Prediction of Fracture
in Plane Elements with Sharp Notches

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ABSTRACT: *In this paper four fracture criteria, adapted for structural components with V-shaped notches under biaxial loading (mode I and II), were used to evaluate the critical loads and directions of crack initiation. For structural elements with sharp notches (i.e. when the stress field singularity for the primary problem geometry is different from the singularity of a crack) those criteria require introduction of certain non-local parameters, e.g. assumption of a finite-length crack or a finite fracture zone. Results of numerical analysis were compared with experimental data. Experiments have been carried out using plane specimens of polymethyl metacrylate (PMMA). Experimental data provide satisfactory agreement with predictions.*

INTRODUCTION

Fracture resistance of structural components with stress concentrators such as sharp notches and cracks can be evaluated using fracture criteria. It would be desirable to have a reliable and general crack criterion that would enable analysis of both crack initiation and crack propagation phenomena in components of any shape (with regular and singular stress fields) under arbitrary loading. The analysis should provide such data as the critical load value, crack initiation site and crack propagation direction. The concepts of fracture mechanics are of particular importance for evaluating safety-operating conditions for components made of brittle materials. For such materials the moment of crack initiation is mostly associated with the final failure of a structural element.

FRACTURE CRITERIA FOR ELEMENTS WITH V-SHAPED NOTCHES UNDER TENSION-SHEAR LOADING

Let us consider an infinite linear elastic plate with a V-shaped notch with a wedge angle of 2β and a system of polar coordinates (r, ϑ), with an origin at the tip of the notch, see Figure 1. The dominant singularity governing the behaviour of the stresses σ_{ij} at the notch-tip region has the form [1]:

$$\sigma_{ij} = \frac{K_{\mathrm{I}}^{\lambda}}{(2\pi r)^{1-\lambda_{\mathrm{I}}}} c_{ij}(\vartheta) + \frac{K_{\mathrm{II}}^{\lambda}}{(2\pi r)^{1-\lambda_{\mathrm{II}}}} d_{ij}(\vartheta), \tag{1}$$

where: K_{I}^{λ} and $K_{\mathrm{II}}^{\lambda}$ are the generalized opening mode (mode I) and sliding mode (mode II) stress intensity factors, λ_{I} and λ_{II} are the exponents of the displacement field for the modes I and II, see Figure 1.

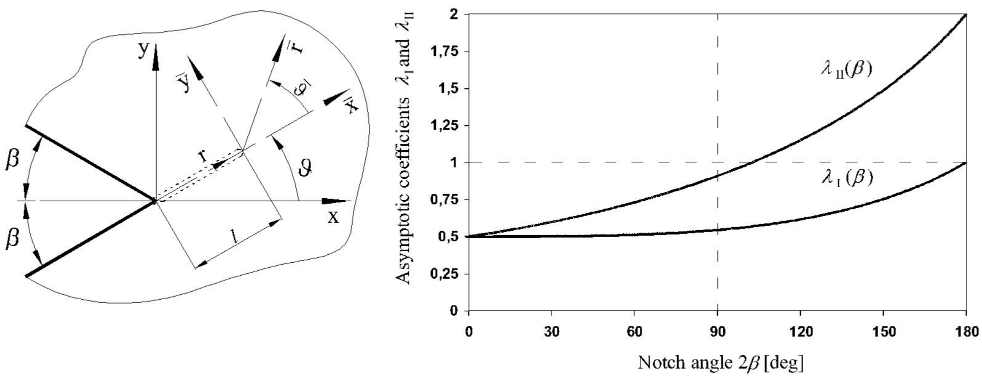

Figure 1: V-shaped notch with a propagating crack of length l and variation of exponents λ_{I} and λ_{II} with notch angle 2β.

The problems of brittle fracture in elements with notches are usually analysed on the grounds of linear elastic fracture mechanics. Modelling of the details of microcracking process is avoided for example by introducing a small equivalent crack, by averaging the elastic stress field around the singular point or by using such parameters as stresses or the strain energy computed at some finite distance from the notch tip [2].

