

# Reference Stress Based $J$ and COD Estimation for Leak-Before-Break Analysis of Pressurised Piping

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***ABSTRACT:** In this paper, extensions of the reference stress based method to estimate  $J$  and COD for through-wall cracked pipes to general problems are given. These problems include circumferential through-wall cracked pipes under combined pressure and bending or under combined axial tension and bending; complex cracked pipes under combined axial tension and bending; axial through-wall cracked pipes under combined pressure and bending. For these cases, the proposed reference stress based  $J$  and COD estimates are validated against published test data and detailed 3-D FE results to show their accuracy and robustness.*

## INTRODUCTION

Pressurised piping is an important element in power plants, and thus application of fracture mechanics analysis to such pressurised piping is important in structural integrity assessment of plant components. One example of such application is the Leak-before-Break (LBB) analysis of piping. In general, application of the LBB procedure requires two steps [1]. Firstly, the crack length corresponding to the (assumed) detectable leakage rate should be calculated for a through-wall cracked (TWC) pipe. For this step, engineering methods to estimate the crack opening displacement (COD) and the leak rate are needed. The second step is to perform the pipe fracture stability analysis, which requires the estimation of the  $J$ -integral.

One popular method to estimate  $J$  and COD for cracked components is the reference stress method [2]. Due to its simplicity, it can be easily extended to complex geometries and loadings, which offers significant advantages in practical application. This method, however, suffers from its accuracy, possibly resulting from the definition of the reference stress. To improve its accuracy, authors have recently proposed the enhanced reference stress approach [3], where the key point is in the definition of the reference

stress, i.e., the reference stress is defined using the optimised reference load, providing best estimates of  $J$ , instead of the plastic limit load.

This paper summarises recent works on application of the enhanced reference stress approach to estimate  $J$  and COD for through-wall cracked pipes.

### REFERENCE STRESS BASED J AND COD ESTIMATES

For a cracked component, the reference stress based  $J$  estimation equation is given by [3]

$$\frac{J}{J_e} = \frac{E\varepsilon_{ref}}{\sigma_{ref}} + \frac{1}{2} \left( \frac{\sigma_{ref}}{\sigma_y} \right)^2 \frac{\sigma_{ref}}{E\varepsilon_{ref}} ; \quad \sigma_{ref} = \frac{Q}{Q_{OR}} \sigma_y \quad (1)$$

where  $J_e$  is the elastic component of  $J$ ,  $J_e = K^2/E'$ ;  $E' = E/(1-\nu^2)$  for plane strain and  $E' = E$  for plane stress;  $E$  denotes Young's modulus; and  $\varepsilon_{ref}$  denotes the true strain at the reference stress  $\sigma_{ref}$ , determined from true stress-strain data;  $\sigma_y$  denotes the 0.2% proof or lower yield stress of the material of interest;  $Q$  denotes the (generalised) load; and  $Q_{OR}$  denotes the optimised reference (generalised) load. Note that the optimised reference load is generally different from the plastic limit load.

The COD ( $\delta$ ), on the other hand, can be estimated from

$$\frac{\delta}{\delta_e} = \begin{cases} \frac{E\varepsilon_{ref}}{\sigma_{ref}} + \frac{1}{2} \left( \frac{\sigma_{ref}}{\sigma_y} \right)^2 \frac{\sigma_{ref}}{E\varepsilon_{ref}} & \text{for } 0 \leq \frac{\sigma_{ref}}{\sigma_y} \leq 1 \\ \left( \frac{\delta}{\delta_e} \right)_{L_r=1} \left( \frac{\sigma_{ref}}{\sigma_y} \right)^{n_1-1} & \text{for } 1 \leq \frac{\sigma_{ref}}{\sigma_y} \end{cases} \quad (2)$$

where  $\delta_e$  denotes the elastic component of  $\delta$ . In Eq. (2),  $(\delta/\delta_e)_{L_r=1}$  denotes the value of  $(\delta/\delta_e)$  at  $\sigma_{ref} = \sigma_y$ , calculated from the first equation in Eq. (3). The strain hardening index  $n_1$  in Eq. (2) should be estimated from

$$n_1 = \frac{\ln[(\varepsilon_{u,t} - \sigma_{u,t}/E)/0.002]}{\ln[\sigma_{u,t}/\sigma_y]} \quad (3)$$

where  $\sigma_{u,t}$  and  $\varepsilon_{u,t}$  denote the true ultimate tensile stress and percentage uniform elongation at  $\sigma = \sigma_u$ , respectively.

