FRACTURE MODEL FOR AN INTERFACE CRACK WITH BRIDGED ZONE

M. N. Perelmuter

Institute for Problems in Mechanics of RAS
Pr. Vernadskogo 101-1, 117526, Moscow, Russia
e-mail: perelm@ipmnet.ru

Abstract: The application of two-parametric fracture criterion to the problem of cracks limit equilibrium in homogeneous media or at an interface of two different materials is considered. The necessary condition of the crack tip limit equilibrium is the equality of the energy release rate and the rate of the energy dissipation by the bonds (the first condition of fracture). The second condition of fracture is the condition of the bond limit stretching at the trailing edge of the bridge zone. Based on these two fracture conditions the regimes of the bridged zone and the crack tip equilibrium and growth are considered. The estimations of the equilibrium size of the bridged zone, the adhesion fracture energy and the critical stress (the interface strength) depend on the crack size are found.

PROBLEM FORMULATION

Let us consider a straight crack of length 2ℓ at an interface of two dissimilar elastic half-planes such that the crack is placed at $|x|\leq\ell$, y=0. Assume that the uniform tensile stresses, σ_o, are applied at infinity normal to the interface. Consider segments of length d (end zones) adjacent to the tips of the crack ($\ell-d\leq|x|\leq\ell$, y=0). In these zones the surfaces of the crack interact with each other, which suppresses the crack opening.

The physical nature of the crack surfaces interaction is generally changed in dependence on the crack scale and distance from the crack tip. The interatomic and intermolecular forces are limiting mechanisms of the surfaces interaction at the small distances from the crack tips while "mechanical" forces prevail at relatively larger distances. These mechanical forces can be caused, e.g., by reinforcing action of fibers in composites or polymer chains connecting the crack surfaces in polymer-polymer joints or polymer joints with other materials (metals, ceramics, etc.).

To mathematically describe the interaction between the surfaces of the crack, we assume that there exist bonds between the surfaces of the crack at the end zone. The law of deformation of these bonds, which is generally nonlinear, is given.

Under the action of external loads, the stresses *Q(x)* appear in the bonds between the surface of the interface crack at the boundary between different materials. These stresses have normal $q_y(x)$ and tangential $q_x(x)$ components

$$Q(x) = q_y(x) - iq_x(x),\, i^2 = -1 \tag{1}$$

The surfaces of the crack are acted on by the normal and tangential stresses are numerically equal to these components.

The opening of the interface crack, *u(x,0)=u(x)*, can be written as follows

$$u(x) = u_y(x) - iu_x(x),\ \ u_y(x) = u_y^+(x) - u_y^-(x),\ \ u_x(x) = u_x^+(x) - u_x^-(x) \tag{2}$$

where u_x, u_y are the projections of the crack opening on the coordinate axes *x* and *y*, respectively; u^+_x, u^+_y and u^-_x, u^-_y denote the components of the displacements of the upper and lower crack surfaces.

The relation between the crack opening and bond tractions (the bond deformation law) is given as follows [1-2]

$$u_i(x) = c_i(x, q_i)q_i(x)\,, \quad c_i(x, q_i) = g_i(x, q_i)\frac{H}{E_B} \quad , \tag{3}$$

where *i=x* or *i=y* for tangential and normal directions; the functions c_i can be considered as the effective bond compliances in the directions of the coordinate axes; γ_i are the dimensionless functions, *H* is a linear scale proportional to the bonding zone thickness; E_B is the effective Young modulus of the bonds.

The crack opening and bonds stresses along the crack end zone can be determined from solution of the singular integral-differential equations system [1-2]. Below is supposed that all these functions have already known.

