Interaction of the mode III interface crack with a thin stiff nonideal interface

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ABSTRACT: *Asymptotic behaviour of the elastic solution in a vicinity of the interface crack tip lying at a nonideal interface is investigated. The interface is assumed to be essentially stiffer in comparison with the bonded elastic materials. Additionally, inhomogeneity of the interface material is taken into account. The simplest case of the Mode III deformation is under consideration. It is shown that the main terms of asymptotic behaviour are practically similar to those in the case of the ideal interface (but with different SIFs in each materials). Moreover, for some special loading there is no difference between the models at all. However, in a general case, additional singular terms of stresses can appear for the stiff nonideal interface model.*

INTRODUCTION

Plane interface crack situated at the ideal interface is a classic problem of fracture mechanics. The *classic ideal interface approach* consists of assumptions that the interface is of zero thickness and the vectors of displacements and tractions satisfy the continuous transmission conditions.

However, in frames of the *ideal interface approach* it is impossible to take into account the mechanical properties of the bimaterial interphase itself at least at the stage of finding distributions of the stress or displacement fields near the crack tip.

There are several nonclassical interface conditions obtained by different techniques taking into consideration geometrical and material properties of thin interphase zones [5-10]. Nevertheless, only in [8-10], first attempts have been independently made to analyse the asymptotic behaviour of the solutions near the interface crack tip situated at the so-called soft (weak) nonideal interfaces. Corresponding transmission conditions were written in the form: $[\sigma_n]\|_\Gamma = 0$, $\left([u] - M(s)\sigma_n\right)\|_\Gamma = 0$, where (*n,s*) were the normal and tangential coordinates along the nonideal interface Γ, while matrix $M(s)$ depended on mechanical and geometrical features of the thin interphase.

In the next section we obtain transmission conditions for thin stiff interphase of the constant thickness consisted of inhomogeneous elastic material. Asymptotic behaviour of the elastic solution near the interface crack tip in the simplest case of mode III deformation is analysed in the third section. Finally, we discuss possible application of the investigated model in fracture mechanics.

EVALUATION OF TRANSMISSION CONDITIONS

Let us consider a bimaterial solid presented in Fig. 1. The shear moduli of the bonded homogeneous isotropic elastic materials are μ_+ and μ_-, respectively.

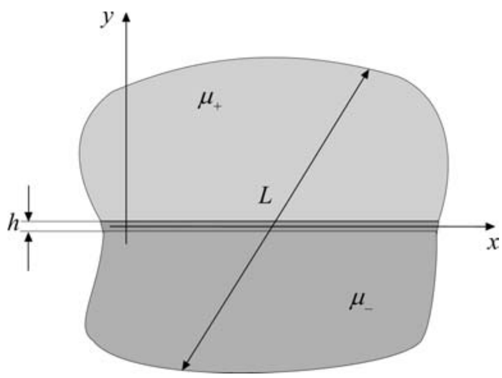

Figure 1. Bimaterial solid with thin stiff interface

It is assumed that a characteristic size of the body L is essentially larger than the thickness of the intermediate zone h. Thus,

$$h = \varepsilon h_0 \,, \quad h_0 \sim L \quad 0 < \varepsilon << 1 \,. \tag{1}$$

Material of the interface is assumed to be inhomogeneous and isotropic with the shear modulus $\mu = \mu(x, y)$ which is essentially stiffer in comparison with both the bonded materials:

$$\mu = \varepsilon^{-1}\mu_0 \,, \qquad \mu_0 \sim \mu_+ , \mu_- \,. \tag{2}$$

To obtain the respective transmission condition let us rescale the variable $y = \varepsilon\xi$ within the interphase $-h/2 < y < h/2$. As a result, displacement $u_z(x, y) = w(\varepsilon, x, \xi)$ has to satisfy the equilibrium equation:

$$D_x\left(\mu_0 D_x w\right) + \varepsilon^{-2} D_\xi\left(\mu_0 D_\xi w\right) = 0 \,, \tag{3}$$

where D_x, D_ξ are the respective partial derivatives. Additionally, the following boundary conditions have to be fulfilled along the respective boundaries of the thin intermediate layer:

$$w\left(\varepsilon, x, \pm\frac{h_0}{2}\right) = u^{\pm}, \quad \frac{1}{\varepsilon^2}\mu_0 D_\xi w\left(\varepsilon, x, \pm\frac{h_0}{2}\right) = \sigma_{yz}^{\pm}. \tag{4}$$

