An assessment of the probability of root failures in load carrying cruciform joints of duplex stainless steel 2205

R. F. Martins[1] and C. M. Branco[2]

[1] New University of Lisbon, FCT/DEMI, 2829 Monte de Caparica, Portugal
[2] ICEMS/IST, Avda. Rovisco Pais, 1049-001 Lisbon, Portugal

ABSTRACT: *Fatigue life data and failure probability assessments in these types of joints, fabricated from duplex stainless steel 2205, are comparatively scarce and, therefore, a research study was initiated to assess the fatigue behaviour of load carrying joints of this material and to obtain probability of failure data using the methods of Probabilistic Fracture Mechanics (PFM). Probability of failure results was obtained both for the situations of unstable fracture and fatigue crack growth. For fatigue crack growth, the parameters used were the design stresses for three values of fatigue life (10^5, $2x10^6$ and 10^7 cycles), using design curves of two codes, and also experimental S-N curve obtained by the authors in the duplex 2205 material and corresponding to the lower fatigue curve for 95% confidence interval. The K solutions were 2dFE J integral analysis, obtained by the authors, and the BS7910 formulation. Sensitivity analysis of the probability of failure results is presented in the paper.*

1. INTRODUCTION

Due to high safety and operating costs related with a structural failure, reliability studies of welded structures were published recently [1]. Probabilistic Fracture Mechanics can be used to quantify the probability of failure of a structure. As referred in [1], the application of PFM requires a considerable level of experience, since incorrect failure predictions can happen, due to non-adequate valuable data values, such as fracture toughness and other materials data, and also defect size and distribution in the structure.

2. FUNDAMENTALS DEFINITIONS OF PFM

A structural failure is due to occur when the defect sizes in the structure are greater than the safe values calculated with Fracture Mechanics. Assuming a non-deterministic variation of both these kinds of defects, the failure probability is calculated from the superposition between the two probability density functions corresponding to the two above-mentioned variables (Fig. 1). The calculated value for the probability of unstable failure of the structure in a given instant is given by the equation,

$$\text{Probability of failure} = \int \eta(x)dx \int g(x)dx \qquad (1)$$

where,

$$\eta(x) := \frac{\beta\left(\frac{x-\gamma}{\alpha}\right)^{(\beta-1)} e^{\left(-\left(\frac{x-\gamma}{\alpha}\right)^{\beta}\right)}}{\alpha} \qquad (2)$$

In equations (1) and (2) η(x) is the probability density function of the defects found in the structure, and g(x) is the probability density function of the critical defects.

2.1 Probability Density Function, η(x), of the defects found in the structure

The values of the parameters used in equations (1) and (2) were taken from ref. [2], and were taken from a Weibull type of fitting of the depth of the defects measured in a great number of welded joints in North Sea oil platforms. These parameters have not taken into account the reliability of the non-destructive method used for the measurement of the defects.

The following values were introduced in equation (2); β=0.80; γ=0.1 and α=1.12 at instant t=0 when the structure enters into service. In service, and since fatigue loading occurs, the initial probability density function will change, since the initial defects will propagate and its size will change (increase), and the geometry will change. Hence, new values of β, γ and η should be introduced in equation (2). Alternatively, the actual propagated dimensions of the initial defect distribution were calculated, assuming that the probability of finding defects with a size equal or below the size of the propagated defects was 100%, as shown in Fig. 1.

2.2 Probability density function of critical defects, g(x)

The probability density function of critical defects of structural high strength steels, g(x), was calculated using 19 valid CTOD values, obtained at the temperature of –20ºC [3].