The enhanced reference stress based  $J$  and  $\delta$  estimates require only two values: the elastic component and the optimised reference load  $Q_{OR}$ . With these two parameters,  $J$  and  $\delta$  can be estimated using stress-strain data of the material of interest. Noting that elastic  $J$  and  $\delta$  values can be easily obtained,

a central point of the proposed method is then to determine the optimised reference load.

In the subsequent sections, solutions for  $Q_{OR}$  are given for TWC pipes under various loading conditions and for complex cracked pipes under combined bending and tension, together with comparisons with the detailed FE results and experimental pipe test data.

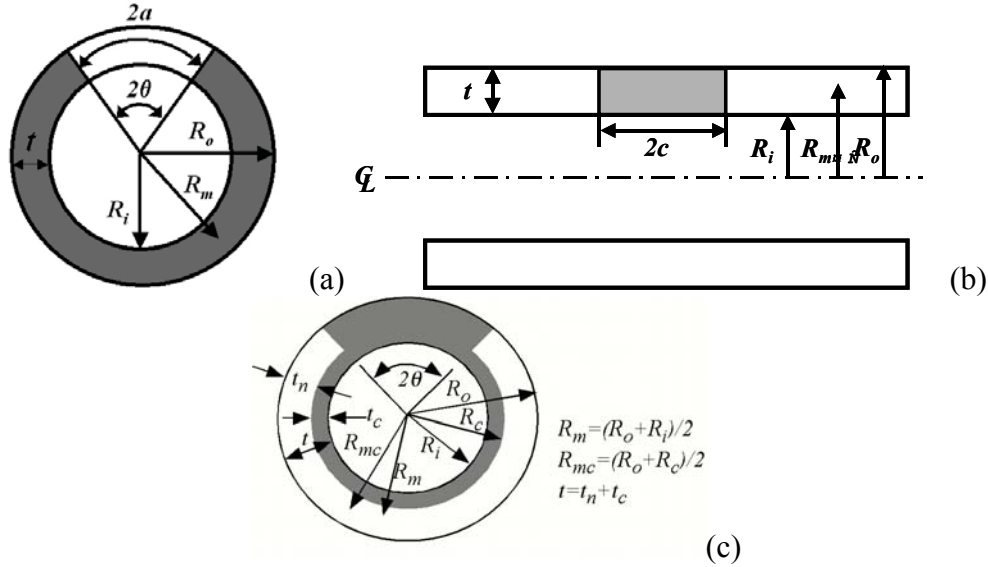


Fig. 1. Schematic illustration of (a) circumferential through-wall crack, (b) axial through-wall crack and (c) complex crack.

## CIRCUMFERENTIAL THROUGH-WALL CRACKS

### *Single Loading (Axial Tension, Internal Pressure, Global Bending)*

For circumferential TWC pipes under axial tension  $P$ , internal pressure  $p$  or global bending  $M$  (Fig. 1a), the optimised reference loads are given by [4]

$$P_{OR} = 2R_m t \sigma_y \left[ 0.82 + 0.75 \left( \frac{\theta}{\pi} \right) + 0.42 \left( \frac{\theta}{\pi} \right)^2 \right] \left[ \pi - \theta - 2 \sin^{-1} \left( \frac{1}{2} \sin \theta \right) \right] \quad (4)$$

$$P_{OR} = 2R_m t \sigma_y \left[ 0.45 + 1.88 \left( \frac{\theta}{\pi} \right) - 0.75 \left( \frac{\theta}{\pi} \right)^2 \right] \left[ \pi - \theta - 2 \sin^{-1} \left( \frac{1}{2} \sin \theta \right) \right] \quad (5)$$

$$M_{OR} = 4R_m^2 t \sigma_y \left[ 0.82 + 0.75 \left( \frac{\theta}{\pi} \right) + 0.42 \left( \frac{\theta}{\pi} \right)^2 \right] \left[ \cos \left( \frac{\theta}{2} \right) - \frac{\sin \theta}{2} \right] \quad (6)$$

Resulting COD predictions are compared with pipe test data in Fig. 2, which shows excellent agreement. In Fig. 3, the maximum loads, predicted according to the proposed method, are compared with pipe test data and the R6 results [5]. It shows that the proposed method gives less conservative maximum loads. More detailed information on pipe test data and more results can be found in Refs. [4, 5-8].