Strain energy release rate criterion (Griffith criterion) defines energy required to create a new free surface (a crack) in a linear elastic solid. It has been modified for multiaxial loading conditions [3-5]. Let us consider a V-shaped notch with a crack of length l propagating from the notch tip in an arbitrary radial direction ϑ, see Figure 1. An approach proposed in Ref. [3] assumes a virtual crack growth $\Delta l \rightarrow 0$ in the direction defined by angle ϑ_0 determined by maximizing the energy release. The energy release rate $G(\vartheta)$ is the work done by the stresses in front of the crack on the displacements of the crack edges, namely:

$$G(\vartheta) = -\lim_{\Delta l \to 0} \frac{1}{\Delta l} \int_0^{\Delta l} [(\bar{\sigma}_{\vartheta\vartheta}(\bar{\vartheta} = 0)\bar{u}_\vartheta(\bar{\vartheta} = \pi) + \bar{\tau}_{r\vartheta}(\bar{\vartheta} = 0)\bar{u}_r(\bar{\vartheta} = \pi)]\mathrm{d}\bar{r}\,, \qquad (2)$$

where ($\bar{r}$, $\bar{\vartheta}$) are polar coordinates with an origin at the crack tip and $\bar{\sigma}_{\vartheta\vartheta}$, $\bar{\tau}_{r\vartheta}$, $\bar{u}_\vartheta$ and $\bar{u}_r$ are the stress and displacement components in the above coordinates.

Taking into consideration singular terms of the asymptotic expansion of the stress and displacement fields near the crack tip one obtains:

$$G(\vartheta) = \frac{1+\kappa}{8\mu}[\bar{K}_{\mathrm{I}}^2(\vartheta) + \bar{K}_{\mathrm{II}}^2(\vartheta)], \qquad (3)$$

where $\bar{K}_{\mathrm{I}}$ and $\bar{K}_{\mathrm{II}}$ are mode I and mode II stress intensity factors for a crack of length l with an origin located at the notch tip.

Fracture will initiate once the energy release rate G in direction $\vartheta = \vartheta_0$ reaches a critical value G_c that is a material constant related to the critical mode I stress intensity factor K_{Ic}:

$$G(\vartheta = \vartheta_0) = G_c = \frac{1+\kappa}{8\mu} K_{\mathrm{Ic}}^2 \qquad (4)$$

The crack length l can be computed by assuming equivalence the maximum normal stress criterion $\sigma_{max} = \sigma_c$ and the energy release rate criterion (Griffith-Irwin condition) $K_I = K_{Ic}$ for a case of a small crack at the semicircular notch root under tensile loading:

$$l = \frac{1}{\pi}\left(\frac{K_{\mathrm{Ic}}}{1.122\sigma_c}\right)^2. \qquad (5)$$

Strain energy density fracture criterion proposed by Sih [6] assumes, that the crack propagation direction ϑ_0 is determined by the minimum value of strain energy density factor S. The factor S is defined as the product of an elastic strain energy dU_S stored in the elementary volume dV and the radial distance r_c measured from the V- notch tip:

$$S = \frac{\mathrm{d}U_{\mathrm{S}}}{\mathrm{d}V} r_{\mathrm{c}} \,. \tag{6}$$

The crack propagation occurs once the factor S reaches a critical value S_{c}:

$$S(\vartheta_0) = S_{\mathrm{c}} = \frac{\kappa - 1}{8\mu} K_{\mathrm{Ic}}^2 \,. \tag{7}$$

The critical radius r_{c} can be determined by applying the maximum normal stress condition for a tensile plane element with symmetric semicircular notches. In such a case for plane stress we obtain:

$$r_{\mathrm{c}} = (1 - \nu) \left(\frac{K_{\mathrm{Ic}}}{\sigma_{\mathrm{c}}} \right)^2 . \tag{8}$$

The strain criterion proposed by McClintock [7] is based on the assumption that crack propagation occurs when the normal strain $\varepsilon_{\vartheta\vartheta}$ at some small distance ρ_{c} measured from crack tip reaches its critical value ε_{c}. For analysis of brittle fracture the stress form of McClintock's criterion is commonly used. The approach assumes that the crack propagation occurs once the circumferential stress $\sigma_{\vartheta\vartheta}$ at a distance $r = \rho_{\mathrm{c}}$ reaches a critical value σ_{c}:

$$\max_{\vartheta} \sigma_{\vartheta\vartheta}(r = \rho_{\mathrm{c}}) = \sigma_{\mathrm{c}} . \tag{9}$$

The maximum of this condition determine direction of crack propagation ϑ_0. The parameter ρ_{c} denotes the distance from notch or crack tip to a point at which the value of the normal stress $\sigma_{\vartheta\vartheta}$ is measured. In order to determine the above parameter the asymptotic solution for a crack under mode I loading together with the Griffith – Irwin criterion can be used:

$$\rho_{\mathrm{c}} = \frac{1}{2\pi} \left(\frac{K_{\mathrm{Ic}}}{\sigma_{\mathrm{c}}} \right)^2 . \tag{10}$$

