Probabilistic Approach For Modeling Of Aggressive Environment Influence On Creep Rupture

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ABSTRACT*: the new method of modelling aggressive environment influence on creep rupture is suggested. This method is based on introducing of a notion of structural elements and postulating elementary creep properties of these ones. The equations of behavior of a specimen as a whole are based on the behavior of the elements. The various problems of uniaxial loading are considered. It is shown that due to failure of some elements the field of stresses may become nonequal and a surface of failure may appear. As a criterion of failure of specimen, the lost of stability in developing of this surface is accepted.*

BASIC EQUATIONS

This approach to modeling of creep rupture is a generalization of the probabilistic model [1]. In more general sense, this approach can be considered as development of structural phenomenological theory of V.V. Bolotin [2]. Material is considered consisting of a set of structural elements. Properties of these elements depend on aggressive environment elements' concentration. This dependency can appear from the one hand in reducing an ultimate stress from the other hand in increasing creep rate. Let's suppose that both mechanisms act independently and lead to weakening strength of construction.

Let's introduce non-dimensional expression for stress $s = \sigma / \sigma_{b0}$, where σ_{b0} is ultimate stress for given material under given temperature in inert environment (i.e. in vacuum). Let's consider non-dimensional expression for concentration of the aggressive environment elements $c = C / C_\infty$, where C is concentration of environment elements and C_∞ is the limit value of concentration. Usually in order to determine concentration of physically active substances the following diffusion equation is used:

$$\frac{\partial c}{\partial t} = div\left[D \cdot grad\left(c\right)\right] \; D \cdot grad\left(c\right)\Big|_{r\in S} = -\gamma\left(1 - c\big|_{r\in S}\right), \; c\left(r,0\right) = 0\,. \qquad (1)$$

It's suggested that under aggressive environment influence a structural element may fail due to damage accumulation, random deviations of ultimate stress and other factors. In this approach, to model this effect a notion of probability of structural element destruction on time interval is introduced. Let's suppose that structural element has the following properties:

1. The element immediately fails if equivalent stress s_p, affecting this element, reaches ultimate stress s_b.
2. The ultimate stress of the element depends on concentration of environment elements in the structural element: $s_b = w(c)$.
3. Being under loading, the element may fail. Probability of this event satisfies the following equation: $q(r,t,t+\Delta t) = g\left(\frac{s_p}{s_b}\left(r,t\right)\right) \cdot \Delta t + o\left(\Delta t\right)$
4. The element is under creep conditions. Creep rate depends on concentration: $\dot{p} = \dot{p}\left(\frac{s_p}{a\left(c\right)}\right)$.

Let's introduce parameter of density of the structural elements $\psi(r,t)$, which characterizes damage of material. It can be shown [1] that according to made suggestions differential equation for $\psi(r,t)$ has form:

$$\frac{\partial \psi}{\partial t}\left(r,t\right) = -\psi(r,t) \cdot g\left(\frac{s_p\left(r,t\right)}{w\left(c\left(r,t\right)\right)}\right) \qquad (2)$$

with initial condition: $\psi\left(r,0\right) = 1$. Parameter ψ varies from 1 (for non-damaged material) to 0 (for completely destructed material). As we postulate properties for structural element at microlayer, let's accept the simplest forms for functions $\dot{p}$, g, a and w: $g(s) = As^n$, $\dot{p}(s) = Bs^m$.

As for functions a and w, let's take linear approximations: $a(c) = 1 - \alpha c,\; w(c) = 1 - \omega c$. So environment influence is described by two parameters: α and ω.

UNIAXIAL LOADING

Let's consider the uniaxial tension problem of thin plate with constant strength P, which experiences aggressive environment influence. Coordinate x is introduced along direction of thickness. Only a half of specimen is considered: from surface ($x=0$) to middle ($x=h$). Nominal stress, which corresponds to loading P, is denoted as s_0.

The creep rate does not depend on the coordinate x, so the next condition takes place:

$$\frac{\partial}{\partial x}\dot{p}\left(\frac{s_p\left(x,t\right)}{a\left(c\right)}\right)=0 \text{ or } s_p\left(x,t\right)=a\left(c\left(x,t\right)\right)\cdot\phi\left(t\right) \tag{3}$$

where $s_p\left(x,t\right)$ is a stress on the structural element and $\phi\left(t\right)$ is unknown function of time. In time, part of the structural elements fail, so loading is distributed among the undestructed elements, at the same time summary applied strength P remains unchanged. This fact can be written through the equation:

$$\int_0^h s_p\psi dx = hs_0$$

This condition can be used to determine function $\phi\left(t\right)$:

$$s_p = s_0 ha\left[\int_0^h a\psi dx\right]^{-1} \text{ or } s_p = \frac{s_0 a}{\overline{\overline{a\psi}}},$$

where $\overline{a\psi}$ is the mean value of the product $a\psi$ in the section. Equation for parameter ψ has form:

$$\frac{\partial\psi}{\partial t}=-\psi\cdot g\left(\frac{s_0}{\overline{\overline{a\psi}}}\cdot\frac{a(c)}{w(c)}\right).$$

Equation for creep rate:

$$\dot{p} = \dot{p}\left(\frac{s_0}{\overline{\overline{a\psi}}}\right).$$

As we can see, increasing or decreasing of parameter ψ (along coordinate x) depends only on a ratio a/w. From physical point of view it is clear that the first elements fail on the surface of the specimen ($x = 0$). So the ratio a/w must be decreasing function of coordinate x. This condition can be written as: $\frac{\partial}{\partial x}\left(\frac{a}{w}\right) \le 0$.

