Similarity Scaling in 3-D Fatigue Crack Closure

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ABSTRACT*: This work describes a 3-D finite element study of mode I fatigue crack growth in the small-scale yielding (SSY) regime with zero T-stress. Under remotely applied, constant amplitude cyclic loading with a ratio $K_{min}/K_{max} = 0$, closure along the crack front and behind the growing crack occurs when model nodes impinge on a frictionless, rigid plane. The material behavior follows a purely kinematic hardening, constitutive model with constant hardening modulus. Dimensional analysis suggests, and the computational results confirm, that the normalized remote opening load value, K_{op}/K_{max}, at each location along the crack front remains unchanged when the peak load (K_{max}), thickness (B) and material flow stress (σ_0) all vary to maintain a fixed value of $\bar{K} = K_{max}/\sigma_0\sqrt{B}$. Through parametric computations at various $\bar{K}$ levels, the results illustrate the effects of normalized peak loads on the through-thickness opening-closing behavior.*

INTRODUCTION

Plasticity induced closure often influences strongly the behavior of fatigue cracks at engineering scales in metallic materials [1]. Current predictive models generally adopt the effective stress-intensity factor ($\Delta K_{eff} = K_{max} - K_{op}$) in a Paris law type relationship to quantify crack growth rates. Here, K_{max} denotes the maximum stress intensity factor (K_I) in a load cycle computed as a through-thickness average value remote from the crack front, while K_{op} represents the value of K_I when the crack opens completely during the load cycle. The numerical studies on plasticity induced crack closure (see McClung [2] for a recent review) seek to predict the opening stress intensity factor K_{op}. However, the majority of these studies consider two-dimensional models; the few studies of three-dimensional effects describe case-by-case, geometry-specific analyses. The present work considers a large subset of 3-D configurations characterized by thin, metallic components (and test specimens) that contain an initially sharp, straight-through crack growing under constant amplitude, mode I fatigue loading. For such configurations, we demonstrate the existence of a *similarity scaling relationship* that couples in a unique way the opening stress intensity factor, K_{op}, to the peak load in each cycle, the specimen thickness and the material flow properties.

In the subset of cases examined, the characteristic in-plane dimensions (*e.g.* the crack length, the remaining ligament, the height etc.) exceed multiples of the specimen thickness B. Over the regime of moderate-to-high cycle fatigue loading, K_{max} remains a comparatively small fraction of the fracture toughness K_c. Consequently, the peak load values produce plastic zones comparable in size to B and much smaller than any distance to a nearby boundary, load application point and the crack length. These conditions, termed small-scale yielding (SSY), lead to a linear-elastic, plane-stress mode I field that encloses the crack front plastic zone (see Fig. 1). The effects of geometry, remote loading (tension *vs.* bending, etc.) and the displacement boundary conditions then impact the crack front material through K_I and the non-singular T-stress [3].

Under SSY conditions and with the material flow stress denoted σ_0, the size of the plastic zone ahead of the fatigue crack grows in proportion to $(K_I/\sigma_0)^2$. For 2-D idealizations of SSY, which have no geometric length-scale, this quantity defines the well-known similarity scaling parameter characterizing the near-tip fields. The 3-D SSY configuration introduces the thickness B into the analysis as a geometric length-scale. This leads naturally to the quantity $\overline{K} = K_{max}/\sigma_0\sqrt{B}$ as a potentially useful, non-dimensional scaling measure of the fatigue loading. This parameter essentially describes the plastic zone size at peak of the load cycle relative to the thickness (see also [4]).

In view of the above observation and anticipating that plastic zone size, thickness, material flow properties and R-ratio (K_{min}/K_{max}) interact in a dimensionally consistent manner to govern the crack opening behavior, we expect to find a relationship of the form

$$\frac{K_{op}}{K_{max}} = F\left(\frac{K_{max}}{\sigma_0\sqrt{B}}, \frac{z}{B}, \frac{\Delta a}{B}; R; \frac{\sigma_0}{E}, \frac{E_T}{E}, \nu\right) \tag{1}$$

where z denotes the distance to the crack front location measured from the centerplane ($z = 0$), E_T defines the constant hardening modulus for use in a kinematically hardening plasticity model and ν symbolizes Poisson's ratio. For simplicity in this initial work, the crack front remains straight during growth (no tunneling) and a single Δa value defines the amount of crack extension. The crack front closes gradually under reversed load beginning at the outside surface, and opens last at the outside surface upon re-loading. The K_{op}/K_{max} value thus varies strongly along the crack front. At extended amounts of growth (steady state) the opening loads remain constant and the dependence on $\Delta a/B$ drops out of Eq. 1.