The non-local stress fracture criterion proposed by Seweryn and Mróz [8] assumes that initiation or propagation of cracking occurs when the mean value of the function R_σ (σ_n, τ_n) of decohesive normal and shear stresses over a segment d_0 reaches its critical value, thus

$$R_f = \max_{(\vartheta,\mathbf{x_0})} \overline{R}_\sigma(\sigma_n,\tau_n) = \max_{(\vartheta,\mathbf{x_0})}\left[\frac{1}{d_0}\int_0^{d_0} R_\sigma(\sigma_n,\tau_n)\mathrm{d}r\right] = 1\,, \tag{11}$$

where: R_f - stress fracture factor; $\overline{R}_\sigma(\sigma_n,\tau_n)$ - non-local stress fracture function; R_σ (σ_n, τ_n) - local stress fracture function; σ_n, τ_n - normal and shear stresses on a physical plane; $\mathbf{x_0}$ – origin of a local polar system (r, ϑ) at an expected crack initiation point; d_0 - length of fracture zone.

For analysis of brittle fracture in plane PMMA samples with V-notches subjected to tension and shear loading it is sufficient to formulate the local fracture function as corresponding to a simple tensile strength condition:

$$R_\sigma(\sigma_n) = \sigma_n / \sigma_c\,. \tag{12}$$

Combining Eq. 12 and Eq. 11 the length of the zone d_0 can be determined by considering a mode I crack case and using the Griffith – Irwin criterion:

$$d_0 = \frac{1}{2\pi}\left(\frac{2K_{\mathrm{Ic}}}{\sigma_{\mathrm{c}}}\right)^2. \tag{13}$$

EXPERIMENTAL VERIFICATION OF FRACTURE CRITERION

Recent experimental investigations [9] were used to verification of presented fracture criterion. Specimens were loaded in combined tension and shear in the special device placed in a tensile machine.
The tensile and shearing load components P and T are calculated as follows:

$$P = F\cos\psi \qquad\qquad T = F\sin\psi\,, \tag{14}$$

where F is the load applied by a tension machine and ψ is the loading angle.

The generalized mode I and mode II stress intensity factors K_I^λ and K_II^λ can be used to characterize singular stress and displacement fields in the notch tip vicinity. In order to illustrate the presented method K_I^λ and K_II^λ have been evaluated using the path independent integral H [10]. The problem geometry, material parameters (E=3300MPa, ν=0.35), boundary conditions and loads are the same as in the experimental investigation. Only in case of the strain energy release rate criterion the values of the standard stress intensity factors for a crack assumed at a notch tip have been computed using the *J*-integral separately for mode I and mode II [2].

The value of the critical rupture stress σ_c has been determined by using tensile specimens with semicircular notches and by computing the stress distribution at the instant of fracture. Have been obtain $\sigma_c = 102.8$ MPa.

The critical value of the stress intensity factor K_{Ic} can be expressed as the mean value of the generalized stress intensity factor K_{Ic}^{λ} obtained for three notch angles 2β= 20°, 40° and 60° (for these angles the asymptotic coefficient λ_I varies insignificantly). It has been $K_{Ic} = 1.202$ MPa m$^{0.5}$.

The non-local parameters occurring in the presented criteria have been calculated for PMMA using the above-mentioned values of σ_c and K_{Ic} : $l = 0.0000347$ m; $d_0 = 0.000087$ m; $\rho_c = 0.0000218$ m; $r_c = 0.0000888$ m.

Figure 2 presents the directions of crack propagation ϑ_0 and the ratios of the critical mixed mode loads F_c to the critical tensile load P_c determined from different criteria in terms of specimen orientation angle ψ for value of the notch angle $2\beta = 40°$, 60° and 80°. For $\psi = 0$ the specimen is subjected to a pure tension, while for $\psi = 90°$ there is a pure shear in the middle cross section. For angles ψ between 0° and 90° there are mixed loading conditions.

CONCLUSIONS

Four presented criteria adapted for brittle fracture analysis of elements with V-shaped notches can be used for determination both the critical load value and direction of crack initiation. The best correlation with experimental data as concerns the crack directions and the values of critical loads has been obtained with the non-local stress fracture criterion and modified McClintock's stress criterion. In order to apply the above criteria one has to determine the stress field for the actual problem geometry and then the calculations are quite simple. Fairly good agreement can be obtained using the strain energy release rate criterion but this approach is more labour consuming. The critical loads determined by basing on the strain energy density fracture criterion agree with the experimental values. Unfortunately the predictions of the direction of crack initiation from the V-notch tip are not accurate, particularly for the case of shear loading.