As concentration c is a decreasing function of coordinate x, we have condition for parameters α and ω: $\alpha \le \omega$.

Let's consider conditions, when the specimen can fail. Condition on the surface is $s_p\left(0,t_1\right) = s_b\left(0,t_1\right) \equiv w\big(c\left(0,t_1\right)\big)$ or in other form:

$$\frac{s_0}{\overline{\overline{a\psi}}\left(t_1\right)} \cdot a\left(0,t_1\right) = w\left(0,t_1\right)$$

Let's suppose that, while specimen accumulates damage, front of destruction $l\left(t\right)$ appears. This front separates completely destructed part ($0 \le x \le l\left(t\right)$) from undestructed one. Time t_1, when the front appears, is determined by solution of the last equation. Analogous relationship takes place in time interval $t_1 \le t \le t^*$:

$$\frac{s_0}{\overline{\overline{a\psi}}\left(t\right)} \cdot a\big(l\left(t\right),t\big) = w\big(l\left(t\right),t\big) \tag{4}$$

After calculating total time derivative of this equation, we can write differential equation for the front:

$$\frac{dl}{dt} = \frac{s_0 \frac{a}{w}\left(\frac{a_c'}{a} - \frac{w_c'}{w}\right)\frac{\partial c}{\partial t} - \frac{\partial}{\partial t}\left[\overline{a\psi}\right]}{-s_0 \frac{a}{w}\left(\frac{a_c'}{a} - \frac{w_c'}{w}\right)\frac{\partial c}{\partial x} - \frac{1}{h}a\psi}$$

Specimen fails if the front loses the stability. This condition can be written as:

$$-s_0 \frac{a}{w}\left(\frac{a'_c}{a}-\frac{w'_c}{w}\right)\frac{\partial c}{\partial x}-\frac{1}{h}a\psi=0 \quad \text{or} \quad \psi^*=-\frac{hs_0}{w}\left(\frac{a'_c}{a}-\frac{w'_c}{w}\right)\frac{\partial c}{\partial x}$$

Here ψ^* is the density of the elements on the front at the moment of rupture t^*. It is worth to note that, if at the moment $t=t_1$ the density of elements $\psi(0,t_1)$ on the surface is larger than ψ^* (from the last equation), then the front does not appear. In such case specimen fails when $t=t^*\equiv t_1$.

CONSTANT CONCENTRATION

The equations can be integrated in case of constant concentration $c=c^*$. Density of the structural elements is function of time in this case (t^* is a time of failure):

$$\frac{d\psi}{dt}=-\frac{A}{w^n(c^*)}\frac{s_0^n}{\psi^{n-1}}, \quad 1-\psi^n=An\left(\frac{s_0}{w(c^*)}\right)^n\cdot t$$

$$t^*=\frac{1}{An}\left[\left(\frac{w(c^*)}{s_0}\right)^n-1\right]$$

The equation for the creep rate of the structural element is:

$$\dot{p}(t)=B\left[\frac{s_0a^{-1}(c^*)}{\left(1-Aw^{-n}(c^*)ns_0^nt\right)^{\frac{1}{n}}}\right]^m$$

$$m\neq n:\quad p(t)=\frac{B}{Aw^{-n}(c^*)(n-m)}a^{-m}(c^*)s_0^{m-n}\left[1-\left(1-Aw^{-n}(c^*)ns_0^nt\right)^{\frac{n-m}{n}}\right]$$

$$m=n:\quad p(t)=\frac{B}{An}\left(\frac{w(c^*)}{a(c^*)}\right)^n\ln\left[1-Aw^{-n}(c^*)ns_0^nt\right]$$

The limit value of the creep strain $p^* = p\left(t^*\right)$:

$$m \neq n: \quad p^* \equiv p(t^*) = \frac{B}{A(m-n)} \frac{w^n(c^*)}{a^m(c^*)}\left[1-\left(\frac{s_0}{w(c^*)}\right)^{m-n}\right]$$

$$m = n: \quad p^* \equiv p(t^*) = -\frac{B}{A}\left(\frac{w(c^*)}{a(c^*)}\right)^n \ln\left(\frac{s_0}{w(c^*)}\right)$$