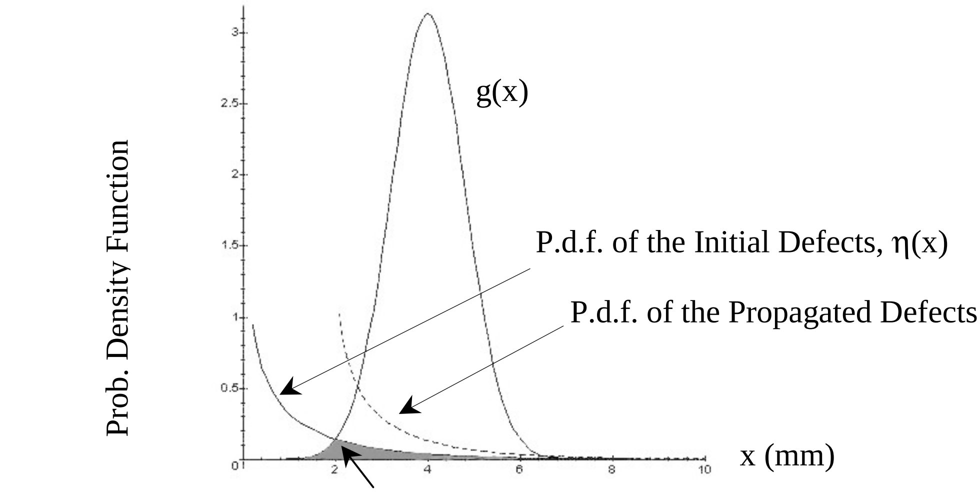

Figure 1: Probabilitity density functions of the initial, actual and critical defects. Probability of failure.

These values were obtained in several European laboratories from 1983 to 1997.

For an initial hypothesis,H_o, CTOD values were calculated with a normal distribution, with a significance level of 0.05 ($\alpha_{\%}$=0.05), i.e. with equations

$$H_0: CTOD \rightarrow N(\mu,\sigma) \quad (3)$$

$$E(CTOD) = \mu = 0.8396 \text{ mm}$$

$$V(CTOD) = \sigma^2 = 0.46267 \text{ mm}$$

$$P.d.f.(CTOD) = \frac{1}{\sigma \times \sqrt{2\pi}} \times e^{\left[-0.5\times\left(\frac{CTOD-\mu}{\sigma}\right)^2\right]} \quad (4)$$

where E (CTOD) and V (CTOD) are, respectively, the expected value and the variance of the CTOD data. The expected value of the sample was considered equal to the mean value of the CTOD values.

In a second hypothesis, the CTOD values were adjusted to a lognormal distribution, and the equations are

$$H_0: CTOD \rightarrow LN(\mu,\sigma) \quad (5)$$

$$E(CTOD) = e^{\left(b+\frac{a^2}{2}\right)} = \mu = 0.8396$$

$$V(CTOD) = \gamma.(\gamma\text{-}1).e^{2b} = \sigma^2 = 0.46267 \text{ com } \gamma = e^{\left(a^2\right)}$$

$$P.d.f.(CTOD) = \frac{1}{CTOD \times a \times \sqrt{2\pi}} \times e^{\left[-0.5 \times \left(\frac{\ln(CTOD)-b}{a}\right)^2\right]} \quad (6)$$

CTOD>0, $\mu \in \Re$, $\sigma>0$

The χ^2 test was used to check the validity of the fitting of the sample distributions to the theoretical ones. The number of classes, k, where the sample size, n, was divided was calculated by the Sturges rule,

$$k \geq \frac{\ln(2n)}{\ln(2)} = 5, k \in N \quad (8)$$

The χ^2 test has shown that the sample values of CTOD could be fitted to a lognormal distribution with a significance level of 0.05. The normal type distribution did not provide satisfactory values, as also confirmed by the plots in Fig. 2.

The probability density function of CTOD, δ_I, was modified to the probability density function of acceptable defects, g(x), using the CTOD design curve. The transformation equations are,

$$a_m = 0.5 \times \frac{E.\sigma_y}{\pi.\sigma_{max}^2} \times \delta_I \quad : \quad \frac{\sigma_{max}}{\sigma_y} < 0.5 \quad (9)$$

$$a_m = 0.5 \times \frac{E}{\pi.(\sigma_{max} - 0.25.\sigma_y)} \times \delta_I \quad : \quad \frac{\sigma_{max}}{\sigma_y} > 0.5 \quad (10)$$

where σ_{max} is the maximum stress in the structure, near the weld detail under analysis, and includes the sum of the applied stresses with the residual stresses; E is the Young's modulus, and σ_y is the yield stress, respectively.

The probability density function of the critical defects, a_m, is therefore given by,

$$a_m = C \times CTOD \text{ com } C>0 \text{ (cte.) e } CTOD \rightarrow LN(\mu,\sigma) \quad (11)$$

$$Y = C.X \Leftrightarrow X = Y/C \therefore \partial X/\partial Y = 1/C \quad (12)$$

where the change in variables in the unidimensional case gives [4]

$$f_Y(Y) = f_X(Y/C) \cdot \left| \frac{1}{C} \right| \qquad (13)$$

where f_x is the probability density function of CTOD in the point $\underline{Y}$/C. Therefore, the probability density function of the critical defects, g(x) gives, finally,

$$g(x) = P.d.f.(a_m) = \frac{1}{(a_m/C) \cdot \sigma \cdot \sqrt{2\pi}} \cdot e^{\left[-0.5 \cdot \left(\frac{\ln(a_m/C)-\mu}{\sigma}\right)^2\right]} \cdot \frac{1}{C} \qquad (14)$$

with $a_m/C>0$, $\mu \in \Re$ and $\sigma>0$.