TWO PARAMETRIC FRACTURE CRITERION

The potential energy of the body containing a crack with bridged zone can be written as follows (in the absence of body forces)

$$\Pi = \int_v w(e_{ij})dv - \int_{s_e} t_i u_i ds + \int_{s_i} f(u)ds\,, \tag{4}$$

where $w(\varepsilon_{ij})$ is the density of the deformation energy in the body volume v, ε_{ij} are the components of the strain tensor; t_i, u_i are the tractions and displacements at the body boundary s_e; $f(u)$ is the density of the strain energy of the bonds in the crack end zones, u is the crack opening in the end zones of area s_i.

The crack limit equilibrium corresponds to the following condition

$$-\frac{\partial \Pi}{\partial \ell} = -\frac{\partial}{\partial \ell}\left[\int_v w(\varepsilon_{ij})dv - \int_{s_e} f_i u_i ds\right] - \frac{\partial}{\partial \ell}\int_{s_i} f(u)ds = 0 \qquad (5)$$

The last term is the rate of the energy absorption in the crack end zone and is associated with the energy necessary to create a unit of its new surface. The remaining terms represent the energy release rate at creation of a new crack surface.

Note, that within the framework of the model the rate of the energy absorption depends on the end zone size and bond characteristics. The equilibrium end zone size is not assumed to be constant. It can be determined from condition (5) while searching for the critical load needs additional conditions of the bond rupture.

The energy release rate in case of an interface crack under the external load σ_0 and the stresses $-Q(x)$ applied to the crack surfaces in the bridged zone can be written as follows [3]

$$G_{tip}(d,\ell) = \left(\frac{\kappa_1+1}{\mu_1} + \frac{\kappa_2+1}{\mu_2}\right)\frac{K_B^2}{16\cosh^2(\pi\beta)}, \quad \alpha = \frac{\mu_2\kappa_1+\mu_1}{\mu_1\kappa_2+\mu_2}; \; \beta = \frac{\ell n\alpha}{2\pi}, \qquad (6)$$

where $K_B = \sqrt{K_{\mathrm{I}}^2 + K_{\mathrm{II}}^2}$ and $K_{I,II} = K_{I,II}^{\infty} - K_{I,II}^{Q}$ ($K_{I,II}^{\infty}$ and $K_{I,II}^{Q}$ are the stress intensity factors due to the external load and the stresses in the crack bridged zone). The stress intensity factors $K_{I,II}$ are determined by [1]

$$K_{\mathrm{I}} + iK_{\mathrm{II}} = \frac{\sigma_0\sqrt{\pi\ell}}{(2\ell)^{i\beta}}\left[\left(1 - \frac{2\cosh(\pi\beta)}{\pi}\int_{1-d/\ell}^{1}\frac{p_y(t)}{\sqrt{1-t^2}}dt\right) + i\left(2\beta - \frac{2\cosh(\pi\beta)}{\pi}\int_{1-d/\ell}^{1}\frac{t\,p_x(t)}{\sqrt{1-t^2}}dt\right)\right], \quad p_y(x) - ip_x(x) = Q(x)\left(\frac{\ell-x}{\ell+x}\right)^{i\beta} \qquad (7)$$

Let us calculate the rate of the energy absorption for an interface crack with bonding. Denote by $U_{bond}(d,\ell)$ the work of bond deformation and by $G_{bond}(d,\ell)$ the rate of the energy absorption per unit thickness of the body. Then

$$U_{bond}(d,\ell) = b \int_{\ell-d}^{\ell} f(u)dx\,, \quad G_{bond}(d,\ell) = -\frac{\partial U_{bond}(d,\ell)}{b\partial\ell} \tag{8}$$

where b is the body thickness.

Note, that differentiation in formula (8) is performed with respect to the upper limit of the integral. Hence, it is assumed that the crack advance is accompanied by the crack end zone increasing such that the trailing edge of the end zone remains unchanged and placed at $x=\ell-d$.

