The influence of notch geometry and non-linearity on the SIF in case of three-point bend marble specimens

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ABSTRACT: *The mechanical behaviour of U-notched specimens made from Dionysos marble and subjected to three-point bending is studied here both experimentally and numerically. The numerical analysis is based on the Displacement Discontinuity Technique. Attention is focused to the relation between LEFM and Linear Elastic Notch Mechanics as well as to the influence of the non-linearity of the stress-strain law of this marble on the values of the Stress Intensity Factors. Discrepancies between theoretical predictions and experimental results are indicated: Contrary to common sense linear approaches are not fully applicable in case of marble, although from a macroscopic point it is considered as a brittle material.*

INTRODUCTION

The present work is part of a project aiming to the mechanical characterization of a series of natural building stones used by ancient Greeks for the erection of some of the most important monuments of the cultural heritage of western civilization. The same or similar materials are used nowadays by the experts working for the conservation and restoration of these monuments.

Among the basic principles of the restoration technique is the mechanical compatibility of the original material with the one used for the substitution of missing parts or completion of damaged ones. It is exactly this principle that has dictated the demand of the scientists, responsible for the restoration of the Parthenon of Athens, for a thorough and systematic investigation of the mechanical behaviour of Dionysos marble, the material substituting, almost exclusively, the extremely durable Pentelik marble, used by ancient Athenians for building up the most cherished monument of the classical antiquity.

Within this frame an experimental and numerical study is presented here related to the behaviour of U-notched prismatic Dionysos marble specimens subjected to three-point bending, in an effort to simulate the present state of a number of architraves (epistyles) of the temple, which are laterally cracked.

THE EXPERIMENTAL PROCEDURE

The material

Dionysos marble consists mainly of calcite (~98%) and contains very small amounts of muscovite, sericite, quartz and chlorite. Its grain size varies around 0.40 mm, its density around 2720 kg/m^3 and its absorption coefficient by weight around 0.11%. Its crystals are polygonic and of almost constant size.

From the mechanical point of view it is a slightly bimodular material (i.e. its Young's modulus in compression exceeds the respective one in tension) and as it happens with most natural building stones it is characterized as a transversely isotropic material the compressive strength of which is almost ten times higher from the tensile one. Its stress-strain law was obtained by Vardoulakis and Kourkoulis [1], with the aid of specially designed direct tension tests with series of loading-unloading loops, as it is shown in Figure 1. Its mechanical constants were recently recapitulated by Exadaktylos et al. [2].

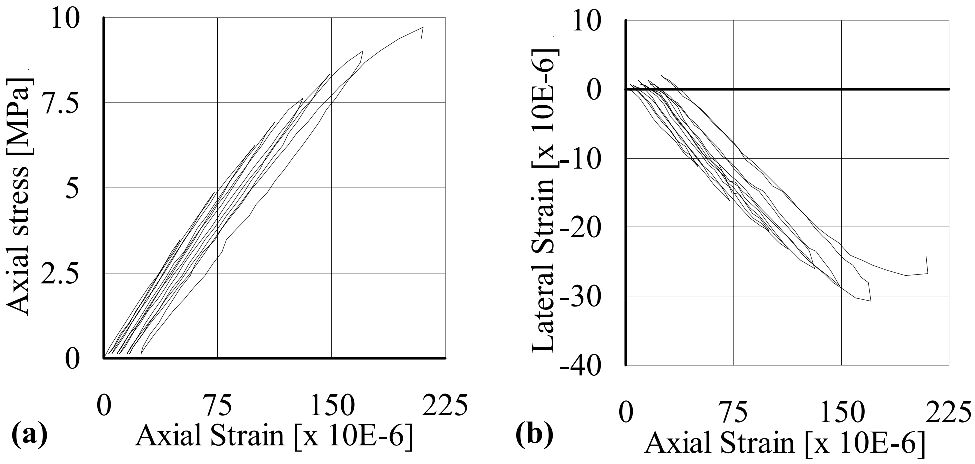

Figure 1: The axial stress (a) and the lateral strain (b) versus the axial strain for Dionysos marble under direct tension.