Since the probability density functions of the initial defects, η(x) and the critical defects, g(x) were both obtained, the probability of failure may be calculated with equation (1). Results obtained for a case study are presented next.

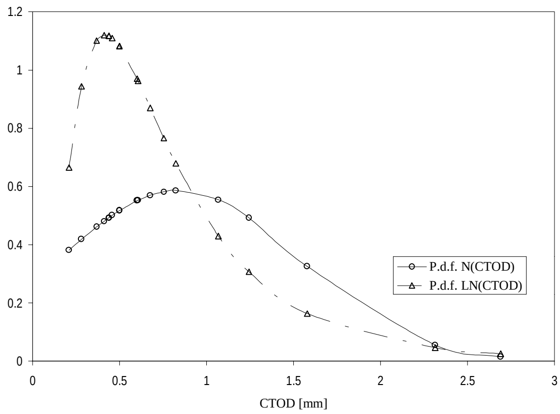

Figure 2: Probability density functions for the CTOD values.

3. RESULTS AND DISCUSSION

For a load carrying cruciform joint with full lack of penetration (Fig. 3a)), failure probabilities were obtained for three values of the fatigue lives, $N=10^5$, $N=2x10^6$ and $N=10^7$ cycles, and assuming crack propagation from the weld root through the weld metal. For the stresses, the values taken from two design curves were chosen, namely the IIW class 45 and an S-N curve experimentally obtained by the authors in previous work [5] and referring to 95% confidence limits for the data. This curve is plotted in Fig. 4.

An initial defect size at weld root, with depth equal to 1.13mm, was taken, and the crack was assumed to propagate in the weld metal (Fig.3b)) at angles of 45, 60, 75 and 90º with the axis of the longitudinal load direction in the main plate (Fig.3c)). The value of initial crack size was obtained by measurements at weld root in the fracture surfaces of the specimens (Fig. 3b)). Values of J and the linear elastic K were obtained with equation (15) and (16). The stress intensity factor equation in BS7910 [6], Annex J, was also used for comparison. Equation (15) was derived from J data obtained with the 2D FE code ABAQUS. The elements at the crack tip have three collapsed nodes and two intermediate nodes at ¼ position.

Details of the mesh are shown in Fig. 3d), and the data was obtained for angles of crack propagation of 45, 60, 75 and 90º and crack sizes of a/w=0.125; 0.25; 0.375 and 0.5, where w is the weld throat length (Fig. 3c)).

$$J=A1.(a/w)^4+A2.(a/w)^3+A3.(a/w)^2+A4.(a/w)+A5 \qquad [N/mm] \qquad (15)$$

$$A1=-1.1524\times10^{-4}\alpha^3+2.33218\times10^{-2}\alpha^2-1.50951\alpha+3.43544\times10^{1}$$

$$A2=1.09272\times10^{-4}\alpha^3-2.18932\times10^{-2}\alpha^2+1.44922\alpha-3.24981\times10^{1}$$

$$A3=-3.40489\times10^{-5}\alpha^3+6.89327\times10^{-3}\alpha^2-4.48804\times10^{-1}\alpha+1.02567\times10^{1}$$

$$A4=3.74761\times10^{-6}\alpha^3-7.07411\times10^{-4}\alpha^2+5.06334\times10^{-2}\alpha-7.70296\times10^{-1}$$

$$A5=3.19\times10^{-1}$$

$$0\le(a/w)\le0.5,\ 45°\le\alpha\le90°$$

$$K^2=\frac{E\,.J}{1-\nu^2}\times\frac{\sigma}{100},\ E \text{ e } \sigma\ [MPa] \qquad (16)$$

Values of the constants C and m of the Paris law were obtained experimentally for a duplex grade stainless steel 2205, loaded in three point bending [5], and the values are given in Table 1.

TABLE 1: Values of C and m of the Paris law

Δσ [MPa]	C [mm/cycle;MPa.m$^{0.5}$]	m
250 / 200 / 120	9.68E-09	2.7816
80 / 60	3.10E-09	3.0811

A comparison between the two assumed S-N curves can be seen in Fig 4. The experimental results give a very good fit with the IIW class 45 design curve for failure from the weld root in load carrying cruciform joints. For this reason, negligible differences were found between the P_f results obtained by the two types of S-N curves. Some of the more significant obtained values of the probability of failure are given in Tables 2 and 3. The remaining results are in [5].