The density of the strain energy of the bonds is equal to

$$f(u) = \int_{o}^{u_y(x)} q_y(u_y)du_y + \int_{o}^{u_x(x)} q_x(u_x)du_x\,, \tag{9}$$

Substituting expression (9) in formula (8) and taking into account that $u_y(\ell)=u_x(\ell)=0$ at the crack tip we obtain

$$G_{bond}(d,\ell) = \int_{\ell-d}^{\ell} \frac{\partial u_y(x)}{\partial\ell} q_y(u_y)dx + \int_{\ell-d}^{\ell} \frac{\partial u_x(x)}{\partial\ell} q_x(u_x)dx \tag{10}$$

Taking into account formulae (6) and (10) the condition of the crack tip limit equilibrium (5) can be rewritten as follows

$$G_{tip}(d,\ell) = G_{bond}(d,\ell). \tag{11}$$

Condition (11) is necessary but insufficient for searching for a limit equilibrium state of the crack tip and the end zone. This condition enables us to determine the end zone size, d_{cr}, such that the crack tip is in an equilibrium at the given level of the external loads.

To search for the limit state of both the crack tip and end zone within the framework of the model one should introduce an additional condition, e.g., the condition of bond limit stretching at the trailing edge of the end zone $x_o=\ell-d_{cr}$

$$u(x_o) = ([u_x(x_o)]^2 + [u_y(x_o)]^2)^{1/2} = \mathbf{d}_{cr} \tag{12}$$

where $\mathbf{d}_{cr}$ is the bond rupture length.

If

$$G_{tip}(d,\ell) \geq G_{bond}(d,\ell) \quad (13)$$

at a certain end zone size, *d*, and

$$u(\ell\,-d) < \boldsymbol{d}_{cr} \quad (14)$$

then the crack length increases with the end zone growth up to the size d_{cr} without bond rupture. This stage of the crack growth can be treated as the system shakedown to the given level of the external loads.

The crack tip advance with simultaneous bond rupture at the trailing edge of the end zone occurs if both conditions

$$u(\ell - d) \geq \boldsymbol{d}_{cr} \quad (15)$$

and (13) are fulfilled.

The regime of bond rupture at the trailing edge of the end zone without the crack tip advance is observed then conditions

$$G_{tip}(d,\ell) < G_{bond}(d,\ell) \quad (16)$$

and (15) are fulfilled. In this case the size of the end zone decreases and tends to the limit value d_{cr} at the given load.

The end zone size and crack length are reserved within the framework of the model if the inequalities (14) and (16) hold.

Thus, the bond rupture characteristics and load level determine the fracture regimes: the crack tip advance with the end zone growth; end zone shortening without the crack tip advance; the crack tip advance and bond rupture at the trailing edge of the end zone.

Autonomy of the end zone [4] appears if the rates of the crack tip advance and bond rupture at the trailing edge of the end zone according to conditions (11) and (12) coincide.

COMPUTATION RESULTS

Solving jointly Eqs. (11-12) we can determine the critical external loads σ_o, the end zone size d_{cr} and the adhesion fracture resistance at the crack limit equilibrium state for given crack length and bond characteristics. The following material parameters are used in the numerical analysis:

$E_1=2\mu_1(1+\nu_1)$=140 GPa, ν=0.278 (silicon), $E_2=2\mu_2(1+\nu_2)$=1GPa, ν_2=0.35 (polymer), $\gamma_{1,2}(s)$=1. The parameters of the bridged model were chosen as follow (see detail in [5]): the intermediate layer size is $H = a\sqrt{N}$ =10^{-6}m, where a=10^{-8}m is the length of a statistical segment of the polymer, N=10000 is the polymerization index, the bond rupture length (critical crack opening) is $\delta_{cr} \approx 5*10^{-6}$m. To evaluate the bond compliance $c=c_y=c_x$ we suppose that the stiffness of each polymer molecules the same for normal and shear loading and one is $K_s = F_{cr}/U_c$, where $F_{cr}=10^{-9}$N is the critical force for bond breaking and $U_c = aN$ is the total length of the free polymer chain after tension (in supposing of a small strain). For temperature *T=323K* and the prescribed values of the polymer characteristics we obtain $K_s \approx 10^{-5}\, N/m$. Supposing that the density of the interface bonds is N_o=0.2*10^{18} m^{-2} we can get the effective stiffness of the interface bond as $K = K_s N_o = 2*10^{12}\, N/m^3$ and the bond compliance $c = 1/K = 0.5*10^{-12}\, m^3/N$. Then, we can compute the effective module of the bonds as $E_b = H/c = 2\,MPa$. The results of the computation for the initial crack size $2L_0$=1 mm are presented. The distribution of the relative critical stress σ_0/σ_{cr} (where $\sigma_{cr} = E_B \delta_{cr}/H$) versus the relative crack size L/L_0 is given in Figure 1. The fracture stress monotonically decreases with the crack size increasing. It should be noted that in the contrary to Griffith theory we have the finite fracture stress for a plate without crack. The distribution of the adhesion fracture resistance (the critical energy release rate, see Eq. 11) $G_{bond}(d_{cr},\ell)/G^{\infty}_{bond}$ (where $G^{\infty}_{bond} = 0.5 u_{cr}\sigma_{cr}$) versus the relative crack size is given in Figure 2. The noticeable changing of the adhesion fracture resistance observes only for the relative short cracks ($L/L_0 < 4$). For the long cracks this value approaches to the steady-state quantity G^{∞}_{bond}. For chosen material parameters we can obtain $G^{\infty}_{bond} = 25$ J/m^2. The relative end zone size ($10^{-3} d_{cr}/H$) at the crack limit equilibrium vs. the relative crack size is given on Figure. 3. For the short cracks (or other word - for the cracks with relative soft bonds) this value changes rapidly but for the long cracks one has gradual changing and for large cracks this value limits to constant quantity as in homogeneous media [6]. This is the confirmation of the end zone autonomy hypothesis [4].

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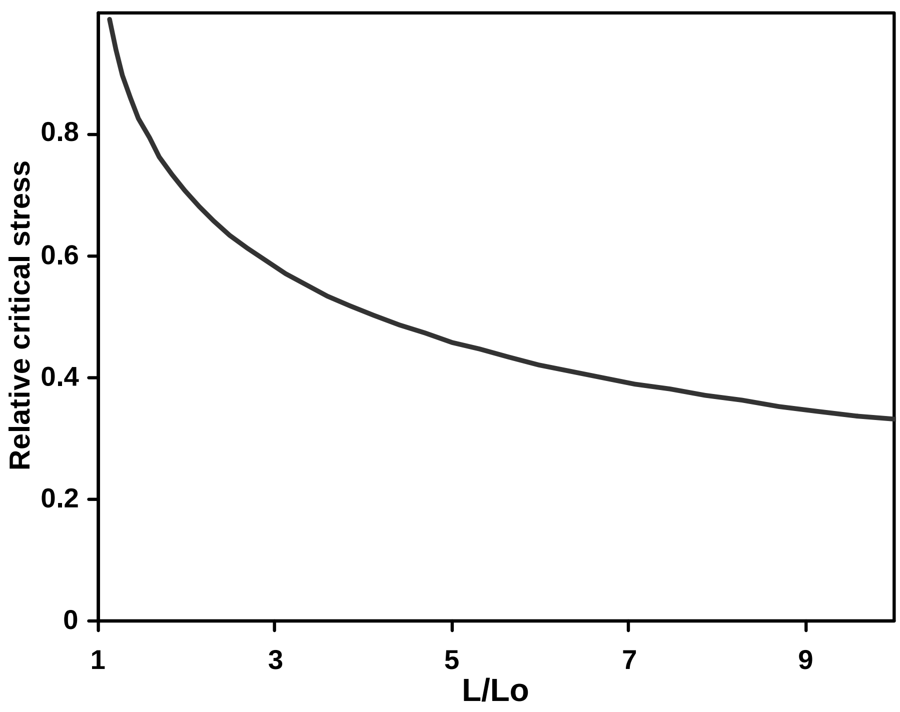

