Damage Process and Crack Propagation in Materials

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ABSTRACT: *A damage evolution approach has been proposed to describe various deformation and failure processes. This approach is based on the concept of damage and synergetic principles. It is postulated that deformation and fracture processes in solids are determined by some general functional law related to the accumulation of damage. A damage evolution equation allows an estimation of the critical time for a solid to reach its critical state at the controlling parameter for the deformation or the failure process under study. Fracture mechanics parameters are accepted as the controlling parameters for the failure processes.*

The damage evolution approach can be seen to be synonymous with the present day laws to analyse (i) fatigue crack growth; (ii) low cycle fatigue life of notched structural components and (iii) stress corrosion. Proposed analytical equations describe the experimental results very well.

INTRODUCTION

A concept of damage evolution in solids has been suggested by Kachanov [1] and Rabotnov [2] for the analysis of damage under creep loading. The concept of damage evolution has been used in different later versions (e.g. [3-5]) for the analysis of various processes of damage accumulation. The accumulation of damage can be associated with a change of continuity Ψ. The continuum parameter Ψ (or damage parameter $D = 1 - \Psi$) has not a physical interpretation. A change of the parameter Ψ means the appearance and growth of cracks and/or voids, and a change in the mechanical and physical properties of a solid. Consequently, the value of Ψ reflects damage evolution (the state) of solids under an external influence. The description of the evolution phenomena in various branches of knowledge can be based on an interdisciplinary or a synergetic branch of science. A typical non-linear evolution equation of the state of autonomous systems can be expressed as a function of state parameters $\mathbf{q}$ of the system and controlling parameters ξ [6]. For practical applications of the evolution approach it is important to choose the vector of state parameters $\mathbf{q}$ of the system and the controlling parameters ξ . A specific form of the function can be obtained from data of basic experiments and an analysis of the system behaviour under the influence of various external factors during time τ .

This paper concentrates on a concept of damage evolution to some problems of crack propagation when the mechanisms of failure do not change in the time period being considered.

DAMAGE PROCESS IN SOLIDS

The evolution approach has been extended to deformation and fracture processes of a mechanical loaded system, i.e. "solid - damage". It is assumed that the accumulation of damage (the system state) is determined by the scalar $0 \le \Psi \le 1$ which is the single state variable $q = \Psi$. The controlling parameters ξ for deformation and failure processes of solids could be stress and strain, the stress intensity factor, temperature and other parameters, which are essential in the consideration of the damage accumulation process.

It is postulated that deformation and fracture processes are governed by some general functional law of damage accumulation [7]. For a simple case the damage evolution law can be formulated as

$$\frac{d\Psi}{d\tau} = -A\left(\frac{\xi}{\Psi}\right)^{n}, \qquad (1)$$

where $A > 0$, $n \ge 0$ are material (the "solid - damage" system) constants for the fracture process under study. The evolution law (1) can be made more precise when the physical and mechanical aspects of a failure process are more clearly understood by examining the fracture mechanisms of the solid and the type of loading under study. The value of Ψ decreases with an increase in time τ during the process of the accumulation of damage in a solid. The value $\Psi = 1$ corresponds to the non-damaged state of a solid when $\tau = 0$, and the value $\Psi = \Psi_c$ corresponds to the critical state when $\tau = \tau_c$, where τ_c is the critical time. So, failure occurs in a solid if the damage reaches the critical value $\Psi = \Psi_c$ at $\tau = \tau_c$. The following relationship can be written as follows by integrating Eq. (1) from $\Psi = 1$ to $\Psi = \Psi_c$

$$\Psi_c^{1+n} = 1 - A(1+n)\int_0^{\tau_c} \xi^n d\tau \ . \tag{2}$$

Taking into account Eq. (1) and equation for the determination of the critical time, the cumulative damage law is expressed in the integral form

$$\int_0^{\tau_c} \frac{d\tau}{\tau_c} = 1 \, , \tag{3}$$

if the controlling parameter ξ is constant. Moreover, the cumulative damage law given by Eq. (3) may be rewritten using number of cycles N or other similar parameters, which are dependent on time of loading, instead of time.

The influence of the controlling parameter ξ on the critical time may be analysed for damage evolution in solids. First it is assumed that the critical value Ψ_c is constant for the deformation and failure process under study, and the critical state of a damaged solid can be reached for various combinations of the controlling parameter and time τ. It has been suggested therefore that the critical value Ψ_c [Eq. (2)] is also reached when the controlling parameter ξ is equal to the critical value ξ_c at some fixed time (or a unit of time) $\tau = \tau^* < \tau_c$, that is

$$\Psi_c^{1+n} = 1 - A(1+n)\xi_c^n \tau^* . \tag{4}$$

The evolution equation at Ψ_c=*const* is derived from Eqs. (2) and (4), namely

$$\int_0^{\tau_c} \xi^n d\tau = \xi_c^n \tau^* . \tag{5}$$

This equation may be rewritten at $\xi = const$ as

$$\tau_c = \tau^* \left(\frac{\xi_c}{\xi} \right)^n . \tag{6}$$

Thus, the damage evolution equation allows one to estimate the critical time for a solid to reach its critical state under the given controlling parameter for the deformation and fracture processes being studied. Examples of the application of the damage evolution equation will be now discussed for various processes of deformation and failure.