As it is concluded from Figure 1, Dionysos marble exhibits a non-linear behaviour with considerable permanent deformations from the early loading steps. Attention should be paid, also, to the rapidly increasing hysteresis loop. The stress-strain curve is accurately enough described by Gerstner's law [3]:

$$\sigma=E(\varepsilon-m\varepsilon^2) \quad (1)$$

with m a numerically determined constant, equal to m=1880 for the specific material. The conclusions for the compression regime are of similar nature [2].

Experimental procedure and results

Three-point bending tests were carried out with centrally U-notched prismatic specimens of length L=0.40 m and square cross-section hxb=0.10x0.10 m^2. The notch depth was α=0.2 cm and its breadth was s=5 mm. The load was applied uniformly along the thickness of the specimens at a rate producing a grip displacement rate not exceeding 10^{-6}m/min. The contact width, d, between the loading roller and the specimen was estimated to about d=0.8 mm. The loading plane coincided with the plane of material symmetry, i.e. the load was perpendicular to the layers of the marble. The strain measurement system included 10 electrical strain gauges (5 orthogonal strain rosettes) rosettes, of the Kyowa KFG-X-120-D16 type (X=1 for the gauge attached at the notch crown and X=2 for the rest ones), arranged as it is shown in Figure 2.

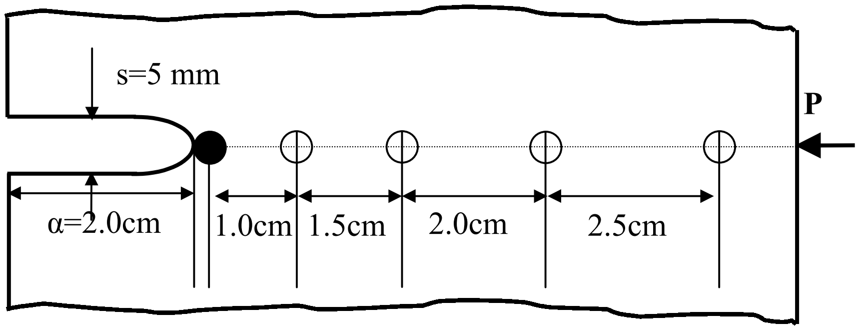

Figure 2: The arrangement of strain gauges.

The fracture load varied around P_{fr}=12.2 kN, with an unexpectedly low standard deviation of about ±5%. Characteristic results are shown in Figure 3, in which the nominal stress, calculated as $\sigma_{nom}=3PL/[2b(h-\alpha)^2]$, is plotted versus the developed strains (either tangential or axial) for the point located at a distance r=10 mm straight ahead from the notch tip. The non-linearity exhibited by the tangential strains is again emphasized, although the point is located well outside the intensively damaged zone developed around the tip, which was estimated to ~8 mm, as it will be proved in the next paragraph.

THE STRESS INTENSITY FACTOR AT THE NOTCH CROWN

The closed full-field solution of the notched three-point bending problem is prohibitively complicated. Thus, only its quantitative features are explored here, with the aid of the Displacement Discontinuity Technique (DDT), based on a solution expressing the stresses and displacements at a point due to a constant displacement discontinuity over a line segment in an elastic body [4].