The 3-D computations described here demonstrate the validity of the above relationship. In particular, the K_{op}/K_{max} *vs.* $\Delta a/B$ response curve at each location along the crack front remains unchanged when K_{max}, thickness (B) and material flow stress (σ_0) all vary to maintain a fixed value of $\overline{K} = K_{max}/\sigma_0\sqrt{B}$. Through computations at various $\overline{K}$ levels, the results illustrate the effects of normalized peak loads on the through-thickness opening-closing behavior.

FINITE ELEMENT PROCEDURES

The SSY model of thickness B consists of an edge crack and a large region of material enclosing the crack front (Fig. 1). The boundary of the domain has a radius, $\overline{R} = 100B$, such that the crack front plastic zone at maximum load remains well-confined within a linear-elastic (plane-stress) region and has negligible interaction with the boundary. The analyses consider various thicknesses B to demonstrate the validity of the proposed similarity scaling of B, σ_0 and K_{max} on K_{op}.

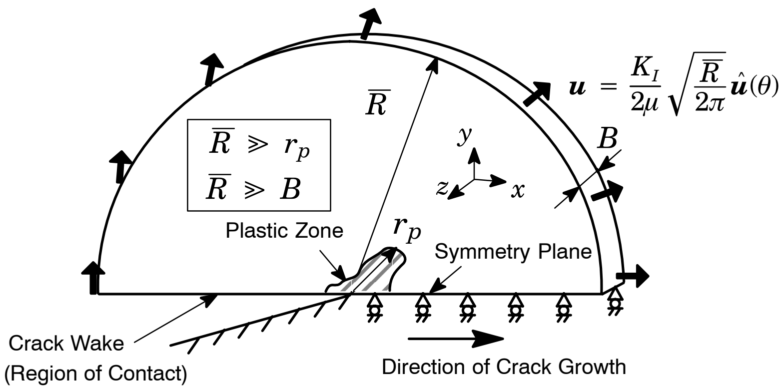

Figure 1: 3-D mode I small-scale yielding framework for modeling fatigue.

Two-fold symmetry of the 3-D mode I configuration allows modeling of only one-quarter of the domain. Preliminary analyses suggested 5 element layers over half thickness with dimensions of $0.25B$, $0.15B$, $0.05B$, $0.03B$ and $0.02B$ – the smallest layer located adjacent to the free surface ($z = 0.5B$). A series of small and identical elements of size L_e ahead of the crack front permits an equal amount of crack growth simultaneously across all layers. A quarter-symmetric model contain typically 11,000 nodes and 9,000 elements with 90 of the smallest elements defined in each layer ahead of the crack front to support growth by node release.

Loading of the model occurs through in-plane displacements $\boldsymbol{u}(u,v)$ imposed on the remote cylindrical boundary according to linear elastic plane-stress field with zero T-stress (Fig. 1). These displacements are imposed uniformly at each through-thickness node location.

A load cycle consists of increasing K_I from zero to a value $K_{\max}$ then decreasing back to zero, i.e. an R-ratio $= 0$. The crack propagates uniformly over the thickness by an amount $\Delta a = L_e$ in each cycle by releasing all (current) crack front nodes in the first unloading step after the peak load. The computational procedures enforce frictionless contact conditions over the symmetry plane ($y = 0$) behind the growing crack front. Variably sized increments specified over a loading cycle provide better resolution ($\Delta K = 0.02K_{max}$) for opening load detection.

The analyses use an incremental, kinematic hardening constitutive model to describe the cyclic, elastic-plastic response of the material. The various analyses described here adopt an elastic modulus $E = 250\sigma_0$ and $500\sigma_0$, where σ_0 denotes the initial yield stress of the material. The constant hardening modulus has an assigned value of $E_T = d\sigma/d\epsilon = E/20$.