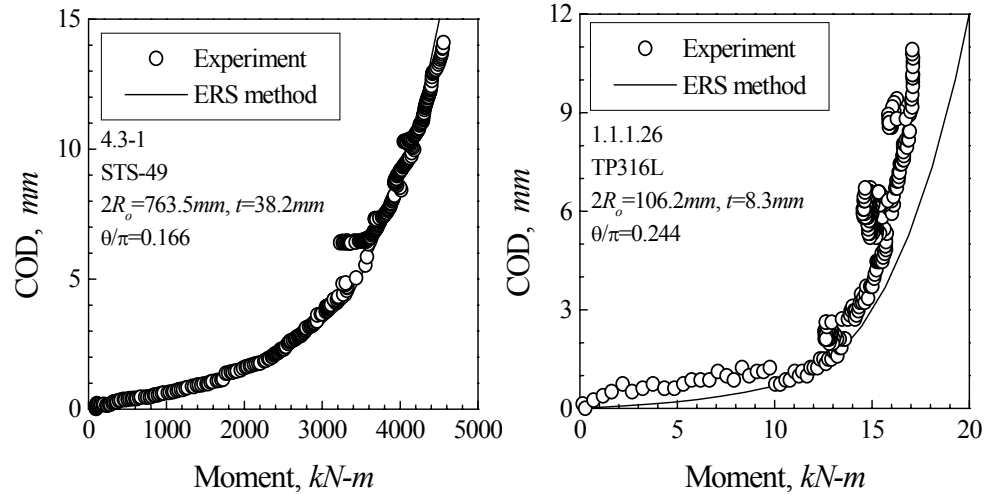


Fig. 2 Comparison of the COD predictions according to the proposed method with pipe test data [4].

### Combined Loading

For a combined axial tension and global bending, the optimised reference load solution can be found from the following yield locus [10]:

$$\left(\frac{P}{P_{OR}}\right)^2 + \frac{M}{M_{OR}} = 1 \quad (7)$$

where  $P_{OR}$  and  $M_{OR}$  are optimised reference load solutions for single loading, see Eqs. (4) and (6). On the other hand, for combined internal pressure and global bending, the similar yield locus can be used [11]:

$$\left(\frac{p}{p_{OR}}\right)^2 + \frac{M}{M_{OR}} = 1 \quad (8)$$

where  $p_{OR}$  denotes the optimised reference pressure solution, given in Eq. (5).

Fig. 4 compares the estimated  $J$  and COD with the results from pipe test data and detailed elastic-plastic FE analysis for combined bending and

tension, showing excellent agreement. More results can be found from Refs. [10,11].

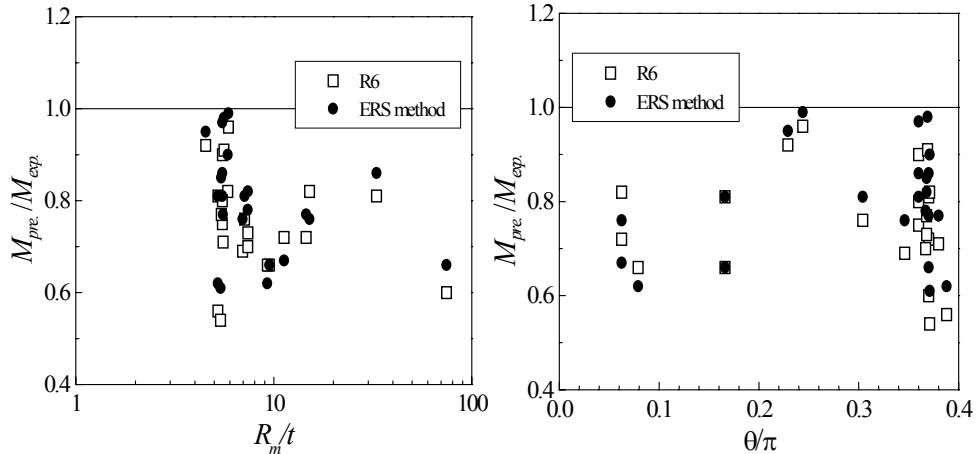


Fig. 3. Comparison of predicted maximum moment from the R6 method and the proposed method with experimental data [9].