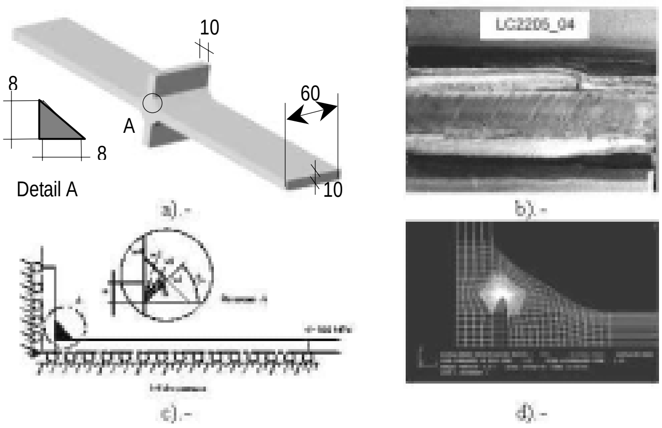

Figure 3: a) Geometry of the non-load carrying cruciform joint with full lack of penetration. b) Macro of the fracture surfaces. Crack propagation from the weld root. c) Boundary conditions and loading in the welded joint of Fig. 3a). Definition of the parameters, α, w and a/w. d) Finite element mesh (deformed and undeformed) to obtain the maximum value of the J integral (case for α=75º and a/w=0.375).

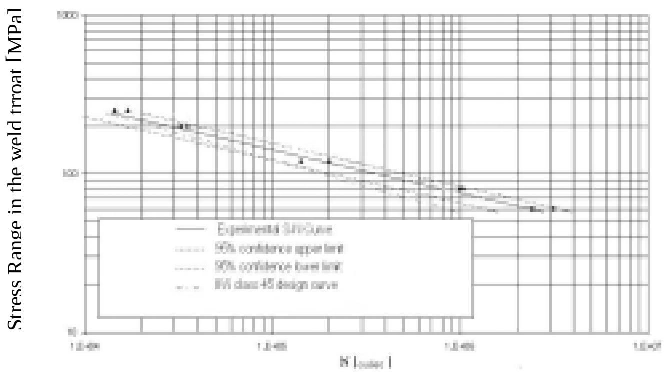

Figure 5: IIW design curve (Class 45) and experimental S-N curve.

TABLE 2 : Initial values of probability of failure, P_f,

IIW Design Curve (Class 45)		S-N Curve (95% lower limit)	
Δσ [MPa]	Initial Prob. Failure	Δσ [MPa]	Initial Prob. Failure
122.1	1.556E-04	141.44	1.512E-04
44.98	7.680E-11	62.84	2.000E-09
26.3	1.310E-15	40.64	5.930E-13

TABLE 3: Probability of failure values for crack propagation between a_i and a_f values of crack depth in the weld metal. α=90º. a_i=1.13mm. Remaining parameters as in Table 2.

	IIW Design Curve (Class 45)						
N[cycles]	a_{final} [mm][(15)]	a_{final} [mm][7]	$\Delta\sigma$ [MPa]	Prob. Failure[(15)]	Prob. Failure[7]	a/w[(15)]	a/w[7]
10^5	2.569	2.543	122.093	1.762E-04	1.758E-04	0.321	0.318
$2x10^6$	1.941	1.936	44.979	8.670E-11	8.660E-11	0.246	0.242
10^7	1.899	1.895	26.304	1.500E-15	1.500E-15	0.237	0.236
	95% lower limit confidence S-N Curve						
N[cycles]	a_{final} [mm][(15)]	a_{final} [mm][7]	$\Delta\sigma$ [MPa]	Prob. Failure[(15)]	Prob. Failure[7]	a/w[(15)]	a/w[7]
10^5	2.555	2.529	121.771	1.711E-04	1.707E-04	0.319	0.316
$2x10^6$	2.896	2.844	54.101	2.550E-09	2.530E-09	0.362	0.355
10^7	4	3.868	34.987	9.440E-13	9.240E-13	0.5	0.4853

4. CONCLUSIONS

- Both for static failure, under essentially brittle fracture conditions, and in failure by fatigue, it is the stress range value that defines the level of magnitude of the obtained probability of failure.
- The stress intensity factor formulations of BS7910 for this type of joints gave lower values of the probability of failure than the P_f values obtained with equations (15), developed in this work.
- Additional work is needed in this area and is currently in progress for several types of mechanical components to get more insight in the assessment of reliability or probability of failure.

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