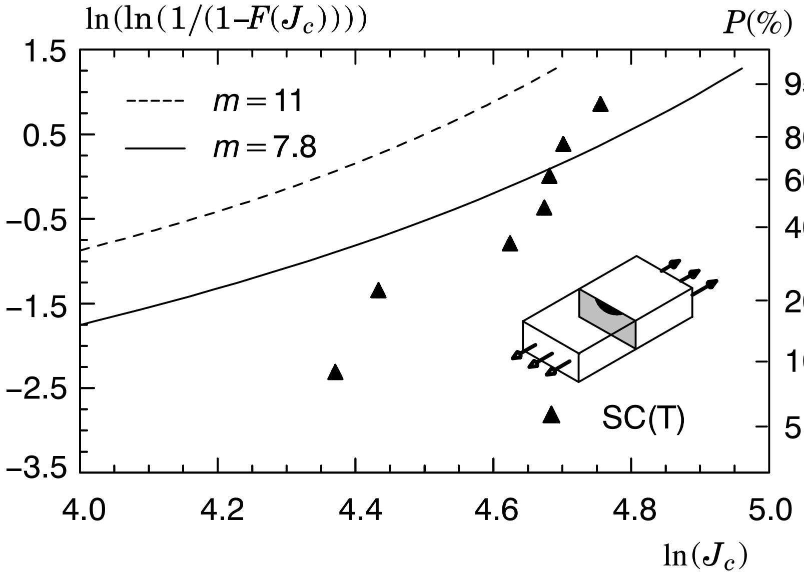
Fracture Predictions Using the Calibrated Weibull Stress Model

To verify the predictive capability of the Weibull stress methodology adopted in the present work, the toughness scaling model based upon the Weibull stress (σ_w) is employed to predict the toughness distribution for the shallow notch SE(B) specimen and the bolt-loaded SC(T) specimen. The Weibull probability plots in Fig. 2 show the predicted distributions of cleavage fracture toughness for both fracture specimens using the calibrated micromechanics model with $m = 7.8$ and 11. The solid symbols in the plots indicate the measured cleavage fracture toughness (J_c) for these specimens. The lines on each figure represents the *predicted* Weibull distribution generated from the statistical distribution (not individual values from tested specimens) of toughness values for the C(T) specimen with $a / W = 0.6$.

The Weibull stress predictions for the shallow notch SE(B) specimen using m=7.8 agrees well with the experimental data for almost the entire toughness range. The predicted distribution for this specimen using m=11 also agrees reasonably well with the experimental, albeit providing slightly conservative failure predictions (*i.e.*, for a given failure probability, the predicted J_c-value is lower than the actual data for the entire toughness range). In contrast, the Weibull stress predictions for the SC(T) specimen display much more sensitivity upon the calibrated m-value. The overall agreement between the predicted distribution for m=7.8 with the experimental data is reasonable, while the calibrated model using $m = 11$ provides numerical predictions which differ significantly from the experimental toughness values. However, such deviation from the experimentally measured distribution should not be pessimistically interpreted. Recall that the toughness distribution for the bolt-loaded SC(T) specimen differs significantly from the toughness distribution for the shallow notch SE(B) specimen even though both specimens have similar levels of characteristic toughness (note the Weibull slopes for the toughness distributions of these specimens displayed on Fig. 1(b)). The large deviation of the Weibull slope for the SC(T) specimen from the SSY value $\alpha = 2$ is most likely associated with the strong tensile field that develops ahead of crack front for this specimen. The tested bolt-loaded crack configuration has a symmetrical geometry (see Fig. 1(a)) which provides a predominantly tensile loading with only small bending moments acting on the crack plane [7]. Consequently, the near-tip stresses relax significantly from the SSY levels with rapid development of plasticity in the crack ligament thereby decreasing the amount of scatter in toughness val-



(a)



(b)

Figure 2 *Cleavage fracture predictions for the tested fracture specimens: (a) shallow notch SE(B) specimen; (b) bolt-loaded SC(T) specimen.*

ues (*i.e.*, increasing the Weibull modulus, α). Such features most likely impose a stronger sensitivity of the prediction process on the calibrated *m*-value.

CONCLUDING REMARKS

This paper describes a new procedure to calibrate the Weibull stress modulus, *m*, which employs SSY values for cleavage fracture toughness measured in the DBT region. The calibration scheme utilizes only one set of fracture toughness data while, at the same time, avoiding the non-uniqueness of calibrated Weibull stress parameters. A thickness correction based upon the weakest link model (WLM) provides the *second* toughness parameter required in the calibration scheme. The proposed methodology predicts the measured statistical distribution of cleavage fracture toughness (J_c) for different fracture specimens which provides a compelling support to the new calibration procedure.

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