Fracture Model of Bonded Joints : Initiation and Propagation

A. Münch[1, 2] and Y. Ousset[2]

[1]Laboratoire de Modélisation Mécanique, Université Paris VI, Paris, France.
[2]Office National d'Etudes et de Recherches Aérospatiales, Châtillon, France.

ABSTRACT : *Let us consider an assembly made of two adherends glued together by a thin adhesive layer of thickness* εh. *Under the assumption that the ratio of the Young's modulus of the adhesive over the Young's modulus of the adherends is of order* ε, *a simplified model of the joint behaviour is obtained by the asymptotic expansion technique. Then, the debonding is studied within the framework of fracture mechanics. If the growth is stable, for a given value of the loading, the crack front at arrest is sought as the minimum of the total energy of the assembly. This minimum is characterised by a non linear equation which is solved by the Newton's method. To this end, the first and the second derivatives of the mechanical energy with respect to a front displacement are derived. Finally, a numerical example is presented highlighting the unstable character of the crack growth at initiation .*

INTRODUCTION

There are three main techniques to put structural elements together : the riveting, the welding and the gluing. This last technique of marginal use creates renewed interest in the repair of damaged structures made of composite materials. At the design level, arises the problem of the gluing performance.

The adhesively bonded assemblies are characterised by both the small thickness of the adhesive layer and the small value of the adhesive Young's modulus compared to the adherends ones. Using these features, a simplified model of the joint behaviour was first developed in the case of linear elasticity in small displacements by the asymptotic expansion technique [1, 2]. To our knowledge, the only one model valid for large displacements is the one of Edlund and Klarbring [3] obtained making a priori hypothesis. We have developed a non linear model using the asymptotic expansion technique [4] which differs from the Edlund and Klarbring model as it does not involve the strain of the adhesive mid-surface. This model is recalled in the next paragraph.

Here, the assembly debonding is studied within the framework of fracture mechanics. There are few works in this field, most of them reduce to the energy release rate computation. Here, a crack growth algorithm previously developed to study delamination growth in layered composite materials [5] is applied. It consists in seeking, for a fixed level of loading and a stable growth, the minimum of the total energy of the assembly (the sum of the mechanical or potential energy and the Griffith's fracture energy) with respect to the crack front location. To this end, the first and second derivatives of the mechanical energy with respect to the front displacement must be computed. Their expressions are given in the next paragraph. Finally, a numerical example is presented and discussed.

DEBONDING MODEL

Problem statement

Let two adherends $\Omega^{+/-}$ be glued by a thin adhesive layer Ω_ε^m of thickness εh. The assembly is clamped on the part Γ_u of its boundary and loaded on the part Γ_t (see figure 1). The equilibrium path is given by the pairs (u, λ) satisfying the following equilibrium equations :

$$\begin{cases} \displaystyle\int_{\Omega^{+/-}} Tr\left(S:\sigma^\varepsilon\cdot\tau\right)+\int_{\Omega_\varepsilon^m} Tr\left(S^m:\sigma^\varepsilon\cdot\tau\right) \\ \qquad\qquad -\displaystyle\int_{\Omega^{+/-}} Tr\left(\tau\cdot\gamma\left(u^\varepsilon\right)\right)-\int_{\Omega_\varepsilon^m} Tr\left(\tau\cdot\gamma\left(u^\varepsilon\right)\right)=0 \qquad \forall\tau \\ \displaystyle\int_{\Omega^{+/-}} Tr\left(\sigma^\varepsilon\cdot\overline{F^\varepsilon}\cdot\nabla v\right)+\int_{\Omega_\varepsilon^m} Tr\left(\sigma^\varepsilon\cdot\overline{F^\varepsilon}\cdot\nabla v\right)=\lambda\int_{\Gamma_t} t^\varepsilon\cdot v \quad \forall v \end{cases} \tag{1}$$

where S is the compliance tensor, σ is the Piola-Kirchhoff stress tensor, γ is the Green-Lagrange strain tensor, $F = I + \nabla u$ is the gradient matrix, $\overline{F}$ is the transpose of F, λ is the loading factor and Tr is the trace operator. For the sake of simplicity, isotropic materials are considered in the theoretical developments.