The finite element computations reported here are performed with the fracture mechanics research code, WARP3D (Gullerud *et al.* [5]). WARP3D has a software architecture that supports parallel execution via explicit message passing (MPI) coupled with shared-memory when available. The global solution procedure uses an implicit, incremental-iterative strategy with Newton iterations to achieve equilibrium at each load increment.

RESULTS

The use of finite-sized load increments, fatigue crack growth in increments of the element size (L_e) and the breaking of contact behind the front in increments of L_e all combine to discretize an otherwise, smooth physical process. This work defines K_{op} at a crack front location when the second node behind the current crack tip loses contact with the symmetry plane. The node immediately behind the crack tip closes prematurely and exhibits an opening load much higher than the other nodes, especially near the centerplane (see also [6-8]).

3-D Similarity Scaling Parameter

Figure 2 shows the evolution of opening load K_{op}/K_{max} with non-dimensional crack growth $\Delta a/B$ at different locations along the crack front computed for models with two different thicknesses, $B_1 = \underline{B}$ and $B_2 = 2 \times \underline{B}$. Here, $R = K_{min}/K_{max} = 0$ in both cases. These two sets of results are generated with different values of the peak loads ($K^{(1)}_{max}$ and $K^{(2)}_{max}$) such that $K^{(1)}_{max}/\sigma_0\sqrt{B_1} = K^{(2)}_{max}/\sigma_0\sqrt{B_2} = 1$. Results for both cases are effectively identical. At the centerplane ($z/B = 0$), K_{op} first increases, reaches a value of about $0.08K_{max}$ and then decreases to a negligibly small value. Consequently, this curve suggests that at steady-state under SSY conditions, little or no closure occurs near the centerplane for a cyclic load with $K_{max}/\sigma_0\sqrt{B} = 1$ and $R=0$. The opening load at a plane located halfway between the centerplane and the free surface ($z/B = 0.25$) shows a similar trend of first increase and then decay. However, a non-zero steady-state value indicates existence of closure in this plane. Opening loads for the other planes increase sharply with crack growth and attain steady-state values at crack extensions $\Delta a \approx 0.3B$. At the free surface, the opening load reaches a steady-state magnitude of $0.46K_{max}$, highlighting the dramatic contrast with the closure behavior on the centerplane.

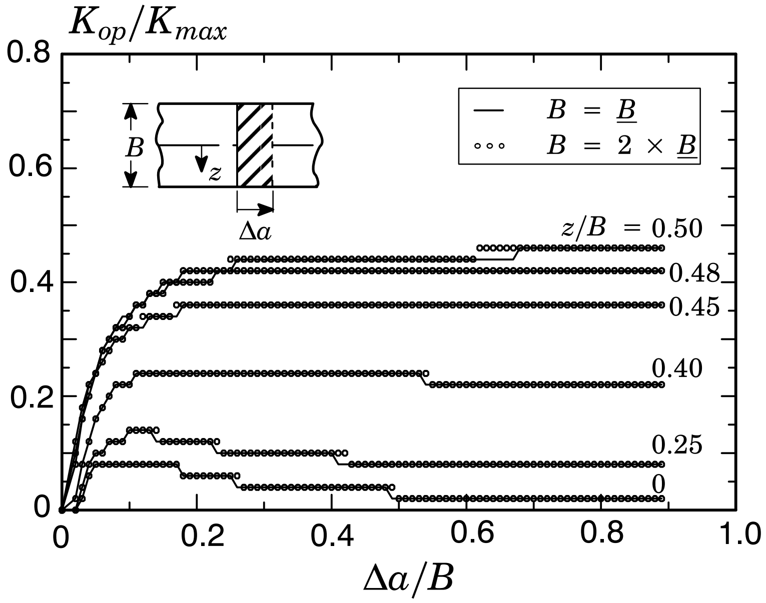

Figure 2: Demonstration of the similarity scaling of normalized opening load at each crack front location when specimens of different thickness are subject to same normalized load $K_{max}/\sigma_0\sqrt{B} = 1$.

