Assessment of the fatigue failure period of a notched component

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ABSTRACT: The total period, N_f to failure is considered, which includes the periods of fatigue macrocrack initiation, N_i , and propagation, N_p , i.e.: $N_f = N_i + N_p$. Fatigue fracture of materials has been modelled as a process of initiation of a macrocrack of length $a_i = d^*$ (the magnitude of d^* is a material constant), which is successively repeated (step-by-step) during its growth. As a result the "local stress range, $\Delta \sigma_y^*$, versus macrocrack initiation period, N_i " relationship, which was established for notched specimens, might be applied to the determination of the "macrocrack growth rate, da/dN, versus effective stress intensity factor range, ΔK_{eff} " relationship and vice versa.

INTRODUCTION

For life time estimation of a structural component with an initial long crack, when at cyclic loading crack of length $a \ge a_i$ grows to the critical length $a = a_c$, macrocrack growth rates are studied only, see Figure 1(*a*).



Figure 1: Material endurance characteristics at various stages of fatigue.

This process is described by the relationship da/dN versus ΔK in form of curve *l* or curve *l'* taking into account the crack closure effect. On the basis of these curves the threshold characteristics ΔK_{th} , ΔK_{th} eff and the period

$$N_p = \int_{a_i}^{c} da / f(\Delta K)$$
 are determined, where $a_c = f(\Delta K_{fc})$. Parameter ΔK_{fc} is

the cyclic fracture toughness, which may be equal, less or larger than the static fracture toughness K_{lc} . The macrocrack period starts when the initial crack reaches the plastic (quasi-elastic) strained portion of the body and can henceforth be described by LEFM conditions. However, when the initial crack is small, $a < a_i$, the growth of microstructurally short and physically small cracks takes place [1]. These events are described by means of curves 2, see Figure 1(a). At $a \ge a_i$ only, when the microcrack transforms to macrocrack behaviour [2], its propagation law is illustrated by curves 1 and 1'. It was shown [2] that the prefracture (process) zone size d^* regulates this transition. The parameters for describing this process are not yet well established, therefore the estimation of the macrocrack initiation period, N_i , which includes the stages of microcrack growth and its transition to the macrocrack of length $a_i = d^*$ was proposed [2]. This involves a twoparameter process and, in the stress approach, it is determined by the local stress range, $\Delta \sigma_y^*$, and the linear structural parameter, d^* , which is a material constant for a given test condition [2]. The relationships $\Delta \sigma_{v}^{*}$ versus N_i and d^* versus N_i , shown in Figure 1 (b) by curves 1 and 2 respectively, establish the threshold magnitude of the local stress range, $(\Delta \sigma_v^*)_{th}$, when, at a stress concentrator, the initiation of the fatigue macrocrack of length $a_i \ge d^*$ is not realized [2]. The relationships $(\Delta \sigma_v^* \text{ v.s.})$ N_i) have such analytical representation [2]:

$$\Delta \sigma_y^* N_i^m = C$$

(1)

Thus, the total period, N_f , to fatigue failure of a notched component may be estimated as

$$N_f = N_i + N_p = \left(\frac{C}{\Delta \sigma_y^*}\right)^{l_m} + \int_{d^*}^{a_c} \frac{da}{f(\Delta K)}.$$
 (2)

So, it is suggested at traditional approach that the processes of fatigue macrocrack initiation and propagation are different, therefore they have been considered separately. However, from our point of view, one essential difference exists: at the macrocrack growth stage the crack closure effect appears, but at the initiation stage ($a_i < d^*$) it is absent [2]. Nevertheless the stages of the fatigue failure of materials, i.e. macrocrack initiation and propagation stages, might be considered as a similar process, hence for their description a unified approach may be applied.

UNIFIED APPROACH FOR ASSESSMENT OF THE PERIOD N_i

A macrocrack is modelled [3] as a sharp notch of root radius ρ , see Figure 2, noting that an analogous consideration has been used earlier. The experimental data [3,4] revealed that for sufficiently small magnitudes of ρ the number of cycles to macrocrack initiation does not depend on the notch root radius, when ρ is less than the certain value, a factor conditioned by the material properties. The following assumption was advanced [3] that such a state takes place when $\rho \leq d^*$, where d^* is the characteristic distance of the prefracture (process) zone, which is independent of ρ [2]. In this case, a macrocrack might be considered as a notch of effective radius $\rho_{eff} = d^*$ and the stress distribution at its tip is suggested to be the same as for the concentrator of radius $\rho = d^*$, see Figure 2(*a*).



Figure 2: Stress distribution near the macrocrack tip (*a*) and a scheme (*b*) of the macrocrack growth.

According to the proposed model [3], during cyclic loading in the vicinity of the existing macrocrack tip the prefracture (process) zone is formed, within which the macrocrack increment of length $\Delta a = d^*$ takes place. It is assumed that the increment formation corresponds to that near the geometrical ($\rho >> d^*$) stress concentrator [2] for the case when a local stress (or strain) range in the process zone is the same both at the notch and macrocrack tips. This process is continuously repeated and the crack for a certain number of cycles N_i propagates a distance d^* step-by-step, see Figure 2(*b*), where N_i is the period to macrocrack initiation of length $a_i = d^*$ at the notch tip [2]. For the given stress-strain state at macrocrack tip, its increment Δa takes place during $\Delta N = N_i$ loading cycles, and its extension is equal to the characteristic distance d^* , i. e. the crack growth rate, da/dN, can be determined by the equation

$$da / dN = \Delta a / \Delta N = d^* / N_i, \qquad (3)$$

where N_i is the initiation period of macrocrack of length $a_i = d^*$, which is assessed on the specimens with geometrical stress concentrators [2]. Thus, the relationships $\Delta \sigma_y^*$ versus N_i and d^* versus N_i established at the macrocrack initiation stage on the notched specimens due to the application of the stress approach [2], determine simultaneously a macrocrack propagation law.