FATIGUE CRACK GROWTH

Fatigue crack growth may be described by an equation of the type of Eq. (1). In this case the stress intensity factor K can be used as the controlling parameter to describe fatigue failure when linear fracture mechanics is valid. It is assumed that the mechanism and the process of fatigue failure remain uninterrupted. Therefore the maximum (or the range) of the stress intensity factor $K_{\max}$ is chosen as the controlling parameter ξ for fatigue crack growth.

Taking into account the functional relation of the value Ψ and the fatigue crack size a and replacing τ with the number of fatigue loading cycles N, Eq. (1) can be expressed as

$$\left(\frac{d\Psi}{da}\right)\frac{da}{dN} = -A\left(\frac{K_{\max}}{\Psi}\right)^{n}. \tag{7}$$

where the value A includes the parameter $dN/d\tau=const$. It has been suggested that a crack increment $(\Delta a_j = a_j - a_{j-1})$ occurs when damage accumulation reaches a critical value in the vicinity of the fatigue crack tip, that is $\Psi = \Psi_c$. Dividing variables in Eq. (7) and taking into account the boundary conditions, the following equation is deduced:

$$\Psi_c^{1+n} = 1 - A(1+n)\int_{a_j-1}^{a_j}\left(\frac{K_{\max}^{n}}{da/dN}\right)da. \tag{8}$$

It is understood that the fatigue crack growth rate da/dN is some average value of the rate during crack incremental growth. Therefore the crack growth rate, as well as the maximum stress intensity factor, can be accepted as constant values in the limits of integration from a_{j-1} to a_j. From Eq. (8) the useful approximate relation is obtained

$$\Psi_c^{1+n} = 1 - A(1+n)\frac{K_{max}^n \Delta a_j}{da / dN}. \qquad (9)$$

The critical value Ψ_c can also be reached after the application of the number of cycles N^* under the controlling parameter $\xi = \xi_c$. From Eqs. (9) and (5) at $\tau^* \to N^*$ the equation for the fatigue crack growth rate can be given as

$$\frac{da}{dN} = V^*\left(\frac{K_{max}}{\xi_c}\right)^n, \qquad V^* = \Delta a_j / N^*. \qquad (10)$$

Considering the value

$$C = \frac{V^*}{\xi_c^n}, \qquad (11)$$

the well-know empirical Paris type law $da / dN = CK_{max}^n$ follows.

It can be seen that the parameters ξ_c and V^* are interdependent for a given interpretation [Eqs. (10) and (11)], namely various rates V^* correspond to various critical parameters ξ_c for the fatigue crack growth diagram ($C = const$). Using Eq. (10), there is a possibility of physically modelling the fatigue crack growth processes when various crack growth mechanisms have been realised. Such parameters as $\Delta a_j, N^*$ and ξ_c will have a defined physical meaning. From this point of view the middle section of the fatigue crack growth diagram can be analysed. Here the failure mechanisms are identified on the basis of the microrelief of the fracture surface. Fatigue crack growth can be accompanied by striation formation on the fatigue fracture surface [8,9]. If the striation spacing coincides with the sub-structural grain size arising in the plastic zone ahead of the crack tip, the value V^* is approximate equal to 10^{-7} m/cycle. The critical parameter ξ_c corresponds to the stress intensity factor, i.e. $\xi_c = K^* = K_{max}$, and the crack increment length Δa_j is equal to the striation spacing and $N^* = 1$. The stress intensity factor K^* is calculated using Young's modulus, the Burgers vector, and the size of the plastic zone ahead of the fatigue crack tip and the grain size of the metal [8].

Apparently, the evolution approach will be useful for fractographic analysis of fatigue crack propagation, because the crack growth equation includes the parameter Δa_j, which characterises the crack growth by discrete jumps.

CRACK PROPAGATION UNDER STRESS CORROSION CRACKING CONDITION

It has been suggested that the process of stress corrosion cracking (SCC) is initiated at some initial (applied) stress intensity factor K_i. Therefore the value K_i is chosen as the controlling parameter. Then Eq. (6) may be rewritten in the following form

$$K_i^n \tau_c = \left(K_{ic}\right)^n \tau^* = const\,. \tag{12}$$

Here τ_c is the time required to initiate the SCC process at the stress intensity factor K_i and τ^* is some standard time (or a unit of time) required to initiate the SCC process at some stress intensity factor $K = K_{ic}$, and n is the constant for a given "solid - environment" system. The critical time τ_c is increased for a reduction of the initial stress intensity factor. From Eq. (12) the following conclusion can be drawn, namely that there is no physical threshold K_{1scc}. Apparently, the experimental value of the threshold stress intensity factor is the value K_{1scc} at some defined time τ_{scc} of testing. A similar situation can be observed in fatigue for the threshold stress intensity factor K_{th}, defined as the value of K_{max} below which the crack does not grow. The fatigue crack growth threshold is determined as the value of K_{max} that corresponds to a certain fatigue crack growth rate V_{th}, which is conventionally assumed to be equal, for example, to 10^{-10} m/cycle.