STEADY-STATE DIFFUSION PROCESS IN THIN SHELL

Let's consider filled with aggressive fluid thin shell. Coordinate x is introduced so that $x=0$ on the internal surface and $x=h$ - on the external surface. Let's suppose that concentration is in steady state and concentration on external surface is equal zero. For simplicity, it's assumed that motion of the front of rupture doesn't affect the distribution of concentration. Thus the concentration satisfies equation:

$$c(x) = 1 - \frac{x}{h}$$

Let's suppose that concentration doesn't affect creep rate immediately, i.e. $a\left(c\right)=1$ or $\alpha = 0$. Function $w(c)$ will be:

$$w(c(x)) = 1 - \omega\left(1-\frac{x}{h}\right), \quad w(x) = \left(1-\omega\right) + \omega\frac{x}{h}$$

Substituting the last expression to (4) we get equation for the front $l\left(t\right)$:

$$\frac{s_0}{\overline{\psi}} = w(l(t)), \quad l(t) = \frac{h}{\omega}\left[\frac{s_0}{\overline{\psi}} - \left(1-\omega\right)\right] \tag{5}$$

The mean value of the density of the structural elements $\overline{\psi}\left(t\right)$ is calculated for the entire section, including destructed part. This mean value is linked with mean value of the density in undestructed part $\tilde{\psi}\left(t\right)$:

$$\bar{\psi}(t)=\frac{1}{h}\int_{l(t)}^{h}\psi dx,\quad \tilde{\psi}(t)=\frac{1}{h-l(t)}\int_{l(t)}^{h}\psi dx,\quad \bar{\psi}=\left[1-\frac{l(t)}{h}\right]\tilde{\psi}$$

Let's represent (5) as a function $\tilde{\psi}(l)$:

$$\tilde{\psi}(l)=\frac{s_0}{\left(1-\frac{l}{h}\right)\left((1-\omega)+\omega\frac{l}{h}\right)} \tag{6}$$

This function is not monotonic: in the beginning it decreases and then begins to increase. This dependence is shown on Figure 1 ($s_0=0.3$, $\omega=0.6$ (about constraints on the constants see below)). Only branch where $\tilde{\psi}_l' \le 0$ relates stable increase of the front $l(t)$. So criterion of failure is relationship $\tilde{\psi}_l'(l^*)=0$:

$$2\omega-1-2\omega\frac{1^*}{h}=0,\quad l^*=h\left(1-\frac{1}{2\omega}\right)$$

where l^* is the limit value of the front $l(t)$.

Let's consider conditions, under which the front appears. First, the front doesn't appear if nominal stress s_0 is higher than the ultimate stress on the surface:

$$s_0\ge\bar{\psi}(0)w(c(0)),\quad s_0\ge(1-\omega)$$

So necessary condition the front to appear is $\omega<1-s_0$. Another limitation to parameter ω follows immediately from (5). To satisfy condition $l^*>0$ we have $\omega>1/2$. Joining both conditions, it is clear that the front can exists only if $s_0<1/2$ for any ω. If $s_0\ge 1-\omega$, the specimen fails immediately after applying the load. If $s_0<1-\omega$, then the front doesn't appear some period of time t_1 (hidden destruction), until equation (5) is satisfied the first time: $\bar{\psi}(t_1)=s_0/(1-\omega)$. If $\omega\le 1/2$ then t_1 is the time of failure.

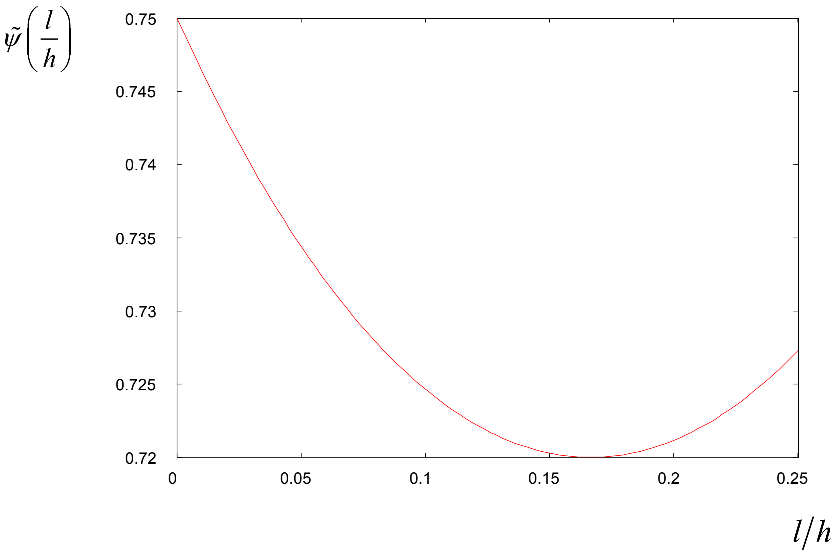

Figure 1: the dependence (6)

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