Figure 1: Fracture stress vs. the relative crack size

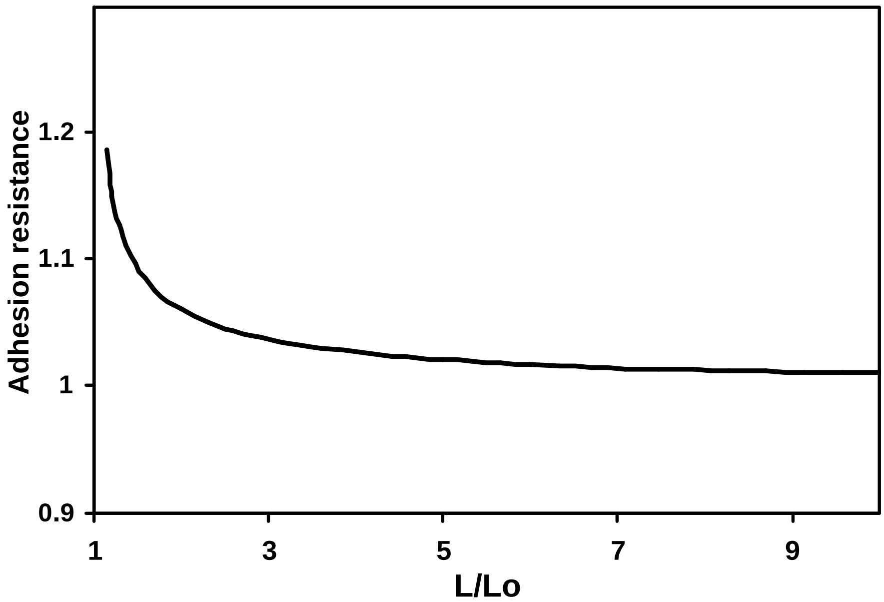

Figure 2: Adhesion fracture resistance vs. the relative crack size

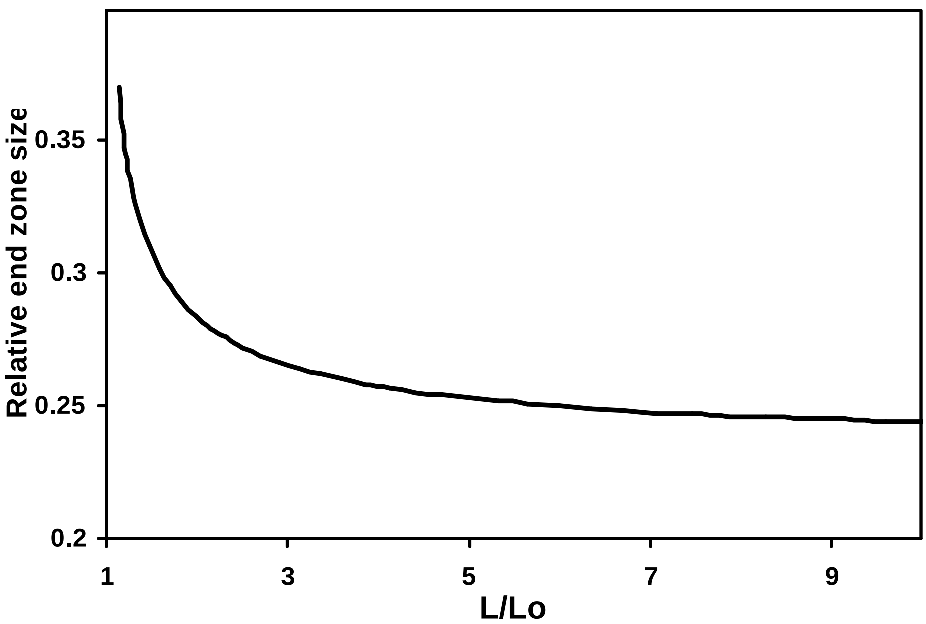

Figure 3: Relative end zone size vs. the relative crack size