Solution of the problem has to be sought in form of series:

$$w(\varepsilon, x, \xi) = \sum_{i=0}^{\infty} \varepsilon^i w_i(x, \xi). \tag{5}$$

Here, $u^{\pm}$ and $\sigma_{yz}^{\pm}$ are displacements and stresses acting along the interface boundaries within the respective materials. They should be represented in the similar series (5). As a result of this procedure, a sequence of the boundary value problems for each function w_i is obtainable. Each of these BVPs is able to be solved only under additional conditions which, in fact, constitutes the sought for transmission conditions. In the case under consideration they take the following form:

$$[u] = 0, \qquad [\sigma_{yz}] + D_x\left(\int_{-h_0/2}^{h_0/2} \mu_0(x,\xi)d\xi \cdot D_x u\right) = 0. \tag{6}$$

Here by $[f]$ we, as usual, understand the jump of a function f across the interface. It is important to note that transmission conditions similar to (6) can also be obtained by applying the standard thin bar approach. Namely, if the thickness and the shear modulus of the thin layer depend only on the variable x, this approach leads to the conditions of the form:

$$[u] = 0, \qquad [\sigma_{yz}] + D_x\big(h(x)\mu(x) \cdot D_x u\big) = 0, \tag{7}$$

which completely coincides with (6) in case $h = const$, $\mu = \mu(x)$. However, asymptotic approach (1) – (6) enables us not only to evaluate the respective transmission conditions, but also to estimate its accuracy. (For example, condition $(6)_1$ is true with the accuracy $O(\varepsilon^2)$). In frames of the approach, the solution can be constructed with an arbitrary accuracy. Moreover, the asymptotic procedure can be slightly corrected to justify transmission conditions for anisotropic, inhomogeneous interface with varying thickness. Natural question is: what the difference with respect to the stress singularity arises between the classical interface crack formulation and the considered stiff nonideal interface approach.

MODELLING PROBLEM

In order to investigate the behaviour of the solution in an arbitrary interface crack problem with the stiff nonideal interface it is enough to solve the respective modelling problem for the infinite bimaterial plane with a semi-infinite interface crack (see figure 2). Materials of the half-planes $y > 0$ and $y < 0$ are homogeneous and isotropic with the shear moduli μ_+ and μ_-.

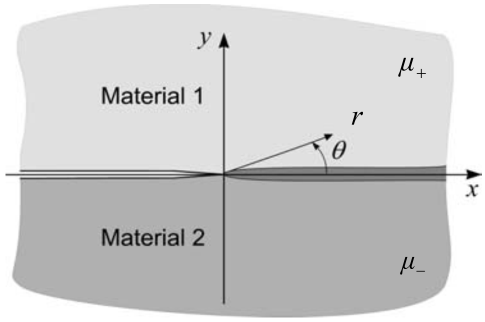

Figure 2. The nonidel interface along the crack line ahead.

Along the crack surfaces tractions are given:

$$\sigma^{\pm}_{\theta z}(r,\pm\pi) = g_{\pm}(r), \qquad \int_0^\infty g_+(r)dr = \int_0^\infty g_-(r)dr \,. \tag{8}$$

Here we have introduced the polar coordinates (r,θ) as shown in the figure 2. Known functions $g_{\pm}(r)$ have to satisfy equilibrium condition $(8)_1$.

Transmission conditions along the nonideal interface follow from (6):

$$[u]\big|_{\theta=0} = 0\,, \qquad \left([\sigma_{\theta z}] + \tau \frac{\partial}{\partial r}\left(r^{\alpha}\frac{\partial}{\partial r}u\right)\right)\bigg|_{\theta=0} = 0\,. \tag{9}$$

Such conditions make possible for us to investigate the asymptotic behaviour of elastic solution for the stiff nonideal interface characterized in a vicinity of the interface crack tip by the condition: $\mu(r)h(r) \sim \tau r^{\alpha}$, $r \to 0$

($0 \le \alpha < \infty$). Here, the case $\alpha = 0$ is the most important one. It corresponds to no damaged interphase of a nonzero thickness near the crack tip. On the other hand, the case $\alpha > 0$ has its own interest. Namely, with these conditions one can investigate the influence of the interface geometry or a damage effect appearing near the crack tip.