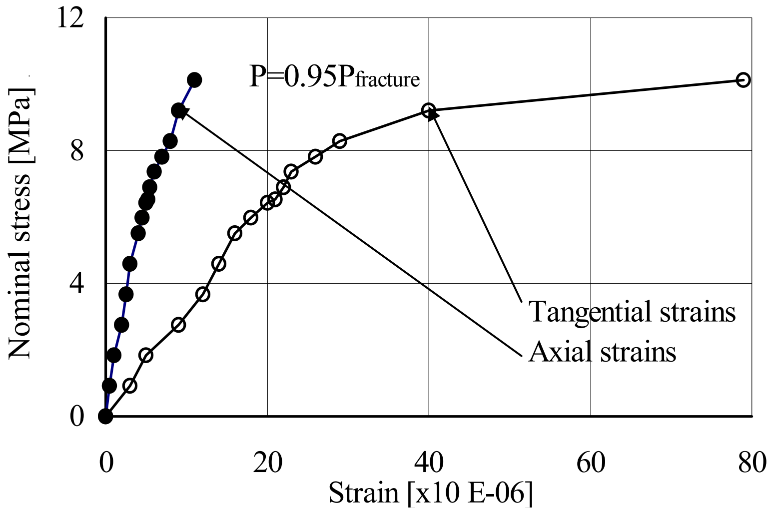

Figure 3: The nominal stress close to the notch crown vs. the developed strains

The DDT, simple and flexible and widely used for analysing engineering problems, is employed here for the calculation of the Stress Intensity Factors (SIFs) at the notch crown in Dionysos marble specimens, in conjunction with the experimental data of the previous paragraph. It is known that concerning *cracked* three-point bend specimens the mode-I SIF, K_I, at the tip of the crack was given analytically by Gross and Srawley [5] in the following form:

$$K_I = \sigma_{nom}\sqrt{\pi\alpha}F(\alpha/h),$$
$$F(\alpha/h) = 1.090 - 1.735(\frac{\alpha}{h}) + 8.20(\frac{\alpha}{h})^2 - 14.18(\frac{\alpha}{h})^3 + 14.57(\frac{\alpha}{h})^4, \quad \frac{\ell}{h} = 4 \qquad (2)$$

The mode-I SIF given in Eq.2 by LEFM is applicable for notched problems only for deep and slender notches (Linear Elastic Notch Mechanics, LENM), as it is shown in Figure 4. In a different case and for the region of the notch tip where r is small compared to other planar dimensions (except for notch breadth, s), the condition of zero traction along the notch crown, which is of finite radius, must be satisfied. Thus, the strain field is given by LENM as:

$$\varepsilon_x(r,\theta) = \frac{1}{\tilde{E}}\frac{K_I}{\sqrt{2\pi r}}\left\{-\frac{1}{1-\nu}\frac{\rho}{2r}\cos\frac{3\theta}{2} + \cos\frac{\theta}{2}\left[\frac{1-2\nu}{1-\nu} - \frac{1}{1-\nu}\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]\right\},$$
$$\varepsilon_y(r,\theta) = \frac{1}{\tilde{E}}\frac{K_I}{\sqrt{2\pi r}}\left\{\frac{1}{1-\nu}\frac{\rho}{2r}\cos\frac{3\theta}{2} + \cos\frac{\theta}{2}\left[\frac{1-2\nu}{1-\nu} + \frac{1}{1-\nu}\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]\right\}, \qquad (3)$$
$$\varepsilon_{xy} = \frac{1}{G}\frac{K_I}{\sqrt{2\pi r}}\left\{-\frac{\rho}{2r}\sin\frac{3\theta}{2} + \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right\}, \qquad r-\rho/2 \to 0,$$

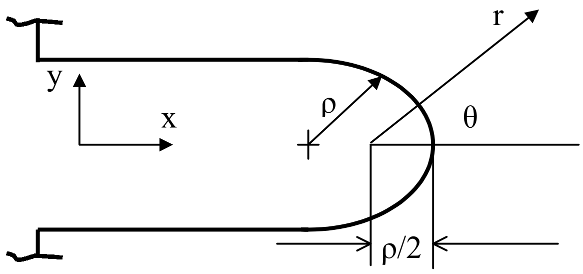

Figure 4: The coordinate systems used in the analysis

where G is the rigidity modulus and $\widetilde{E}=E/(1-\nu)^2$ for plane strain conditions. It is exactly the first terms of Eqs.(3) which fulfil the additional condition, mentioned above, for zero tractions along the notch crown.

For the experimental determination of the SIF the results for the tangential strain of the gauge attached closest to the notch tip (i.e. $r-\rho/2\to 0$, the filled symbol in Figure 2) were employed in conjunction with the second of Eqs.(3):

$$K_I = E'\left\{\frac{1}{1-\nu}\frac{\rho}{2r}+\frac{1-2\nu}{1-\nu}\right\}^{-1}\sqrt{2\pi r}\,\varepsilon_y(r,0^o) \qquad (4)$$

However, independently from the method used for the determination of SIF (either experimentally by strain gauges or numerically by employing the DDT technique), the results are accurate enough only in case the apparent values of SIF are plotted as a function of the distance from the notch crown. When data near to but away from the crack tip are considered, there is some contribution to the strain data from the non-singular part of the strain field.