Asymptotic model of the joint behaviour

It is assumed that the Young's modulus satisfy the relations :

$$\frac{E^-}{E^+} = O(1) \quad ; \quad \frac{E^m}{E^+} = O(\varepsilon) \tag{2}$$

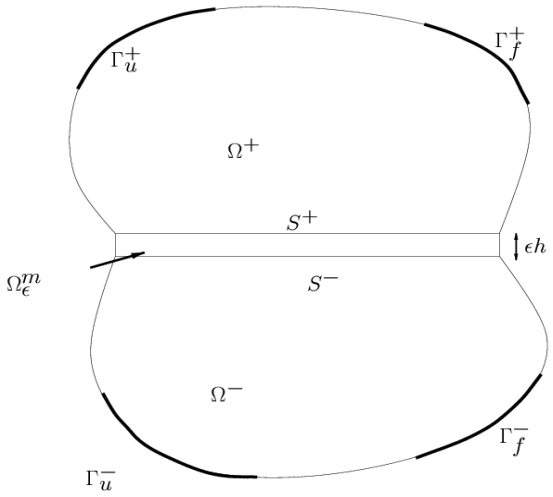

Figure 1 : Geometrical notations.

Making first the variable change $x_3 = x_3^\varepsilon / \varepsilon$ in the adhesive and the scaling $t^\varepsilon = \varepsilon t$ of the applied load, then developing the solution as a power series expansion of ε and identifying the terms of the same order , it is found that the leading term (u,σ) of the expansion satisfies the following relations in the adhesive layer :

(i) The displacement field is linear in x_3 :

$$u = \overline{u}(x_1, x_2) + x_3 < u > (x_1, x_2) \tag{3}$$

where :

$$\overline{u} = \frac{u^+ + u^-}{2} \quad ; \quad < u > = u^+ - u^- \quad ; \quad u^{+/-} = u\Big|_{S^{+/-}}$$

(ii) The stress components are given by :

$$\begin{aligned} \sigma_{ii} &= R^m_{ii33}\left(< u_3 > + \frac{1}{2} < u_k >^2 \right) \\ \sigma_{12} &= 0 \quad ; \quad \sigma_{\alpha 3} = R^m_{\alpha 3 \alpha 3} < u_\alpha > \end{aligned} \tag{4}$$

where R^m is the material stiffness tensor (the Latin indices take the values 1, 2, 3 whereas the Greek indices take the values 1 and 2).
(iii) Integrating with respect to x_3, the adhesive layer reduces to its mid-surface Σ. After returning to the initial variables, the following expression of the adhesive strain energy is obtained :

$$W^m(u,u) = \frac{1}{2\varepsilon h}\int_\Sigma \left[R^m_{\alpha 3 \alpha 3} <u_\alpha>^2 + R^m_{3333}\left(<u_3> + \frac{1}{2\varepsilon h} <u_k>^2 \right)^2 \right] \quad (5)$$

Debonding model
The debonding algorithm is the one previously described to simulate delamination growth in composite structural elements [5]. It consists in minimising the total energy E of the assembly with respect to the crack front position. E is the sum of the mechanical energy J and the Griffith's fracture energy D. This minimum is computed using the Newton's method. To this end, the first and second derivatives of the energies must be computed. The derivatives of D are given in [5]. For the sake of brevity, only the parts of the J derivatives related to the adhesive are derived here. The other terms can be found in [6].

Energy release rate
Any front displacement can be depicted by a plane vector field $\theta = (\theta_1, \theta_2, 0)$ defined in a small neighbourhood of the front γ_f. The energy release rate associated to the front displacement θ has the following expression :

$$g(\theta) = g^{+/-}(\theta) - \frac{1}{2}\int_\Sigma \left[\sigma_{\alpha 3} <u_\alpha> + \sigma_{33}\left(<u_3> + \frac{1}{2\varepsilon h} <u_k>^2 \right) \right] \nabla \cdot \theta \quad (6)$$

which can be written as a curvilinear integral along γ_f, giving the following expression of the crack force G (local energy release rate) :

$$\begin{aligned} g(\theta) &= \int_{\gamma_f} G\theta \cdot \nu \\ &= \frac{1}{2\varepsilon h}\int_{\gamma_f} \left[R^m_{\alpha 3 \alpha 3} <u_\alpha>^2 + R^m_{3333}\left(<u_3> + \frac{1}{2\varepsilon h} <u_k>^2 \right)^2 \right] \theta \cdot \nu \end{aligned} \quad (7)$$