The curves of K_{op}/K_{max} in Fig. 2 show differences in closure behavior at each crack front location for the two thickness of less than or equal to the resolution of load increments specified in the analyses (the numerical solution detects crack opening after solution for a load increment and not during the increment). Analyses with other thickness values also yield results in very close agreement with these curves, again to within the load increment resolution. These analyses establish that the variation of opening load K_{op}/K_{max} with crack growth $\Delta a/B$ scales with the load measure $K_I/\sigma_0\sqrt{B}$.

Effect of Normalized Peak Load

Figure 3 shows the evolution of opening stress-intensity factor across the crack front for $K_{max}/\sigma_0\sqrt{B} = 2$. As $K_{max}/\sigma_0\sqrt{B}$ increases from 1 to 2, the steady-state value of K_{op}/K_{max} at $z/B = 0$ increases from 0.02 (indicating little or no closure) to 0.28. The plane halfway to the free surface ($z/B = 0.25$) also shows an increase in opening load. In contrast, K_{op}/K_{max} values over the outer 5% of the thickness decrease with increased normalized load from a high of 0.46 to 0.40.

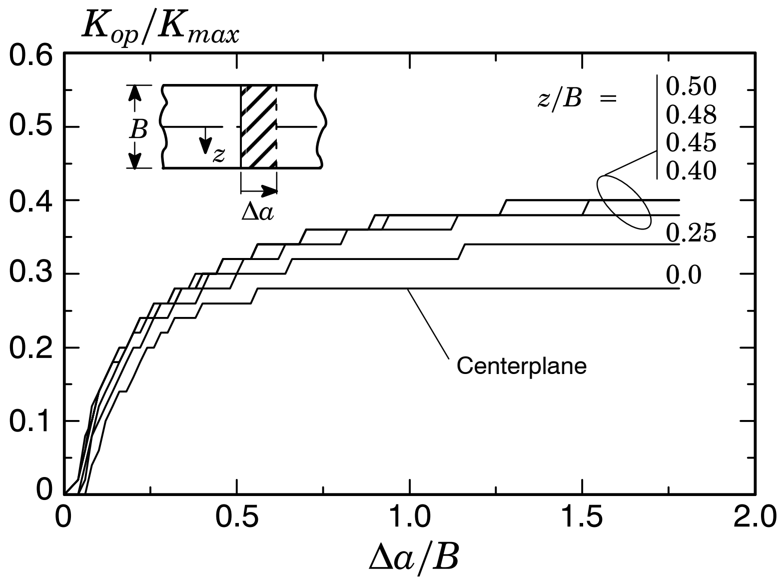

Figure 3: Evolution of normalized opening load for $K_{max}/\sigma_0\sqrt{B} = 2$.

An increase in $K_{max}/\sigma_0\sqrt{B}$ from 1 to 2 grows the plastic zone size relative to thickness, r_p^f/B, from 0.2 to 1.0. The results in Figs. 2 and 3 then indicate a decreasing, through-thickness variation in closure behavior as r_p^f/B grows larger. The increased K_{op}/K_{max} values at the centerplane coincide with growth of the mid-thickness plastic zone out of the near front region of strong, local 3-D effects and into the region of essentially plane-stress conditions.

CONCLUSIONS

This paper describes a 3-D numerical study of mode I fatigue crack growth in the small-scale yielding (SSY) regime with zero T-stress. The work presented here supports the following conclusions for constant amplitude, cyclic loading with $R = K_{min}/K_{max} = 0$:

(1) Under SSY conditions, the computational results demonstrate that the normalized value of the opening stress-intensity factor, K_{op}/K_{max} at each location along the front remains unchanged when the peak load (K_{max}), thickness (B) and material flow stress (σ_0) all vary to maintain a fixed value of $\overline{K} = K_{max}/\sigma_0\sqrt{B}$.
(2) The closure behavior shows the strongest 3-D effects the for lowest loading level considered, $\overline{K} = 1$, which causes a mid-thickness plastic zone size on the crack plane of $\approx 0.2 \times B$ at peak load. The mid-thickness region of the model shows little or no crack closure while the outside surfaces have K_{op}/K_{max} values characteristic of a plane-stress model. At the maximum loading level considered, $\overline{K} = 2$ (mid-thickness plastic zone size $\approx 1 \times B$), K_{op}/K_{max} values at the centerplane increase sharply while outside surface values remain nearly unchanged.

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