Corrosion crack propagation (the kinetic diagram of SCC) can be expressed by the function [7]

$$\frac{da}{d\tau} = V_0 \left(\frac{K}{K_i} \right)^n, \qquad K \geq K_i. \tag{13}$$

Here $V_0 = \Delta a_j / \tau_c$, where Δa_j is the incremental length extension of a corrosion crack and K is the current stress intensity factor. Equation (13) describes the experimental results on stable crack propagation in the steel under the influence of distilled water (Figure 1). The following important considerations need to be noted.

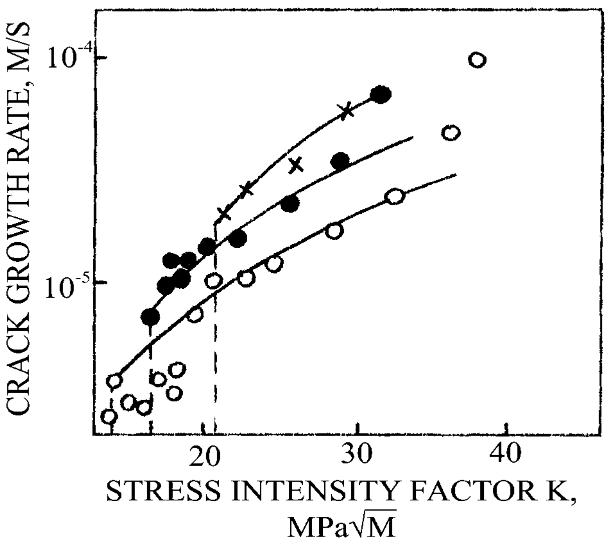

Figure 1: Stress corrosion cracking rates in 50X steel under the influence of distilled water. Predicted crack growth behaviour [Eq. (13)] and experimental data [10].

The SCC kinetic diagram depends on the initial stress intensity factor, namely the fastest corrosion crack growth rates correspond to higher K_i levels [10]. A clear explanation of this phenomenon is found out in a SCC model based on Eq. (13). If the value K_i is increased, then the critical SCC time is reduced due to the expression $K_i^n \tau_c = const$ that leads to the increase of the value V_0. It is possible that the length of the crack growth increment is also increased. As a result, corrosion crack growth occurs at higher rates in terms of $da / d\tau$.

CONCLUSIONS

A damage evolution approach, which has been proposed to describe various deformation and failure processes, leads to a description of fatigue crack growth and stress corrosion cracking. This approach is based on the generalised concept of damage.

It has been shown that the critical parameter ξ_c and the value V^* are interconnected with the Paris type law of fatigue crack growth. It will be very useful to use this connection in fractographic analyses and the physical modelling of fatigue crack propagation.

The corrosion crack growth rate can be calculated using a stress corrosion cracking model. The proposed equation allows one to describe the experimental results for stable crack propagation, showing that the fastest corrosion crack growth rates correspond to the higher initial stress intensity factor levels. The dependence of the corrosion crack propagation versus the initial stress intensity factor has been explained.

REFERENCES

1. Kachanov, L.M. (1986) *Introduction in Continuous Damage Mechanics*. Nijhoff, Dordrecht.
2. Rabotnov, Yu.N. (1969) *Creep Problems in Structural Members.* North-Holland Publishing Company, Amsterdam.
3. Murakami, S. (1988) *Trans. ASME. J. Appl. Mech.* **55**, 280.
4. Altenbach, H. and Zolochevsky, A. (1996) *Fatigue Fract. Engng Mater. Struct.* **19**, 1207.
5. Bhattacharya, B. and Ellingwood, B. (1998) *Int. J. Fatigue* **20**, 631.
6. Haken, H. (1983) *Advanced Synergetic: Instability Hierarchies of Self-Organizing Systems and Devices*. Springer-Verlag, Berlin.
7. Matvienko, Yu.G. (1990) *Physicochemical Mechanics of Materials* **26** (N 1), 31 (in Russian).
8. Grinberg, N.M, Serdyuk, V.A. and Malinkina, T.I. (1991) *Structure and Fatigue Strength of Magnesium Alloys.* Metallurgia, Chelabinsk (in Russian).
9. Botvina, L.R. (1989) *Failure Kinetic of Structural Materials.* Nauka, Moscow (in Russian).
10. Romaniv, O.N. and Nikiforchin, G.N. (1986) *Corrosion Fracture Mechanics of Structural Alloys*. Metallurgia, Moscow (in Russian).