Displacement and stress first derivatives

The displacement and stress first derivatives $\left(u^{(1)},\sigma^{(1)}\right)$ must be computed. They are solutions of the following linear problem :

$$\begin{cases} \sigma_{\alpha 3}^{(1)} = \dfrac{R^m_{\alpha 3\alpha 3}}{2\varepsilon h}<u_\alpha^{(1)}> \;;\; \sigma_{33}^{(1)} = \dfrac{R^m_{3333}}{2\varepsilon h}\left(<u_3^{(1)}> + \dfrac{1}{2\varepsilon h}<u_k><u_k^{(1)}> \right) \\ K_T^{+/-}(u)\left(u^{(1)},v\right) + \displaystyle\int_\Sigma \left[\sigma_{\alpha 3}^{(1)}<v_\alpha> + \sigma_{33}^{(1)}\left(<v_3> + \dfrac{1}{2\varepsilon h}<u_k><v_k> \right) \right. \\ \qquad \left. + \sigma_{33}<u_k^{(1)}><v_k> \right] \qquad \forall v \in V \\ \qquad = F^{+/-}(u,\theta)\cdot v - \displaystyle\int_\Sigma \left[\sigma_{\alpha 3}<v_\alpha> + \sigma_{33}\left(<v_3> + \dfrac{1}{2\varepsilon h}<u_k><v_k> \right) \right] \nabla\cdot\theta \end{cases} \tag{8}$$

where $K_T^{+/-}(u)\left(u^{(1)},v\right)$ is the tangent stiffness operator associated to the adherends.

E gy o v v

Finally, the second derivative of the mechanical energy has the following expression :

$$\begin{aligned} J^{(2)}(\theta,\theta) &= J^{(2)+/-}(\theta,\theta) + \frac{1}{2\varepsilon h}\int_\Sigma R^m_{\alpha 3\alpha 3}<u_\alpha^{(1)}><u_\alpha>\nabla\cdot\theta \\ &+ \frac{1}{2\varepsilon h}\int_\Sigma R^m_{3333}\left(<u_3^{(1)}> + \frac{1}{2\varepsilon h}<u_k^{(1)}><u_k> \right)\left(<u_3> + \frac{1}{2\varepsilon h}<u_i>^2 \right)\nabla\cdot\theta \\ &+ \frac{1}{2\varepsilon h}\int_\Sigma \left[R^m_{\alpha 3\alpha 3}<u_\alpha>^2 + R^m_{3333}\left(<u_3> + \frac{1}{2\varepsilon h}<u_i>^2 \right) \right] \det(\nabla\theta) \end{aligned} \tag{9}$$

where $\det(\nabla\theta) = \theta_{1,1}\theta_{2,2} - \theta_{1,2}\theta_{2,1}$.

Remark : Eq. 7 shows that the crack force does not vanish if there is not a crack, so it would be tempting to take this expression to define a critical energy release rate for crack initiation. Unfortunately, it depends on the adhesive layer thickness and tends to zero with ε. However, for a given value of the thickness, Eqs. 4 and 7 allow to relate a stress criterion to an energy release rate criterion more convenient for computations.

NUMERICAL APPROXIMATION AND NUMERICAL EXAMPLE

Numerical approximation

The adherends are modelled using sixteen-nodes parallelepipedal elements with shape functions that are polynomials of second order in the in-plane variables and linear in the out-of-plane variable. The mid-surface Σ is modelled using eight-nodes serendipity elements.

The crack front is approximated by B_3-splines. The front displacement θ is then defined in a small neighbourhood of the front as a product of B_3-splines in the curvilinear co-ordinate and a cubic bell-shaped function in the transverse co-ordinate [7].

Numerical example

The numerical study concerned a DCB specimen constituted of two arms glued together with an adhesive. The arms had a length of 80 mm, a width of 40 mm and a thickness of 4 mm. Two thickness of the adhesive layer were considered : 0.1 mm and 0.5 mm. The materials were isotropic and the material characteristics were the following :

Adherends

Young's modulus : 150000 MPa ; Poisson's ratio : 0.3

Adhesive

Young's modulus : 4000 MPa ; Poisson's ratio : 0.35

Critical energy release rate : 0.28 N/mm

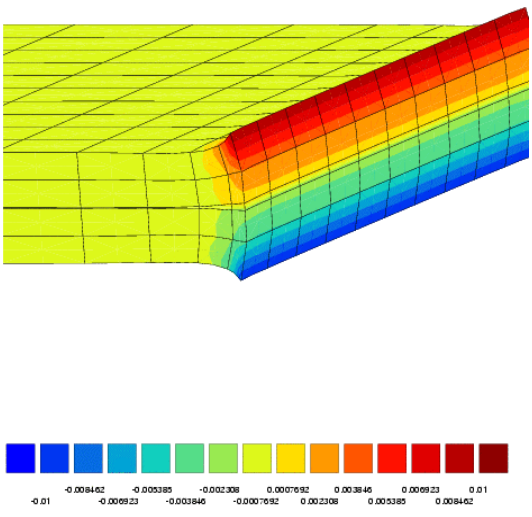

Figure 2 : Deformation of the loaded end before crack initiation.