In order to account for these effects, one should, also, include in Eqs.(3) the components ε^0_{ij} (i,j=x,y), where ε^0_{ij} may be regarded as the Taylor series expansion of the non-singular strains in the immediate vicinity of the notch tip. By computing the component $\varepsilon_y(r,0^o)$, from the second of Eqs.(3), and normalizing both sides the “apparent” SIF, K_{AP}, is obtained in the form:

$$K_{AP} = (1-\nu)\widetilde{E}\sqrt{2\pi r}\,\varepsilon_y$$
$$K_{AP}r = \frac{\rho}{2}K_I + (1-2\nu)K_I r + (1-\nu)\widetilde{E}\sqrt{2\pi}\,\varepsilon^0_y r^{3/2}, \qquad (5)$$

It is observed from Eq.(5) that in case the normalized apparent SIF, K_{AP}, is multiplied by the radial distance, r (i.e. the factor $K_{AP}r$), it varies in the form:

$$K_{AP}\,r = a+br+cr^{3/2} \qquad (6)$$

Taking advantage of the above observation, one can use the numerical data for the strains in front of the notch crown, as they are given in Figure 5, for the determination of the SIFs in conjunction with the analysis presented above. It is concluded from Figure 5 that the strain gradient is significant within only a distance r′≈ρ, (r′=r-ρ/2), where ρ is the notch radius of curvature.

Considering the data of Figure 5 and after appropriate normalization a curve of the form of Eq.6 is fitted on them by regression analysis, using a commercial code. The results are shown in Figure 6. Then the SIF is estimated from either the intercept of the fitted curve with the vertical axis, or by the proportionality constant of the linear term, or by the mean value of the two.

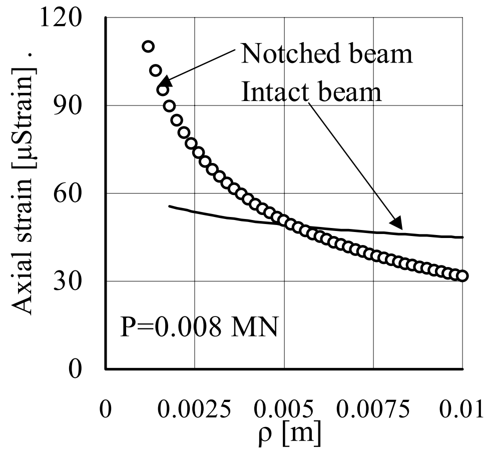

Figure 5: The axial strain versus the distance form the notch tip

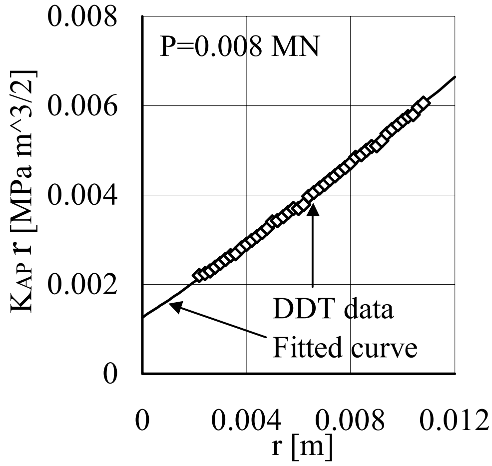

Figure 6: Approximating K_I from DDT analysis results

The values of SIF computed from the numerical strain data are presented in Table 1, for three characteristic tests of the present series and for the same load level P=0.008 MN. From this table it may be seen that the maximum relative difference between the LEFM and the DDT is about 10%. It is thus concluded that the influence of notch width employed in the present series of experiments does not appreciably influence the validity of LEFM predictions.