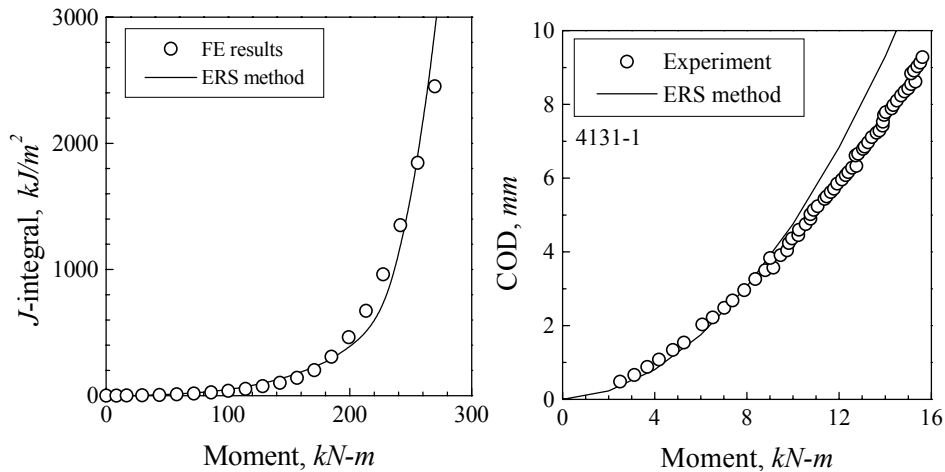


Fig. 4. Comparison of  $J$  and COD from FE results and pipe test data for combined tension and bending, with those estimated using the proposed method [10].

### AXIAL THROUGH-WALL CRACKS

Consider axial TWC pipes under internal pressure  $p$  (Fig. 1b). Dimension for the pipe and the crack is also given in Fig. 1b, and the normalised crack length parameter  $\rho$ , defined by

$$\rho = \frac{c}{\sqrt{R_m t}} \quad (9)$$

The optimised reference pressure  $p_{OR}$  is given by

$$p_{OR} = \frac{2}{\sqrt{3}} \sigma_y \left( \frac{t}{R_m} \right) \frac{\psi(\rho)}{\sqrt{1 + 0.34\rho + 1.34\rho^2}} \quad (10)$$

$$\psi(\rho) = \begin{cases} -0.06\rho^2 + 0.21\rho + 0.82 & \text{for } \rho < 1.5 \\ 1 & \text{for } \rho \geq 1.5 \end{cases}$$

Noting that a global bending moment has only a slight effect on plastic limit load for axial TWC pipes [12], the proposed  $J$  and COD estimation equations for internal pressure can be equally applied to combined pressure and global bending loading.

Fig. 5 compares the estimated  $J$  and COD with the results from detailed elastic-plastic FE analysis for combined pressure and tension, showing good agreement. More results can be found from Ref. [13].

### COMPLEX CRACKS

A complex crack consists of a fully circumferential, internal surface crack in a pipe and a through-wall crack in the same plane as the surface crack (Fig. 1c). The optimised reference loads are given by [14]

$$P_{OR} = 2\pi R_{mc} \sigma_y t_n \left[ 0.82 + 0.75 \left( \frac{\theta}{\pi} \right) + 0.42 \left( \frac{\theta}{\pi} \right)^2 \right] \times \left[ 1 - \frac{\theta}{\pi} - \frac{(1+\eta)}{\pi} \sin^{-1} \left[ \frac{\sin \theta}{(1+\eta)} \right] \right] \quad (11)$$

$$M_{OR} = 4R_{mc}^2 \sigma_y t_n \left[ 0.82 + 0.75 \left( \frac{\theta}{\pi} \right) + 0.42 \left( \frac{\theta}{\pi} \right)^2 \right] \times \left[ -\frac{1}{2} \sin \theta + \frac{(1+\eta)}{2} \sin \left( \frac{\pi - \theta}{1+\eta} \right) \right] \quad (12)$$

Relevant dimension can be found from Fig. 1c.

Fig. 6 compares the estimated CODs using the proposed method with pipe test data for complex cracked pipes under bending, showing good agreement. More results can be found from Ref. [14].

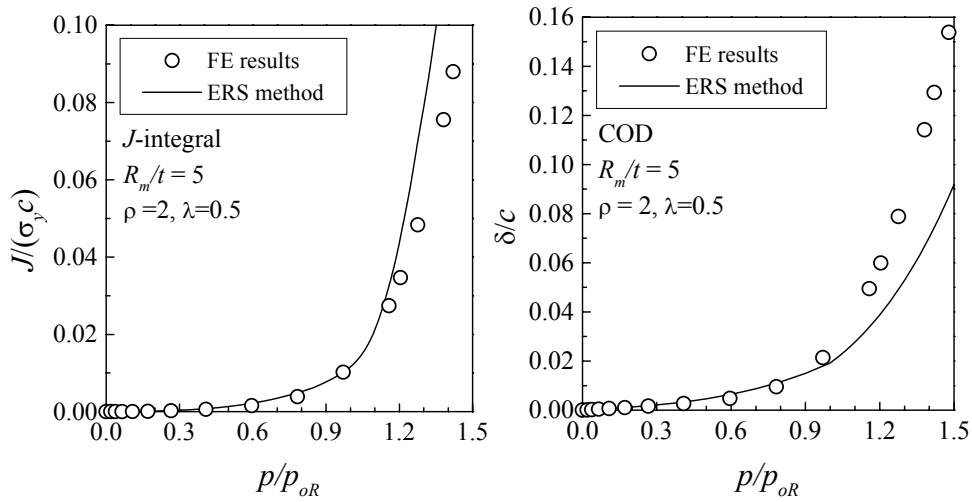


Fig. 5. Comparison of FE  $J$  and COD results for axial TWC pipes under combined internal pressure and bending with the proposed method [13].