Solution of the problem

Applying the standard Mellin transform technique to the Laplace equation within the respective domain on taking into account the boundary and transmission conditions one can eventually obtain a functional equation for the problem under consideration:

$$\tau_* F(s+\alpha-1) + \frac{1+\mu_*}{s} \tan \pi s F(s) = -\frac{1}{s\mu_- \cos \pi s}\left[G_+(s) - G_-(s)\right]. \quad (10)$$

Here $G_+(s) = \tilde{g}_\pm(s+1)$ is the standard Mellin transform while

$$\tau_* = \tau / \mu_-, \quad \mu_* = \mu_+ / \mu_-, \quad (11)$$

are newly introduced dimensionless parameters. Moreover, $G_+(0) = G_-(0)$ due to the equilibrium condition $(8)_2$, so the right-hand (10) is an analytical function at zero point.

Similarly as it has been done in [8,9] we are able to prove that the functional equation (10) has a unique solution for any value of the parameter α. This solution is an analytical function in some strip $-\omega_0 < \Re s < \omega_\infty$ containing the imaginary axis. The values of parameters $\omega_0, \omega_\infty > 0$ depend on α and play an important role in the further analysis.

If $0 \le \alpha < 1$ then the function $F(s)$ has simple poles in the points: $s = -\omega_0 = \alpha - 1.5$ and $s = \omega_\infty = 1$. Its next poles in the left half-plane are situated at points $s = -\omega_j = (j+1)(\alpha-1) - 1/2$ ($j = 1,2,...$). As a result, some of them in the case $0.5 \le \alpha < 1$ belong to the strip $-1 \le \Re s < 0$. Moreover, the number of such terms tends to infinity as $\alpha \to 1$.

In the case $\alpha = 1$ one can conclude that the equation (10) is a linear one and $\omega_0 = \omega_\infty = \omega(\tau_*, \mu_*)$. Here, $\omega = \omega(\tau_*, \mu_*) \in (0.5,1)$ is the unique positive solution of equation $\tau_* s \cos \pi s + (1+\mu_*)\sin \pi s = 0$ within the interval $(0,1)$. Moreover, $\omega \to 1$ as $\tau_* \to 0$ and $\omega \to 0.5$ as $\tau_* \to \infty$.

Finally, if $\alpha > 1$ then the function $F(s)$ has the only simple poles in the points: $s = -\omega_0 = -1$, $s = \omega_\infty = \alpha - 1/2$.

Asymptotics of the elastic solution

When the solution of the functional equation (10) and its properties are known, the sought-for displacement has to be calculated from the inverse Mellin transform in the respective half-planes ($\pm\theta \in [0,\pi]$):

$$u^{\pm}(r,\theta) = \frac{1}{2\pi i}\int_{-i\infty+\delta}^{i\infty+\delta}\left\{F(s)\cos[s(\theta \mp \pi)] + \frac{G_{\pm}(s)}{\mu_{\pm}}\sin(s\theta)\right\}\frac{r^{-s}ds}{s\cos(\pi s)}, \quad (12)$$

where $\delta > 0$ is a small arbitrary value. Using the information about possible poles of the function $F(s)$, one can write the asymptotics of the elastic solution near the crack tip in the following common manner ($\pm\theta \in [0,\pi]$):

$$u^{\pm} = u_0 + \frac{2k_3^{\pm}}{\mu_{\pm}}\sqrt{r}\sin\frac{\theta}{2} + f(\alpha,r,\theta) + O(r^{1+\varepsilon}), \quad r \to 0, \quad (13)$$

where $u_0 = F(0)$, $k_3^{\pm} = \pi^{-1}\left[G_{\pm}(-1/2) \mp \mu_{\pm}F(-1/2)\right]$, while the function $f(\alpha,r,\theta) = o(\sqrt{r})$ takes different forms depending on value of α:

$$f(\alpha,r,\theta) = \begin{cases} 0 & 0 \le \alpha < 0.5, \\ \sum\limits_{j=0} k_3^{(j)} r^{\omega_j}\cos[\omega_j(\theta \mp \pi)] & 0.5 \le \alpha < 1, \\ k_3^{*} r^{\omega}\cos[\omega(\theta \mp \pi)] & \alpha = 1, \\ Cx & 1 < \alpha < \infty. \end{cases} \quad (14)$$