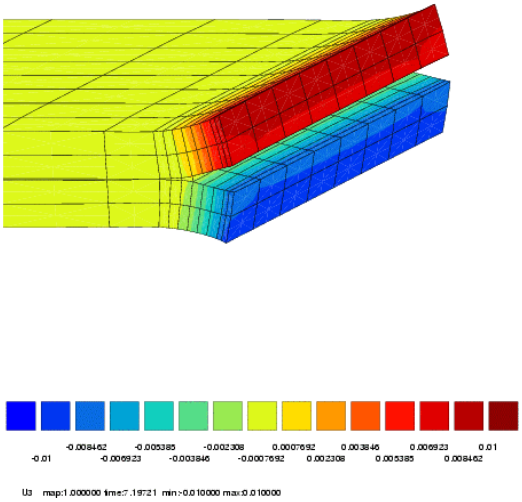

Figure 3 : Deformation of the loaded end after Crack initiation.

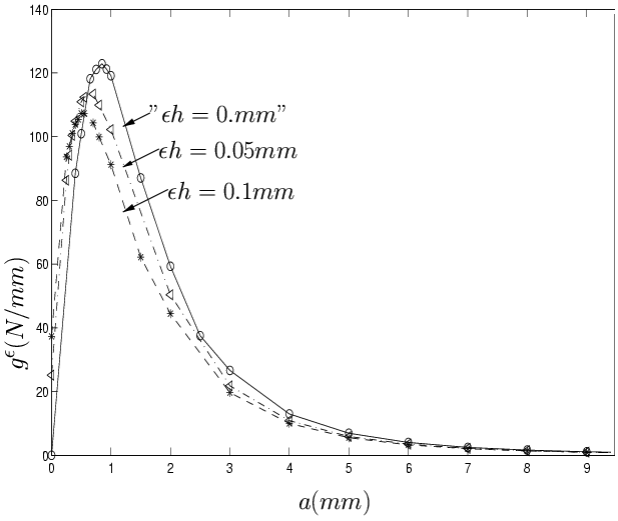

Figure 4 : Crack force as a function of the crack length.

The figure 2 shows the deformation of loaded end of the specimen just before the crack initiation, whereas the figure 3 shows this deformation after initiation at the stable position of the crack front. The figure 4 depicts the variations of the crack force G at the front mid-point as a function of the crack length for a constant value of the prescribed normal displacement at the end of the upper arm. As previously mentioned, G is not zero when there is not a crack but tends to zero with the layer thickness. Moreover, it can be observed that these curves are first increasing, then decreasing. This shows firstly that the crack at initiation has an unstable growth and secondly that

the initial crack has a minimal length. Finally, the front locations for increasing values of the prescribed normal displacement are reported in the figure 5.

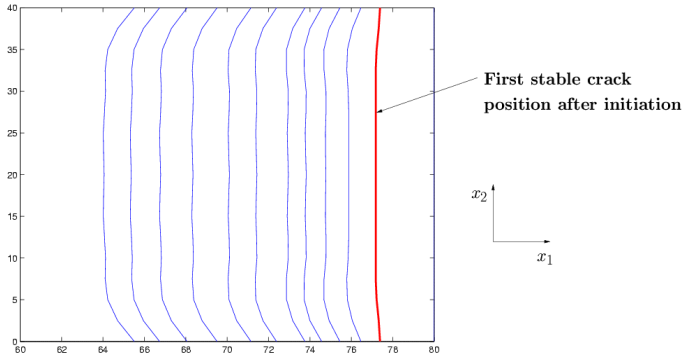

Figure 5 : Crack front positions during growth.

CONCLUSION

We have presented a numerical model to study the debonding of adhesively assemblies. Based on a fracture mechanics approach, it consists in seeking the minimum of the total energy of the assembly. The used Newton's algorithm involving the second derivative of the energy was proving efficient with a maximum of five front iterations between two front positions at arrest. The numerical example allowed to show the unstable character of the crack growth at initiation.

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