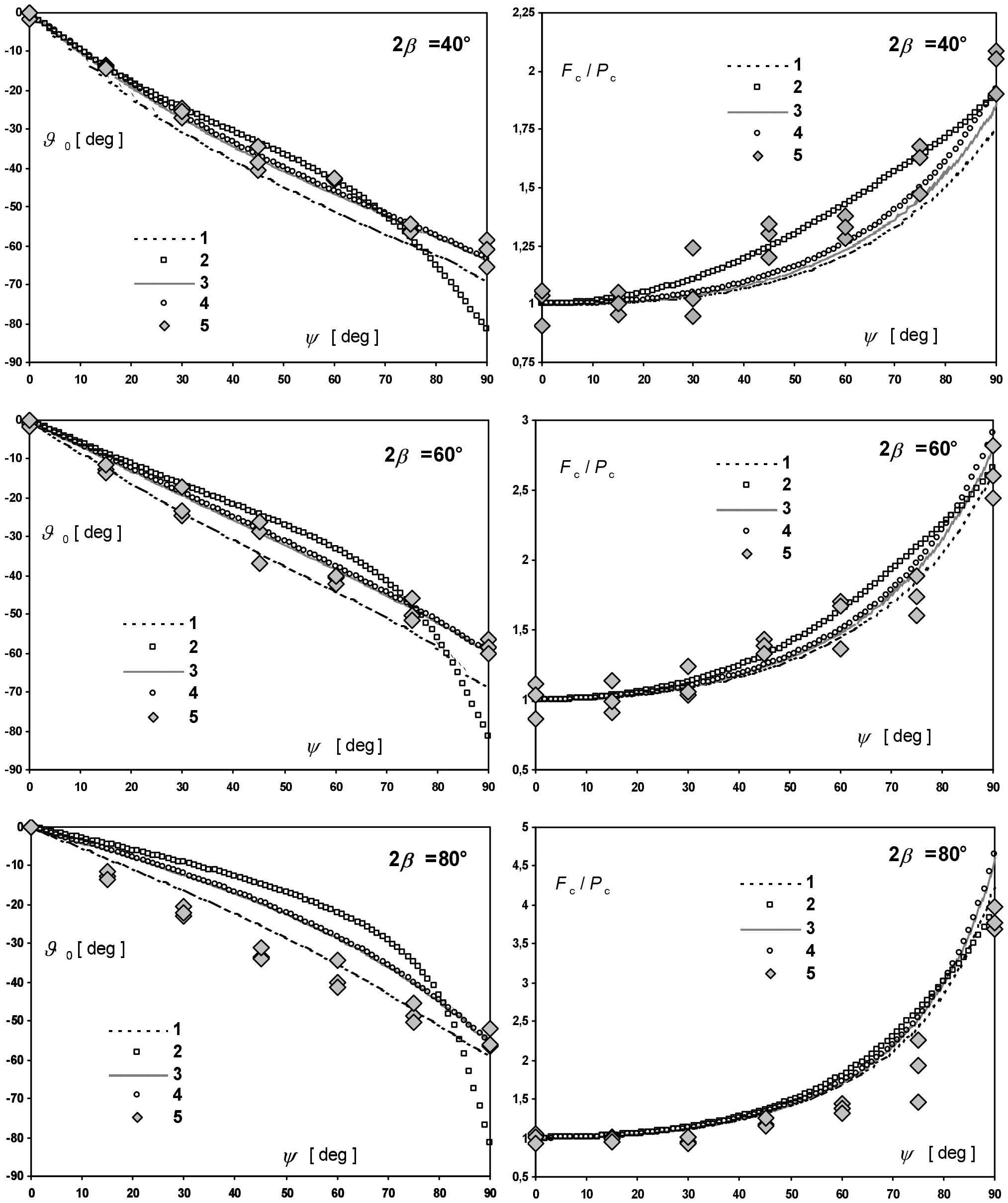

Figure 2: Directions of crack initiation ϑ_0 and critical values of load F_c/P_c for specimens with $2\beta = 40°$, $60°$ and $80°$ in function of specimen orientations angle ψ in an experimental device; key: 1 - strain energy release rate criterion, 2 - strain energy density criterion, 3 - modified McClintock's stress criterion, 4 - non-local stress criterion, 5 - experimental data.

Suitability of presented criteria for fracture analysis of elements with V-notches is illustrated in Table 1.

TABLE 1: Evaluation of brittle fracture criteria

Fracture criterion	Physical justification	Simplicity of calculations	Agreement with experiment
non-local stress criterion	■ ■	■ ■ ■	■ ■ ■
modified McClintock's stress criterion	■ ■	■ ■ ■	■ ■ ■
strain energy release rate criterion – analysis of assumed crack propagation	■	■	■ ■ ■
strain energy density criterion	■	■ ■ ■	■ ■

■ ■ ■ - very good; ■ ■ - good; ■ - sufficiently

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ACKNOWLEDGEMENTS: *The investigation described in the paper is a part of the research project No. 8 T07A 009 20 sponsored by the Polish State Committee for Scientific Research.*