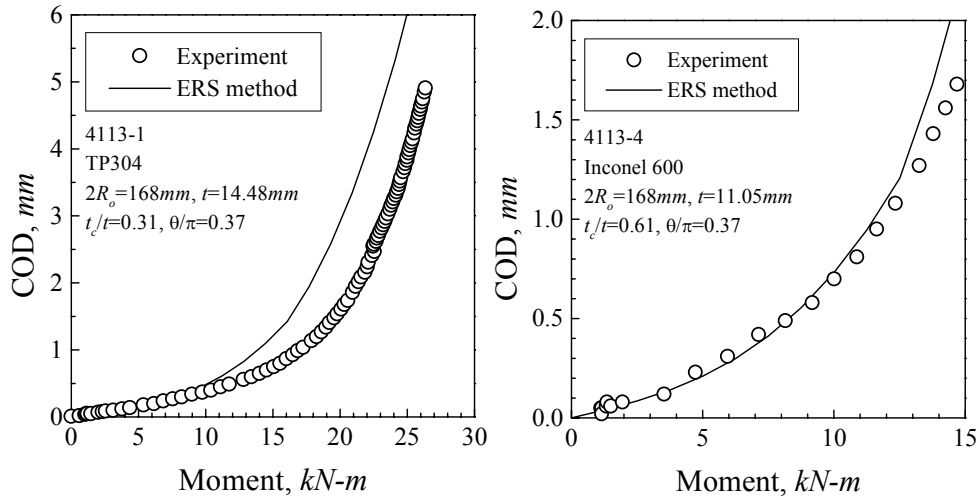


Fig. 6 Comparison of the COD predictions according to the enhanced reference stress method with pipe test data for complex cracked pipes under bending [14].

## CONCLUSIONS

This paper extends the reference stress based method to estimate  $J$  and COD for TWC pipes to general problems, such as combined pressure and bending or under combined axial tension and bending; complex cracked pipes under combined axial tension and bending; axial cracked pipes under combined

pressure and bending. Validation against extensive published test data and detailed 3-D finite element results shows their accuracy and robustness.

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## **REFERENCES**

- [1] NUREG 1061 (1984). Evaluation of potential for pipe break, Volume 3. USNRC.
- [2] Ainsworth RA. Engineering Fracture Mechanics 1984; 19: 633-642.
- [3] Y-J Kim and P J Budden, 2000, Proc ECF 13.
- [4] Y-J Kim and P J Budden, 2001, Reference stress approximations for J and COD for circumferential TWC pipes. Int J Fract (accepted).
- [5] R6: Assessment of the integrity of structures containing defects, revision 3. British Energy Generation Ltd., 1999.
- [6] Y-J Kim, N-S Kim and Y-J Kim, 2001. Fatigue Fract Eng Mat Structures 24, 243-254.
- [7] Y-J Kim, N-S Kim and Y-J Kim, 2001. Fatigue Fract Eng Mat Structures 24, 617-624.
- [8] Y-J Kim, N-S Kim and Y-J Kim 2002, Estimations of creep fracture mechanics parameters for TWC pipes and FE validation. Fatigue Fract Eng Mat Structures (submitted).
- [9] Y-J Kim, D-J Shim, N-S Kim and Y-J Kim, 2002, Elastic-plastic fracture mechanics assessment for test data for circumferential cracked pipes. Eng Frac Mech (submitted).
- [10] Y-J Kim, N-S Kim and Y-J Kim, 2002, Eng Frac Mech, 69, 367-388.
- [11] Y-J Kim, N-S Kim and Y-J Kim, 2002, Pressure induced hoop stress effect on fracture analysis of circumferential TWC pipes, Eng Frac Mech (in press).
- [12] Miller A G. Int Jnl of Pressure Vessels and Piping 1988; 32: 191-327.
- [13] Y-J Kim, N-S Kim, Y-J Park and Y-J Kim, 2002, Elastic-plastic J and COD estimates for axial TWC pipes, Int J Pres Vess Piping (in press).
- [14] Y-J Kim, N-S Kim and Y-J Kim, 2001, International Journal of Fracture, 111, 71-86.