TABLE 1: Comparison of mode-I SIF values as predicted by LEFM and LENM

LEFM (Eq.2) ($MPa\sqrt{m}$)	DDT & LENM (Eq.5) ($MPa\sqrt{m}$)	Relative difference (%)
1.1212	1.0310	8.8
	1.0276	9.1
	1.0254	9.3

As a second step, the SIFs computed from Eq.4, by using the measurements from the rectangular gauge rosette attached just in front of the notch tip are plotted in Figure 7, together with the respective LEFM predictions. As it is noted from this plot the values of the SIF follows closely those predicted by LEFM only for load levels below 30% of the fracture load. From this point on they deviate monotonically in a strongly non-linear fashion towards higher values than those predicted by the classical Fracture Mechanics theory.

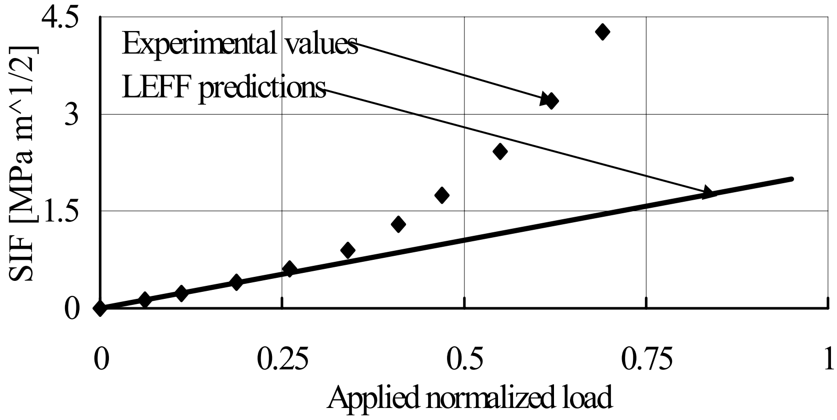

Figure 7: Experimental and theoretical variation of the SIF

The last observation could be attributed to the non-linearity of the stress-strain law of Dionysos marble as well as to the fact that the strain gauge lies within an intensively damaged zone or micro-cracking zone developed ahead of the notch tip. Indeed, considering the mean value of critical mode-I SIF determined form the present series of tests (K_{IC}≈1.85 MPa $m^{1/2}$), and for a tensile strength σ_t=9.2 MPa [1] (for the anisotropy direction characterizing the specimens), one may estimate the size of the process zone in plane strain as:

$$r_p \approx 0.2(K_{IC} / \sigma_t)^2 \cong 0.008\text{m} \tag{7}$$

which indicates, as the SIF-data of Figure 7, that the near-tip strain measurements of the present series of experiments lie inside the process zone (considering that the overall diameter of the respective rosette was about 4 mm).

DISCUSSION AND CONCLUSIONS

A series of three-point bending experiments with U-notched specimens made from Dionysos marble was carried out. The strain field developed ahead from the notch tip, was recorded as a function of the externally applied load.

The experimental results were used for the determination of the SIF with the aid of a numerical analysis based on a linear elastic, isotropic displacement discontinuity model. The outcome of the analysis was compared with the one obtained by LEFM. It was concluded that, as far as it concerns the SIF, the discrepancies did not exceed 10%, indicating that on a first approximation LEFM could be used for the description of notched marble behaviour.

On the other hand, the load-strain measurements have shown that the material outside the process zone behaves in a linear manner for a larger portion of the stress-strain curve, if compared with the intact tension specimens, obviously due to the stress concentration around the notch crown. However, beyond a certain load-level (about 60% of the fracture load one) the relation becomes again non-linear, due to damage, until the peak load is reached.

Concerning the immediate vicinity of the notch crown, the measurements reveal that the stress-strain behaviour is linear only up to stress levels about 20-30% of the failure stress and then it becomes strongly non-linear. This non-linearity should be attributed to the development of a zone of intense micro-cracking (process yield zone), of the order of 8 mm for Dionysos marble.

It is to be emphasized, however, that a global description of the behaviour of notched marble specimens should include, also, the influence of the anisotropy as well as of the microstructure characterizing it: Recent bending tests, with intact Dionysos marble beams, revealed that the classical beam theory is inadequate, unless the influence of microstructure is taken into account with the aid of the characteristic internal lengths of the material [6,7].

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