In figure 3 the respective distribution of all stress singularity exponents λ_k ($\lambda_{main} = -0.5$ and $\lambda_j = \omega_j - 1$) is shown together with the so-called T-stress (when respective $\lambda_j = 0$) as a function of the parameter α. Let us note that T-stress appears not only when $\alpha > 1$, but also in the cases when $\alpha = \alpha_j \equiv 1 - 1/(2j)$, $j = 1,2,\ldots$.

Let us remind that in case of the ideal interface the only main stress singularity $\lambda_{main} = -0.5$ as well as T-stress appear. However, in case of the soft nonideal interface elastic solution for the interface crack exhibits a more complicated behaviour [8].

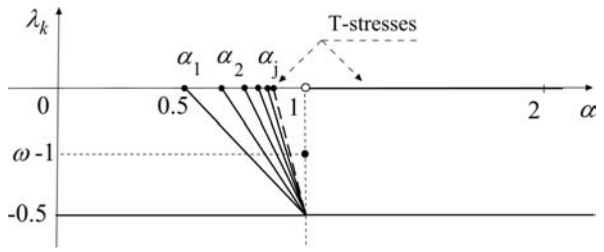

Figure 3. Distribution of the stress singularity exponents

DISCUSSIONS AND CONCLUSIONS

First of all let us note that in the case of the symmetrical loading ($G_+(s) = G_-(s)$) the equation (10) has the only trivial zero solution. As a result, the interface crack problem with the *stiff nonideal interface* coincides completely in this case with the solution for the *ideal interface* regardless of the values of α and τ under the mode III deformation. However, for an arbitrary geometry of the finite bimaterial structure such a condition is impossible to be realized.

When $\alpha = 0$ (stiff nonideal interface of a constant local thickness and the local material homogeneity near the crack tip) the only classical square root singularity arises (without any T-stress), and the next term in asymptotic expansion (13) can be estimated as $O(r\sqrt{r})$. Hence, one can investigate in which material the interface crack will propagate (SIFs in the bonded materials are different, in general) by any of the classic fracture mechanics criteria.

However, the thickness of the interphase varies and can vanish at some point where no adhesive presents between the materials. This is a natural consequence of the fact that the surfaces of the matched materials are not identical. This means that $h(r) \to 0$ ($r \to 0$) in local coordinates connected with the point. The other effect can also appear when the shear modulus of the interphase material tends to zero due to a damage accumulation near the crack tip within the stiff interface. Both these cases lead to the same stiff nonideal interface model with some $\alpha > 0$. Unfortunately, the value of α is unknown a priori. However, in cases $0 < \alpha < 0.5$ one can conclude from (14) that the model under consideration gives practically the same result with respect to the stress singularity as in the case $\alpha = 0$ (see figure 3). The only difference being the values of SIFs.

If $\alpha > 1$ the asymptotic expansion (13) coincides with that for the ideal interface and the only respective constants C determining the T-stress are different, in general. (Thus, this case can be naturally called *the almost ideal stiff interface* as it has been done in [8] for *the almost soft interface*).

As a result, the influence of a possible damage effect or local interphase geometry on the stress singularity would be visible only if $0.5 < \alpha < 1$. If we assume a linear changing of elastic properties of the damaged interphase then $\alpha = 1$ and from (10) and (13) it follows that the main SIF is identical to that for the ideal interface. However, the next singularity $\lambda = \omega - 1$ depends essentially on parameters τ_*, μ_* disappears as $\tau_* \to 0$, as could be expected and becomes comparable with the main one when $\tau_* \to \infty$.

Finally, it is important to investigate Mode I and II interface problem for the stiff nonideal interface (6) in order to compare results with those for the soft interface [9] and, particularly, to answer the question whether the classic stress oscillation appears there (and for what value of α) or not.

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