Figure 3: A scheme for the calculation of the fatigue macrocrack propagation curve.

Consequently, it is possible by means of a sufficiently simple calculation to plot the fatigue macrocrack growth rates in case of equality of the local stress range, $\Delta \sigma_y^*$, within the process zone d^* for both notch and macrocrack tip, see Figure 3. It is supposed that for each pair of points A_i and A'_i (i = 1, 2, 3 ... n) on the macrocrack initiation resistance curves, see lines 1 and 2 in Figure 3(*a*), the correspondent point B_i can be obtained on the crack propagation curve, see line 3 in Figure 3(*b*). Let us consider this procedure in detail.

The principal condition that has to be used as a basis for the proposed calculation scheme is the equality of the local stress range, $\Delta \sigma_y^*$, in the vicinity of both notch and macrocrack tip. Since the stress intensity range, ΔK , is the parameter of the macrocrack propagation stage, see Figure 3(*b*), it is necessary to establish the relationship between $\Delta \sigma_y^*$ and ΔK range near the macrocrack tip. For this purpose the well-known approximate formula can be used

$$\Delta \sigma_{v}(0) = 2\Delta K / \sqrt{\pi \rho} = 1.128 \Delta K / \sqrt{\rho}, \qquad (4)$$

where $\Delta \sigma_y(0)$ is the maximum stress range at the tip of the sharp notch of radius ρ ; ΔK is the stress intensity factor range for a crack of equivalent length. The effective radius of the macrocrack tip, as was mentioned above, see Figure 2(*b*), is given by $\rho_{eff} = d^*$ and the maximum stress range equal $\Delta \sigma_y^*$. Taking into account such a consideration and Eq. 4 then the following formulation can be used for $\Delta \sigma_y^*$ (without closure near the crack tip)

$$\Delta \sigma_y^* = 1.128 \Delta K_{eff} / \sqrt{d^*}, \qquad \Delta K_{eff} = 0.886 \Delta \sigma_y^* \sqrt{d^*}.$$
(5)

Now, after the values of $\Delta \sigma_y^*$, N_i , d^* (co-ordinate of points A_1 and A'_1 in Figure 3(*a*)) were established and using Eqs 3 and 5 the corresponding values of da/dN and ΔK_{eff} (co-ordinate of point B_1 in Figure 3(*b*)) can be calculated. Carrying out the similar calculations for points A_2 , A_3 ... A_n , the

co-ordinate of points B_2 , B_3 ... B_n can be established, i. e. the crack growth rate curve da/dN versus ΔK_{eff} can be determined from this procedure.

When testing the mild 08kp steel, it was revealed that the curve (the solid line in Figure 4), calculated via the scheme in Figure 3 using the data established at the macrocrack initiation stage at stress ratio R = 0.1, coincides with the experimental curve da/dN versus ΔK_{eff} , established at R = 0.1 within the low amplitude and threshold regions (see symbols \bullet in Figure 4). It coincides as well with the crack growth rates da/dN versus ΔK established from tests at R = 0.7 (see symbols Δ). Obviously, this coincidence can be explained by the absence of the crack closure effect at the macrocrack initiation stage, contrary to the propagation stage [2], when it disappears in the threshold region only at stress ratio $0.5 \leq R \leq 0.7$.



Figure 4: Prediction (solid curve) and experimental values (symbols) of the fatigue macrocrack growth rate in mild 08kp steel.

The above described correlation between the macrocrack initiation and propagation stages, see Figure 3 and 4, allows one to perform a reverse calculation scheme: assessment of the period N_i to fatigue macrocrack initiation near the stress concentrator using the da/dN versus ΔK_{eff} macrocrack propagation curve. Taking into account the specimen geometry, see Figure 5, for the given load range ΔP the nominal stress range $\Delta \sigma_N = f(\Delta P, W, t)$ is calculated, where W and t is the specimen width and thickness, respectively. Then, the local stress range $\Delta \sigma_y^*$ near the tip of concentrator can be determined for the given value of ρ

$$\Delta \sigma_{y}^{*} = K_{f} \cdot \Delta \sigma_{N} = \left(1 + \frac{K_{t} - 1}{1 + \sqrt{d^{*} / \rho_{eff}}}\right) \Delta \sigma_{N}, \qquad (6)$$

where K_f – fatigue stress concentration factor estimated by means of K_t , d^* and ρ_{eff} values [4]. Hence, by Eq. 5 the value of ΔK_{eff} is calculated, which makes it possible to estimate the corresponding value of da/dN from the experimental curve da/dN versus ΔK_{eff} , which analytically can be represented for given material by equation

$$da/dN = B(\Delta K_{eff} - \Delta K_{th \ eff})^n.$$
⁽⁷⁾

Then from Eq. 3 we have

$$N_i = d^* / (da / dN),$$
 (8)

thus the number of cycles N_i to initiation of a macrocrack of length $a_i = d^*$ can be assessed.



Figure 5: Prediction of the number of cycles N_i to fatigue macrocrack initiation (length $a_i = d^*$) at the given load range ΔP (curves *1*, *2*) and the corresponding experimental data (symbols) for a compact disk specimen of $\rho = 0.75$ mm (curve *1* and symbol

O); $\rho = 4.0 \text{ mm}$ (curve 2 and symbol \bullet) of aluminium 1420T1 alloy

It is shown, see Figure 5, that the results of calculation and experimental data are in good agreement.

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